Impact assessment of layer moduli on measured surface deflections via sensitivity analysis

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ABSTRACT
The integrity of pavement-subgrade systems are routinely evaluated using non-destructive testing techniques such as the falling weight deflectometer (FWD). Back-calculation of pavement layer moduli using non-destructive test data is challenging due to the insensitivity of displacement measurements to the stiffness characteristics of some layers. This paper describes a rigorous finite element technique for identifying the sensitivity of surface displacements to the elastic modulus of each layer. The paper begins by introducing the notion of contribution ratio and then presents a sensitivity analysis framework that is used to quantify deflection and stress field sensitivity to layer modulus. An example is presented to demonstrate the sensitivities.

RÉSUMÉ
L'intégrité des systèmes des couches de fondation des chaussées peuvent être évaluées d'une façon périodique en utilisant les techniques des tests non-destructifs comme par exemple le déflectomètre de la masse tombante (DMT). Les calculs du module de la couche de chaussée en se basant sur les tests non-destructif révèlent une certaine difficulté en raison de l'insensibilité des mesures du déplacement par rapport à la rigidité de certaines couches. Cet article décrit une technique rigoureuse qui se base sur la méthode des éléments finis pour identifier la sensibilité de déplacement des couches de surface par rapport au module élastique de chaque couche. Le document commence par l'introduction de la notion de facteur de contribution puis présente un cadre d'analyse de la sensibilité qui peut être utilisée pour quantifier la sensibilité du champ de déformation et de contraintes par rapport au module de la couche.
Un exemple est introduit pour démontrer les sensibilités.

1 INTRODUCTION

It is well-known that pavement-subgrade systems are complex consisting of materials that are sensitive to moisture, temperature and stress state. The material properties in these systems vary not only systematically but also randomly, which makes it difficult to provide high quality predictions of pavement response to traffic and environmental loading. Strictly speaking, good predictions are only possible for problems in which the load paths are not complex and the variability in properties is not large. As a result, while the use of sophisticated models can help the engineer better understand the problem that is being analyzed, the quality of predictions by these models for providing better estimates of performance parameters that are tied to remaining serviceability life is often undermined due to the inability to capture details that depend on the random variations in properties in space and over time.

Given seasonal changes in temperature and precipitation, as well as cyclical traffic loading, pavement-subgrade systems undergo a continuous evolution in properties that influence the structural integrity of pavements. An important component of pavement management is keeping track of the evolution of changes in the structural performance of pavements, which is often accomplished using the falling weight deflectometer (FWD) together with sophisticated mechanical models to determine the changes in the apparent layer moduli. The ability to back-calculate properties from response data becomes increasingly difficult when the models become increasingly complex. As pointed out by Stolle and Hein (Bush and Baladi 1989), the two main limitations for obtaining good quality estimates of the moduli are:

- solutions to back-calculation are nonunique; and
- idealized models used in analyses yield systematic errors.

The reader is referred to special ASTM technical publications of Bush and Baladi (1989) and Tayabji and Lukanen (2000) addressing this subject matter.

The objective of this paper is to summarize some of the research carried out at McMaster University on the influence of random and systematic variations in elastic moduli on forward and back-calculation predictions within the context of pavement structures. We begin by defining the problem of interest and then move onto addressing the influence of systematic errors on the analysis of pavement response data, as well as the effect of random variations in elastic modulus on measured response. Thereafter, a finite element methodology for performing sensitivity analysis is described. The methodology enables the engineer to determine the sensitivity of deflection and stress or strain predictions to variations in the stiffness characteristics of the various components that make up a pavement-subgrade system.

2 PROBLEM DEFINITION AND BACKGROUND

Regardless of the level of sophistication, models for analysing the response of a pavement to surface loading only provide an approximation to the real pavement-subgrade system. Figure 1 shows schematics that illustrate the difference between a "real" system and an idealized one. The difficulty that the analyst encounters is the inability to properly define geometry and the spatial and temporal distribution of material properties. When evaluating a pavement structure given response data, our ability to accurately back-calculate material properties such as the layer moduli are influenced by both systematic and random errors. This usually implies that estimates for subgrade modulus are better than those for the pavement layers, which are of most interest to the highway engineer since they directly influence the prediction of the performance parameters, such as the tensile strain at the bottom of the asphalt layer, which is required when determining remaining fatigue life via damage analysis.

