Numerical modelling of three dimensional groundwater flow with spreadsheets

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ABSTRACT
Spreadsheets are often used by researchers in various engineering sciences to make numerical models by using Finite Difference Method (FDM). The spreadsheets can be used to simulate water flow in porous area and Finite Difference Method (FDM) is one of operations which can be done by iteration ability of spreadsheets.
In this paper, three dimensional groundwater flow is modeled using spreadsheets capable of meshing and iteration. This model utilizes the new technique, named Location Block Method (LBM), which is replaced by three dimensional matrices in current models. In this method, mass is divided to consecutive sheets with thickness of one distance step (delta z). The results of each iteration in this block are recorded and they will be used for the next iteration. This model is based on 3D Laplas equation and is able to measure the value of water surface, pressure and velocity for all grids and the amount of flux discharge for each element. Some advantages of this model are user friendly, giving physical perception to user, ease in importing data, and ease of changing data in each element of each single distance block, quick access to all output data in all distances and appropriate accuracy and educational aspect. Comparing the results of this model with SEEP/W model and seep 3D shows that this model has appropriate accuracy in spite of simplicity.

RÉSUMÉ
Feuilles de calcul sont souvent utilisés par les chercheurs dans différentes sciences de l'ingénieur à faire des modèles numériques en utilisant la méthode de la différence finis (FDM). Feuilles de calcul peuvent être utilisés pour simuler l'écoulement de l'eau en zone poreuse et la méthode de la différence finis (FDM) est l'une des opérations qui peuvent être faites par capacité d' itération des feuilles de calcul. Dans ce papier, l'écoulement des eaux souterraines en trois dimensions est modélisé en utilisant des feuilles de calcul capable de maillage et d'itération. Ce modèle utilise la nouvelle technique, La Méthode des Blocs Locales (MBL), qui a remplacé le matrice de trois dimensions dans les modèles actuels. Dans cette méthode, la masse, est divisée en des feuilles consecutives avec l'épaisseur d'une distance de l'étape (delta z). Les résultats de chaque itération dans ce bloc sont enregistrés et ils seront utilisés pour la prochaine itération. Ce modèle est basé sur 3D équation de Laplace et est capable de mesurer la valeur de l'eau de surface, la pression et la vitesse pour tous les réseaux et la quantité de flux de décharge pour chaque élément. Quelques avantages de ce modèle sont faciles à utiliser, en donnant la perception physique à l'utilisateur, la facilité à l'importation de données, et l'aise dans l'évolution des données dans chaque élément de chaque bloc de distance, l'accès rapide à toutes les données de sortie dans toutes les distances et la précision et l'aspect éducatif. En comparant les résultats de ce modèle avec SEEP/W et SEEP3D, on comprend que ce modèle a beaucoup de précision en dépit de sa simplicité.

1 INTRODUCTION

Foundation of making numerical model by spreadsheet is based on this supposition which each cell of sheets can be introducer of one node of porous environment. Too many researchers in various engineering sciences have made numerical models by using Finite Difference Method (FDM) in spreadsheets. (Crowley 1986) explained the manner of numerical solving by using spreadsheet and introduced that as an appropriate tool for making model based on Laplace equation. He described advantages and disadvantages of using spreadsheet for modelling in finite difference method. (Eid 1986) showed that how is possible to shift discrete equations to spreadsheet. (Tontiwachwuthikul 1989) made a numerical model in Lotus-123 spreadsheet for investigating manner of gas absorption into chemical absorbent materials, and by comparing results with results which had been obtained from a FORTRAN code, shows that spreadsheet's results are a little less than results of FORTRAN code. (Conner 1992) utilized spreadsheet for simulating heat equilibrium and heat equilibrium temperature of making paper paste process. (Ravella 1993) utilized spreadsheet for solving differential equations. By using a numerical model in spreadsheet, he could produce a simple and attentive method for initial designing of heating reactors. He compared spreadsheet's results with a FORTRAN code and finally showed that his method in addition to is easy and user friendly, is very attentive. (Bornt 1995) made a numerical model in spreadsheet for analyzing amount of temperature decreasing in water conveyance pipes. He showed that spreadsheet is very useful for user to understand subject matter. (Kharab 1995-1997) in four papers engaged in manner of producing numerical models by using spreadsheet. For this reason he utilized macro programming in Lotus-123 spreadsheet. (Mokheimer 1997) engaged in investigation on manner of water motion and heat transfer between two parallel plates. He supposed that water pass with fixed flow between two plates which have different temperatures, then hi simulated heat transfer and water flow by using spreadsheet. (Antar 1998) produced numerical model for flow in boundary layers by using spreadsheet. (Bardet and Tobita 2002) engaged in investigation on manner of water flow in free aquifers by introducing new method. (Nakhaei 2003) introduced spreadsheet as an appropriate tool for simulating groundwater flow, and explained the manner of producing steady flow models in spreadsheet for surrounded, anisotropic and nonhomogeneous aquifers. (Shokri 2007) uses Spreadsheets for unsteady groundwater modeling in confine aquifers with implicit and explicit method.

Because of using simplicity and easy understanding of spreadsheet for 3D groundwater it can be helpful for professional and educational projects. In spreadsheet, against vogue models, it is possible to change dominant equations easily. Simplicity of importing, changing amounts in each element of each block, and easy access to all output data is its advantage.

2 Model's Theory

Firstly seven elements like figure 1 are assumed. Entrance discharge come from element 1 to element 2 is shown by q_{21}. By using Darcy law its quantity reach as follow.

$$q_{21} = k_{21} \frac{\varphi_2 - \varphi_1}{\Delta z} \Delta x \Delta y \quad [2]$$

In mentioned equation φ1 (m) is total water head in element 1, φ2 (m) total water head in element 2, and K21(m/s) hydraulic conductivity ratio from element 1 to element 2 which is reachable by below equation.

$$k_{21} = \frac{2k_1k_2}{k_1 + k_2} \quad [1]$$

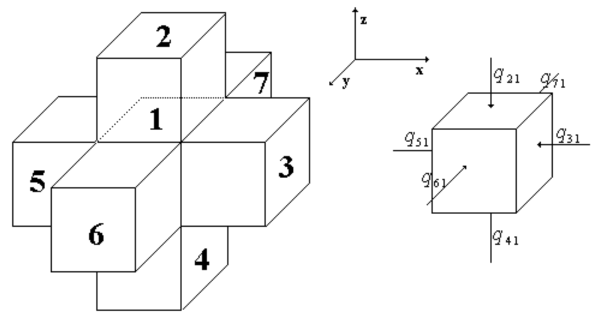


Figure1: showing of entrance discharge from nearby elements 1 to element 1

Sum of entrance discharge from nearby elements1 to elemnt1 is zero because the condition assumed unsteady. By using equation 1, for element1 can be shown:

$$q_{21} + q_{31} + q_{41} + q_{51} + q_{61} + q_{71} = 0 \quad [3]$$

$$\begin{aligned} &k_{21} \frac{\varphi_2 - \varphi_1}{\Delta z} \Delta x \Delta y + k_{31} \frac{\varphi_3 - \varphi_1}{\Delta x} \Delta y \Delta z \\ &+ k_{41} \frac{\varphi_4 - \varphi_1}{\Delta z} \Delta x \Delta y + k_{51} \frac{\varphi_5 - \varphi_1}{\Delta x} \Delta y \Delta z \\ &+ k_{61} \frac{\varphi_6 - \varphi_1}{\Delta y} \Delta x \Delta z + k_{71} \frac{\varphi_7 - \varphi_1}{\Delta y} \Delta x \Delta z = 0 \end{aligned} \quad [4]$$

By assuming same distance interval ($\Delta x = \Delta y = \Delta z$) equation 2 can be simplified as below.

$$\varphi_1 = \frac{k_{21}\varphi_2 + k_{31}\varphi_3 + k_{41}\varphi_4 + k_{51}\varphi_5 + k_{61}\varphi_6 + k_{71}\varphi_7}{k_{21} + k_{31} + k_{41} + k_{51} + k_{61} + k_{71}} \quad [5]$$

Equation 5 can be modified as below by using equation 2.