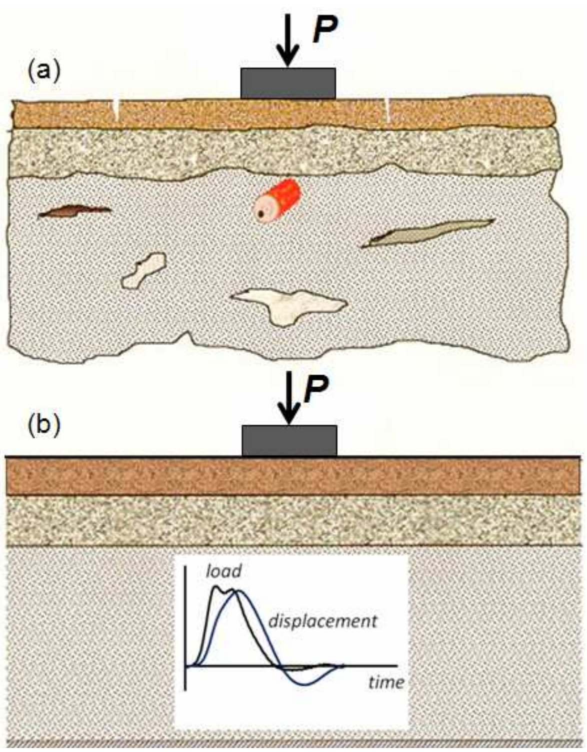

Figure 1. Illustration of (a) actual problem versus (b) idealized pavement-subgrade system

As indicated previously the FWD test is popular for measuring the response of a pavement to impact loading. An important systematic error when analyzing the response data, apart from the assumption of linear elastic isotropic stress-strain behaviour, is the assumption that the pavement-subgrade system can be analyzed using elastostatic models together with the peak load and displacements that are measured at various offsets (Parvini 1997). Figure 1 (b) illustrates typical load and displacement histories. The fact that the peaks do not occur at the same time reflects the importance of inertial in the pavement response. As discussed by Stolle and Guo (2005), the static behaviour can be separated from the overall behaviour to enhance the back-calculation capabilities of the elastostatic models, although this is rarely done. Since the filtered response denoted by $\bar{x}$ corresponds to zero frequency, this mode may be interpreted as a pseudo-static behavior. For the case of discrete data, in which the measurements are equally spaced in time $\Delta t = T/N$, the pseudo-static response is given by the average $\bar{x} = \frac{1}{N}\sum_{k=1}^{N} x_k$, where N corresponds to the number of data points over interval T.

Even if the geometry and loading history of a boundary-valued problem are well defined, accurate predictions are only possible if the point-to-point spatial variability of properties is known. To examine the impact of natural random variations of measured soil properties on the geotechnical engineer's ability to accurately model nonlinear material behaviour, Stolle et al. (2004) performed a series of vacuum triaxial tests on "identically-prepared" dry samples of Ottawa Sand (ASTM C109). The samples were tested under a constant effective confining stress of 50 kPa. Figure 2 summarizes the stress-strain results of 12 tests, showing the average relation represented by a hyperbolic equation fitted to the data, as well as the variability denoted by the one standard deviation limits. Scrutiny of the figure clearly reveals a natural variation in constitutive behaviour even though extreme care was taken when preparing the specimens.

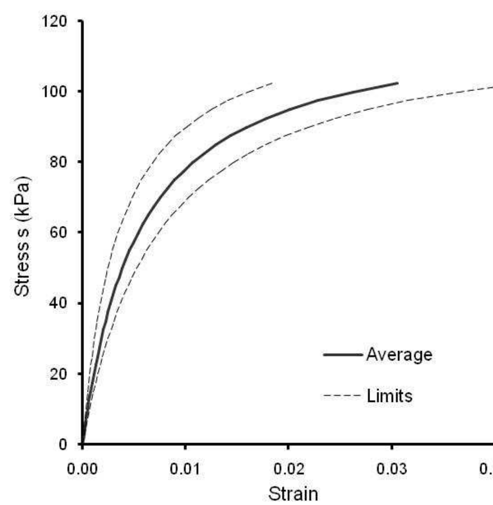

Figure 2. Variation in stress strain response of Ottawa sand.