$$\varphi_{i,j,k}=\frac{\frac{2k_{i+1,j,k}k_{i,j,k}}{k_{i+1,j,k}+k_{i,j,k}}\varphi_{i+1,j,k}+\frac{2k_{i-1,j,k}k_{i,j,k}}{k_{i-1,j,k}+k_{i,j,k}}\varphi_{i-1,j,k}+}{\frac{2k_{i+1,j,k}k_{i,j,k}}{k_{i+1,j,k}+k_{i,j,k}}+\frac{2k_{i-1,j,k}k_{i,j,k}}{k_{i-1,j,k}+k_{i,j,k}}+}$$

$$\frac{\frac{2k_{i,j+1,k}k_{i,j,k}}{k_{i,j+1,k}+k_{i,j,k}}\varphi_{i,j+1,k}+\frac{2k_{i,j-1,k}k_{i,j,k}}{k_{i,j-1,k}+k_{i,j,k}}\varphi_{i,j-1,k}+}{\frac{2k_{i,j+1,k}k_{i,j,k}}{k_{i,j+1,k}+k_{i,j,k}}+\frac{2k_{i,j-1,k}k_{i,j,k}}{k_{i,j-1,k}+k_{i,j,k}}+} \qquad [6]$$

$$\frac{\frac{2k_{i,j,k+1}k_{i,j,k}}{k_{i,j,k+1}+k_{i,j,k}}\varphi_{i,j,k+1}+\frac{2k_{i,j,k-1}k_{i,j,k}}{k_{i,j,k-1}+k_{i,j,k}}\varphi_{i,j,k-1}}{\frac{2k_{i,j,k+1}k_{i,j,k}}{k_{i,j,k+1}+k_{i,j,k}}+\frac{2k_{i,j,k-1}k_{i,j,k}}{k_{i,j,k-1}+k_{i,j,k}}}$$

Equation 6 is a complicated equation and analytical solutions can not solve such these equations but it is possible to solve it by numerical methods. For solving equation 6 a characteristic of groundwater motion must be known as boundary conditions in boundary. Generally boundary conditions can be divided in two main branches, Dirichlet boundary conditions is included boundaries with constant water head and Newman boundary condition is included boundaries without flow and with known flow. Total head in constant head boundary (Dirichlet boundary) as definite amounts import easily in each boundary cell. For Newman boundaries, by using Darcy low it is possible to write:

$$Q_n = k\frac{\partial\varphi}{\partial n}A \qquad [7]$$

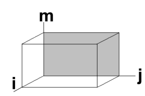
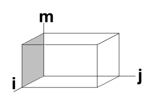
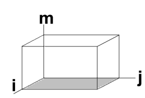
In this equation *n* is vertical to surface vector and by regards to condition of Newman boundary, can be in *x*, *y,* and *z* directions. Equation 7 after separating by central difference method will be changed in equation 8.

$$\varphi_{n+1}-\varphi_{n-1}=\frac{2Q_n}{k\Delta n} \qquad [8]$$

By placing equation 8 in equation 5, equation for known flow boundary conditions will be brought in table 1.

Without flows boundary (impermeable layer) is a branch of Newman boundary. By omitting the discharge's term of equations of table 1 these equations will change to impermeable boundary equation.

Table1. Newman Dominant equations for known flow boundary conditions

Schematic condition of boundary	Newman dominant equations
	$\varphi_{i,j,m}=\frac{1}{6}(2\varphi_{i+1,j,m}+\varphi_{i,j+1,m}+\varphi_{i,j-1,m}+\varphi_{i,j,m+1}+\varphi_{i,j,m-1}+\frac{2Q_i}{k\Delta x})$
	$\varphi_{i,j,m}=\frac{1}{6}(\varphi_{i+1,j,m}+\varphi_{i-1,j,m}+2\varphi_{i,j+1,m}+\varphi_{i,j,m+1}+\varphi_{i,j,m-1}+\frac{2Q_j}{k\Delta x})$
	$\varphi_{i,j,m}=\frac{1}{6}(\varphi_{i+1,j,m}+\varphi_{i-1,j,m}+\varphi_{i,j+1,m}+\varphi_{i,j,m+1}+2\varphi_{i,j,m-1}+\frac{2Q_j}{k\Delta x})$

3 MANNER OF PRODUCING LBN MODEL

The method that will be explained for 3D groundwater modeling in spreadsheets in this paper is called as Location Block Method (LBM). For this purpose firstly a plan of land is made on spreadsheets. By considering on accuracy, Δx can be determined then meshes and first block and elements will be made. For separating in z direction with regards to height of mass (D) and interval location (Δz), number of blocks are D/Δz. Therefore D/Δz new block must be created like first block. Thus each block is exhibited as a horizontal of mass surface with Δz thickness. With this simply mentioned method union porous domain will change to separate domain. After separating except boundaries, each element must have an individual formula from equation 6. For boundary element, equations on table 1 must be used.

With regarding to equation 6 and figure 2 each element can be seen, is related to 4 other elements in its block and 2 elements in other blocks. In figure 2 a simply porous mass and its separating are shown.