The variability in constitutive behavior can be accommodated introducing probability into an analysis, assuming that the independent variables have random values. For example, if we consider stress $\sigma = f(G)$ to depend on a random variable G, the expected value of stress $E[\sigma]$ and its variance $V[\sigma]$, for a given strain history, are defined using perturbation analysis as

$$E\left[\sigma\right] = f\left(\overline{G}\right) + \frac{f''\left(\overline{G}\right)}{2} V\left[G\right] \quad [1]$$

$$V\left[\sigma\right] = \left(f_{,G}\left(\overline{G}\right)\right)^2 V\left[G\right] \quad [2]$$

with $p(G)$ being the probability density function; see, for example, Kleiber and Hien (1992). From eq.1, it is clear that the average response of stress is not obtained by merely substituting the value of the average random variable into the function.

Two useful measures of variability are the standard deviation, which is the square root of the variance, and the coefficient of variation CV, defined as the standard deviation divided by the mean.

Whether one is dealing with systematic or random variations, it is important to be capable of evaluating the sensitivity of the predictions to the uncertainty of the independent variables. The following section focus on sensitivity analysis for the elastodynamic equilibrium of a pavement-subgrade system, in which the layer moduli are random variables that are not correlated. For the case in which a variable is random, we will see that the sensitivity is related to the coefficient of variation of the variable.

3 SENSITIVITY ANALYSIS

3.1 Contribution Ratio for Displacement

As discussed by Stolle and Pavini (2002), let us consider the frequency domain, finite element representation of the elastodynamic equilibrium

$$\mathbf{Ku} = \mathbf{F} + \omega^2\mathbf{Mu} \quad [3]$$

where $\mathbf{K} = (1+2\xi i)\mathbf{K}_s$, which depends on the hysteretic damping ratio ξ, and on static stiffness $\mathbf{K}_s$, $\mathbf{M}$ is the mass matrix, and $\mathbf{u}$ and $\mathbf{F}$ are the complex-valued displacement and load vector corresponding to angular loading frequency (ω). One observes that all forms of loading, including inertial, appear on the right-hand side with the system being on the left.

Let us subdivide our domain into n distinct layers, each characterized by its modulus E_i. One may decompose the stiffness matrix, (Stolle 2002), according to

$$\mathbf{K} = \sum_{i=1}^{n}\mathbf{K}_i \quad [4]$$

in which $\mathbf{K}_i$ is the stiffness contribution to the total stiffness $\mathbf{K}$ of all finite elements having modulus E_i. Similarly, the total displacement $\mathbf{u}$ may be decomposed according to

$$\mathbf{u} = \sum_{i=1}^{n}\mathbf{u}_i \quad [5]$$

where $\mathbf{u}_i$ is defined from the following sequence of operations

$$\mathbf{K}\sum_{i=1}^{n}\mathbf{u}_i = \sum_{i=1}^{n}\mathbf{K}_i\mathbf{u} \quad \Rightarrow \quad \mathbf{u}_i = \mathbf{K}^{-1}\mathbf{K}_i\mathbf{u}\,. \quad [6]$$

In other words, $\mathbf{u}_i$ is the displacement contribution associated with layer 'i'. By recognizing that $\mathbf{K}_{,i} = \dfrac{\mathbf{K}_i}{E_i}$ and $\mathbf{u}_{,i} = -\mathbf{K}^{-1}\mathbf{K}_{,i}\,\mathbf{u}$ for the case where the load $\mathbf{F}$ is independent of E_i, one may write

$$\mathbf{u}_i = -E_i\mathbf{u}_{,i} \quad [7]$$

with the notation $(\cdot)_{,i}$ implying differentiation of quantity $(\cdot)$ with respect to E_i. This indicates that the displacement contribution $\mathbf{u}_i$ is related to the rate of change of the displacement with respect to modulus E_i.

Given that u_{ik} denotes the deflection contribution of all the elements with elastic modulus E_i to the total deflection for the k^{th} degree of freedom u_k, Stolle and Pavini (2002) define contribution ratio

$$CR_k = \left|\frac{u_{ik}}{u_k}\right| \quad [8]$$

to quantify the importance of a layer's material stiffness to surface deflections. Given the definition of $\mathbf{u}_i$ and assuming that all u_{ik} have the same sign, then one would expect $\sum CR_k = 1$, when summed over all i. There are situations however when a layer provides a negative contribution to u_k, which implies that the sum of the contribution ratios can exceed one. Parvini (1997) showed for stochastic elastostatic finite element analysis, that CR_k is related to the coefficient of variation of u_k and that of E_i by the equation; i.e.

$$CR_k = \frac{CV(u_k)}{CV(E_i)}\,. \quad [9]$$

This equation indicates that the sensitivity of displacement k to modulus of layer i is related to the statistical uncertainties associated with the modulus of that layer. A displacement is highly correlated to an elastic modulus if the corresponding CR_k is large, and weakly correlated when the CR_k is small. The notion of contribution ratio provides the engineer with a tool to determine whether a measurement can be realistically used to estimate an in-situ modulus when attempting to estimate in-situ layer moduli using response data.