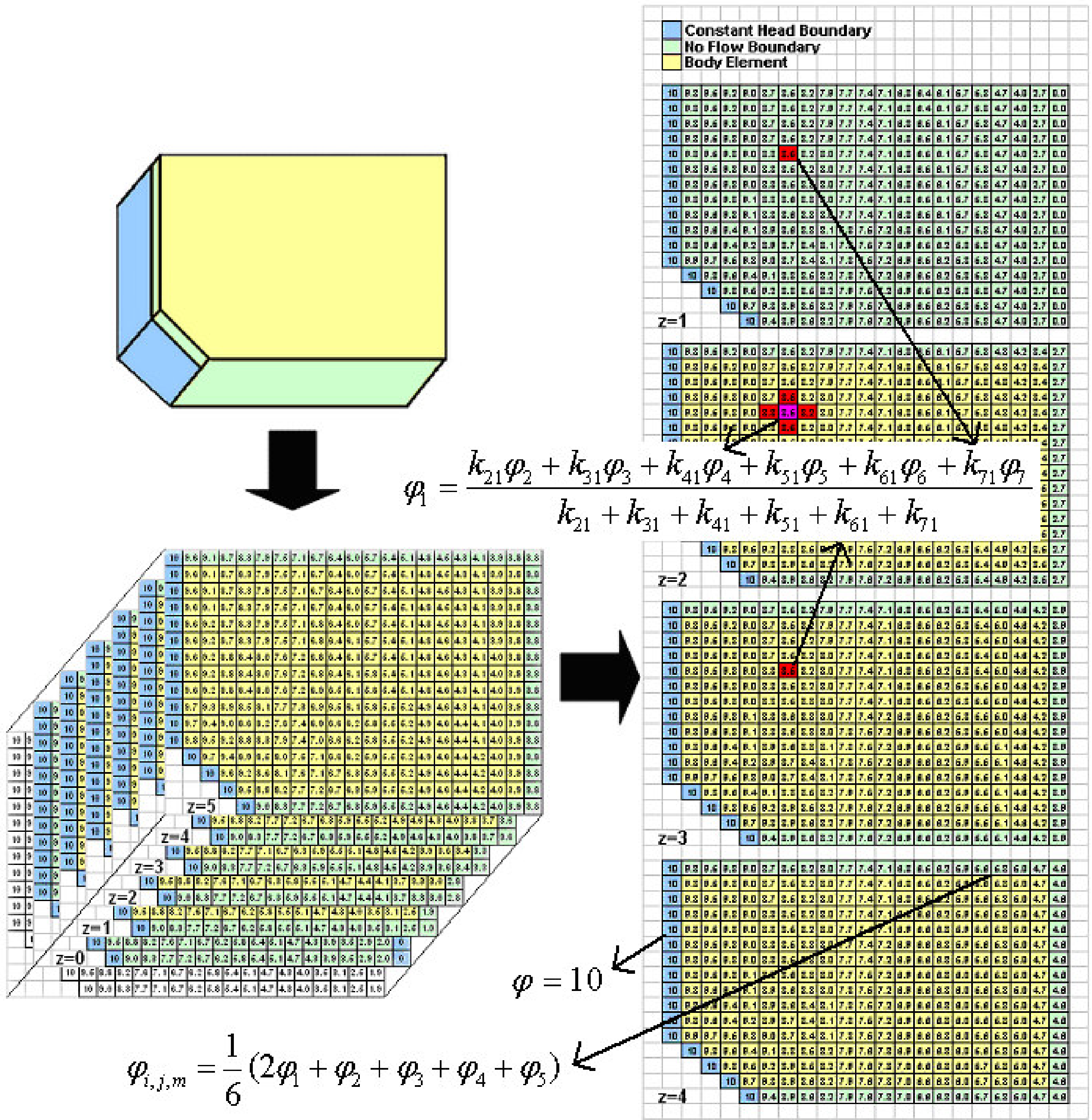


Figure2: a simplified sample that reveal how a union mass can change to separate mass

For producing model, it is possible to use ability of doing iterative calculation of spreadsheets. For example for doing iteration action in Excel spreadsheet, at first, before producing cells, should go to *Tools/Option/Calculation* and then active *Iteration* key, then dismiss calculations from automatic mode and import in manual mode and assign maximum iterations and maximum change. By this effort, after producing blocks and writing formulations, it is possible to run model by pressing *F9* key. By considering in maximum change and iteration, model work with Gauss-Seidel method (The results of each iteration in this block are recorded and they will be used for the next iteration) until it can not find any changes in each elements.