3.2 Contribution Ratio for Stress and Strain

Once the displacement contributions $\mathbf{u}_i$ are obtained, it is possible to quantify the influence of a layer modulus on strain $\boldsymbol{\varepsilon}_i$ and stress $\boldsymbol{\sigma}_i$ contributions via $\boldsymbol{\varepsilon}_i = \mathbf{B}\mathbf{u}_i$ and $\boldsymbol{\sigma}_i = \mathbf{D}\boldsymbol{\varepsilon}_i$, in which **B** and **D** are the usual strain-displacement and constitutive matrices appearing in $\boldsymbol{\varepsilon} = \mathbf{B}\mathbf{u}$ and $\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}$. The contribution ratio for strain (or stress) is obtained by dividing the strain (or stress) contribution by the corresponding total value for the particular component being evaluated; i.e., symbolically written, one has $CR = \varepsilon_i / \varepsilon$ (or $CR = \sigma_i / \sigma$).

4 NUMERICAL EXAMPLE

For situations in which the subgrade provides poor support for a pavement structure, a subgrade cap layer can be constructed to provide protection and reduce the impact of the surface load on the weak foundation; see, for example, Stolle and Hein (2002). Such layers typically consist of granular material with particle sizes ranging from sand fill to crushed rock. The problem presented in this section examines the influence of a cap layer on the behaviour of a pavement-subgrade system, and demonstrates how contribution ratio can be used to determine the sensitivity of a response to material stiffness and identify changes in the mechanics of a system. The displacements and stresses of a pavement structure supported by a weak subgrade are compared to those for the system, in which a cap is constructed to improve pavement performance. Comparisons are made for two loading cases: quasi-static loading; and 25 Hz steady-state harmonic loading. It should be noted that that this loading frequency is at the upper end of the range of frequencies that dominate a FWD test.

Figure 3 shows the finite element mesh of the problem, consisting of 1020 four-noded, with Table 1 summarizing the elastic properties and layer thicknesses. The 40 kN surface load was applied uniformly over a circular area of 0.15 m radius, yielding an average pressure of 566 kPa. For the elastodynamic analysis, a density corresponding to a unit weight of 20 kN/m^3 was assumed along with a damping ratio of 5 percent. Absorbing boundaries were introduced at the artificial vertical boundaries on the right hand side to reduce the impact of reflected waves; see, for example, Parvini (1997).

Representative contribution ratios (CR) for surface displacement at various offset corresponding to typical locations for FWD geophones are presented in Figures 4 and 5 for elastostatic and elastodynamic loading, respectively, with contribution ratios for vertical centreline stress at the bottom of the asphalt, base, subbase and cap layers being illustrated in Figure 6. The legend for all figures that follow is provided with Figure 4.

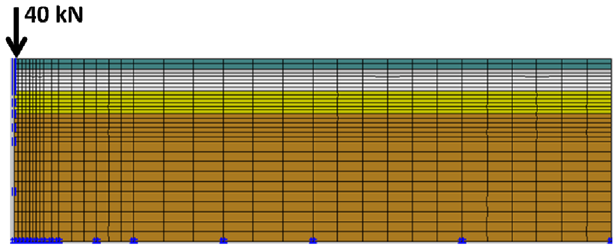

Figure 3: Finite element mesh used for axisymmetrical analysis.

Table 1. Standard parameters for system.

Layer	Thickness (cm)	E (MPa)	Poisson's Ratio
Subgrade	300 – Cap thickness	30*	0.45
Cap	30	150	0.35
Subbase	30	200	0.35
Base	15	250	0.35
Asphalt	20	4000	0.35

4.1 Effect of Subgrade Cap on Surface Deflections

The contribution ratios can be used to quantify the significance of each layer with respect to the observed surface displacement. Figure 4 shows the accumulated contribution ratios of surface deflection at various offsets for the elastostatic loading, comparing the case of no cap with that of a system strengthened with a cap immediately above the subgrade. A comparison of the numbers above the bars, which represent the total displacement's, clearly indicates that the stiffer cap material reduces surface deflection, as one might expect. A close comparison of the contributions for the cases of cap and no cap clearly indicates that the cap reduces the influence of the subgrade, as one might expect. The significance of the pavement layers with respect to surface deflection actually slightly decreases with the introduction of the cap.

An examination of Figure 4 clearly shows that for both cases the measured displacements at large offset reflect the stiffness of the subgrade, with measurements close to the load capturing the properties of the pavement structure. The overall small contribution ratios of the asphalt and base layer beyond the loading plate clearly indicate why it is difficult to estimate their moduli from deflection data, when using *in-situ* tests and back-calculation strategies that rely on elastostatic models. The fact that the presence of the cap layer only has a small influence on the combined CR's for cap and subgrade when compared to that of the subgrade without cap indicates why it may be difficult to separate the influence of the cap and subgrade on surface measurements, particularly when one considers that there are systematic errors due to neglecting the effects of inertia.

4.2 Effect of Inertia

The contribution ratios for elastodynamic analysis are complex-valued. While the sum of the real components adds up to one, the corresponding sum for the imaginary components adds up to zero. Figure 5 summarizes the

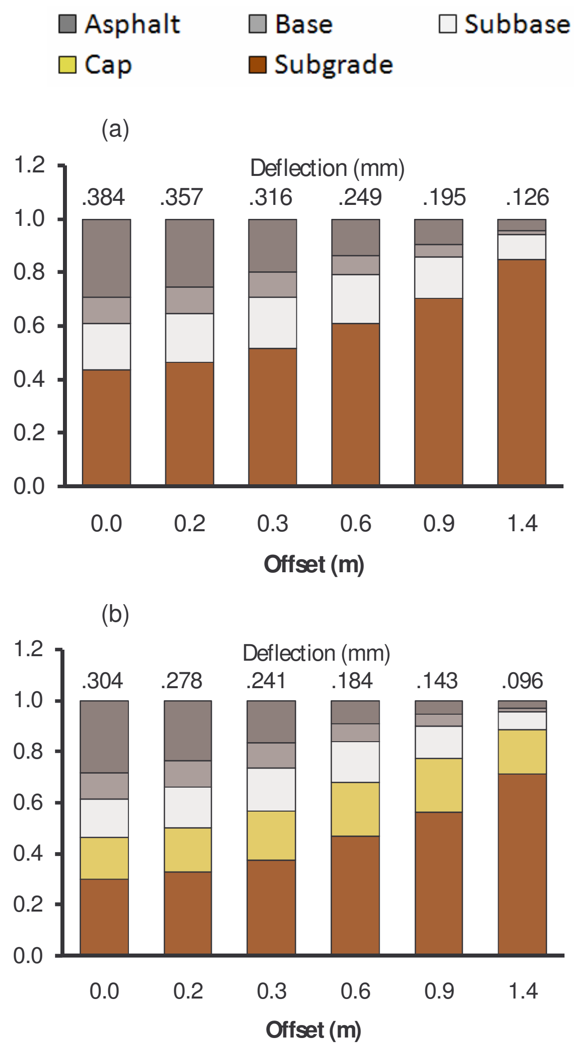

Figure 4. Surface deflection CR for elastostatic loading: (a) without cap; and (b) with cap.

real component of the CR's as a function of offset corresponding to a loading frequency of 25 Hz. Unlike the response corresponding to the static case, the pavement response, including that at larger offset, is more influenced by the properties of the pavement layers. One observes that the presence of the cap changes the response characteristics of the system, such that subgrade, which has negative CR's, has a much smaller influence of surface displacements close to the load. By using dynamic loading and proper data interpretation techniques, more information is available to allow for system identification. By assuming an equivalent static behaviour, information is not only lost, the wrong conclusions may be reached due to the systematic errors.