4 Verification

Lambe and Vitman problem is used for verification of LBM. Figure 3.a presents a flow net solution for seepage flow in the foundation of a concrete dam with a cutoff wall as presented by Lambe and Whitman (1969). The hydraulic conductivity of the homogeneous foundation material is 1x10-3 feet/min and the dimensions of the problem are shown in Figure 3.b

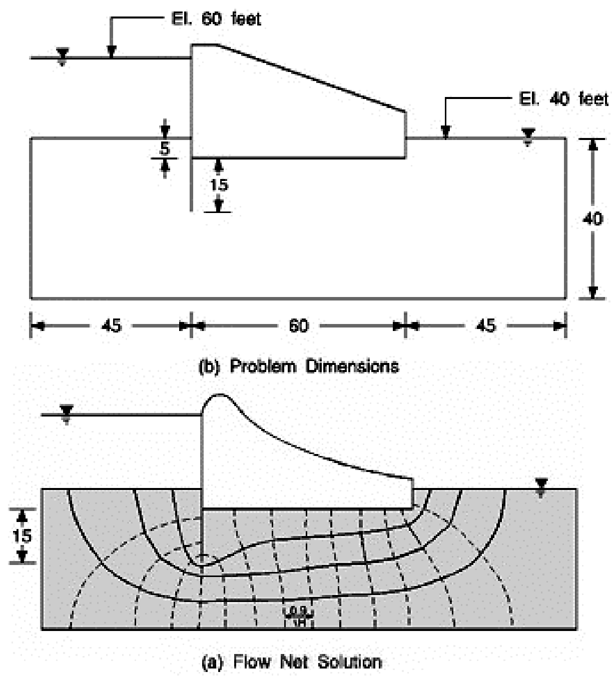


Figure3: flow net and total head contour (Lambe, T.W. Whitman, R.V. 1969)

Based on the flow net solution, the seepage under the dam is 5.76x10-3 ft3/min/ft, the uplift pressure at the downstream toe, (Point A), is 7.1 feet, and the exit gradient is 0.34.

The steady state seepage of the above foundation cutoff problem has been analyzed by LBM, SEEP/W, and SEEP3D. In LBM and SEEP3D width of dam is assumed 40 ft.

For creating LBM model 10 same blocks are used. The boundary nodes of the upstream and downstream surfaces are designed as constant head boundaries with total head equals to 60 feet and 40 feet respectively. No flow boundary conditions are assumed for all other boundaries (table 1). In figure 4 some of this blocks and total head counter are brought. The numerical mesh used for these analyses are shown in figure 4 to 6.

There are 15 contours at intervals of 1.429, beginning at a minimum value of 40. The number of contours is the same as the number of equipotential lines in the original flow net. Head contours computed by LBM are essentially the same as the equipotential lines in analytical method (flownet), SEEP/W head contours, and SEEP3D head contours.

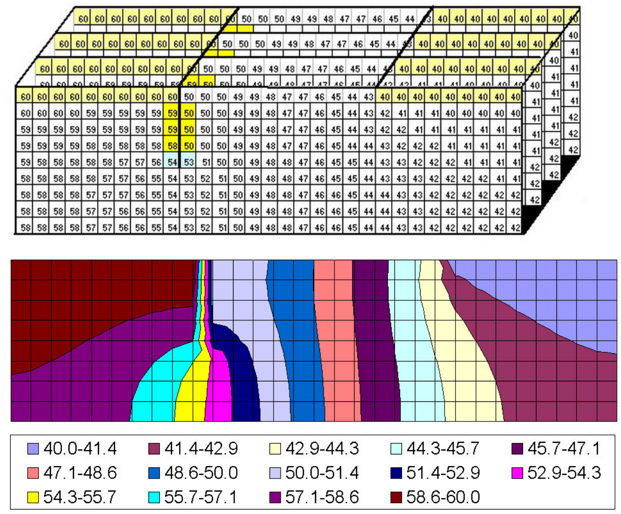


Figure4: Some of LBM blokes and total head contour with LBM

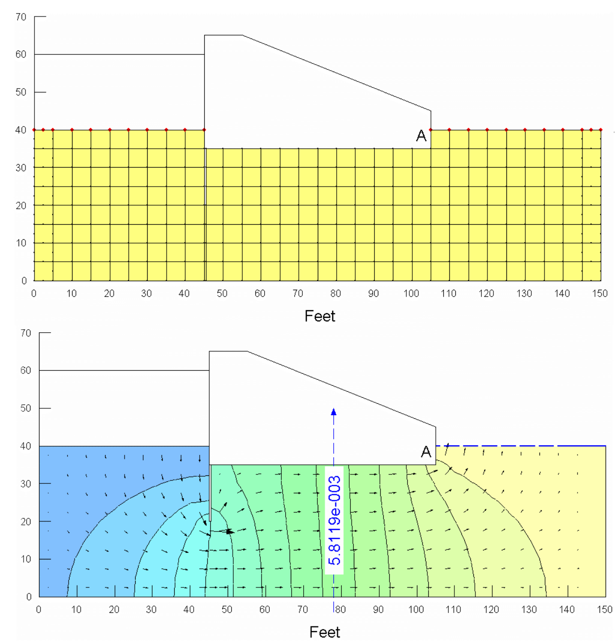


Figure5: domain mesh and total head contour with SEEP/W

The total seepage through the 40 ft. foundation is 0.232ft3/min. When converted to seepage per unit width, the computed seepage from LBM is 5.80x10-3 ft3/min/ft.

With regarding to the result of LBM and other method can find a very close relationship between them. For more accuracy result of LBM, quantity of discharge under dam, upper pressure at A, and exit gradient at downstream toe are compared with flow net, SEEP/W results, and SEEP3D result in table 2.

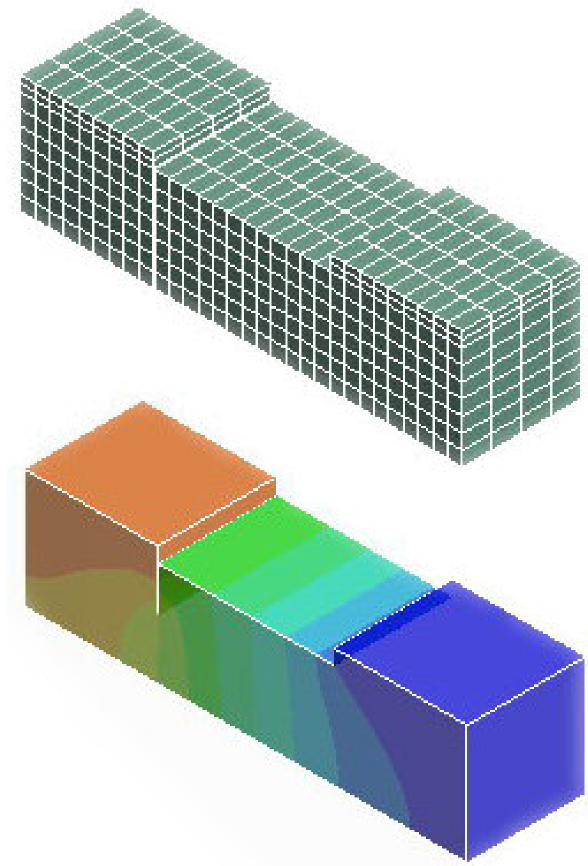


Figure6: domain mesh and total head contour with SEEP3D

Table2: Compare between LBM and other method results

	Total Seepage (ft3/min/ft)	Uplift Pressure Head at A (ft)	Exit Gradient at Downstream Toe
LBM	$5.8*10^{-2}$	6.57	0.31
SEEP3D	$5.67*10^{-2}$	6.97	0.35
SEEP/W	$5.81*10^{-2}$	6.91	0.34
Flow Net	$5.76*10^{-2}$	7.1	0.33

The uplift pressure head at Point A is computed as:

$$P = \varphi - z \quad [9]$$

In mentioned equation P is Pressure Head and z is Elevation.

5 CONCLUSION

5.1 Compared between LBM and SEEP/W, SEEP3D, and net flow show that error of LBM result is too less and it has adequate accuracy for simulating in other domain. This method gives a great ability to spreadsheet for 3D modeling of groundwater. However in small domain like earth dams this model can give an acceptable respond.

5.2 Some advantages of this model are below
-User friendly
-Giving physical perception to user who makes it
-Easy importing data
-Easy accessing and changing data in each element of each single distance block
-Quick access to all output data in all distances,
-Because of these advantages it can use for professional and educational projects.

5.3 Unfortunately because of spreadsheets nature the only method that is usable in LBM is finite difference.

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