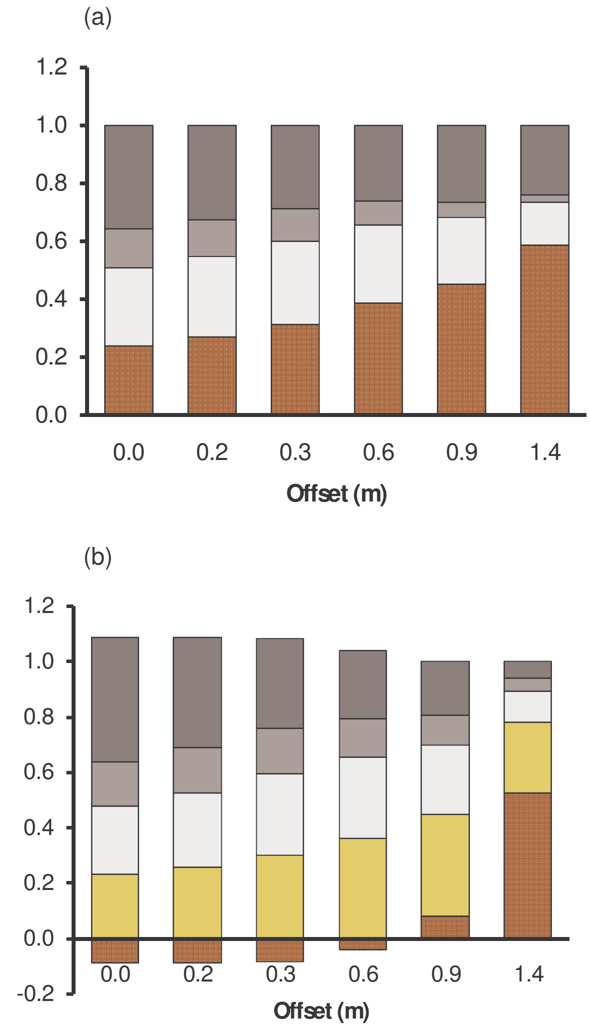

Figure 5. Surface deflection CR for elastodynamic analysis (24 Hz): (a) without cap; and (b) with cap

4.3 Contribution Ratio for Stress

Figure 6 summarizes contribution ratios for stresses at four locations under the load. To interpret this figure, it is best to begin with the CR's for the asphalt (column on extreme right). First of all, the magnitude of stress is not influenced much by the presence of the cap. This is expected due to the point in question being close to the load. One also observes similar contribution ratios from the base and subbase layers for the two cases. As one moves away from the load, the effect of a cap is not too important with regard to the contribution ratios corresponding to the asphalt, base and subbase layers, until one is at the location corresponding to the cap layer (far left column).

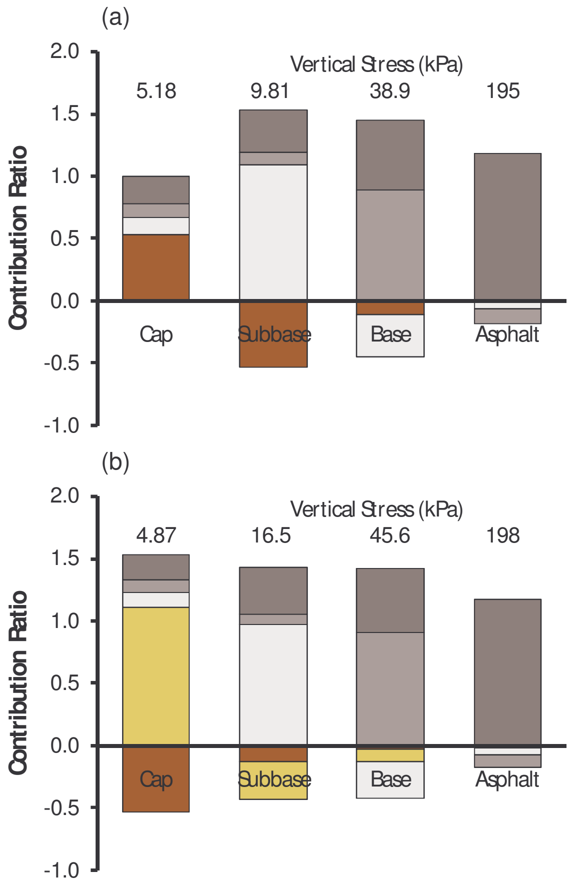

Figure 6. CR's for stress assuming static analysis: (a) without cap and (b) including cap layer

5 CONCLUDING REMARKS

This paper has provided a review of the effect of random properties on response and the notion of contribution ratio and its application to interpreting the response of pavement-subgrade systems. Although pavement engineers have for a long time had a feel for the overall sensitivity of surface deflections to the stiffness characteristics of the layers, the details regarding the sensitivities have largely alluded them.

A methodology to quantify this feel in terms of a tangible measure; that is, the contribution ratio. It was shown that analysis details are important for system identification purposes, a main conclusion being that a pavement structure responds differently to an impact load when compared to a pseudo-static load.

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