Visualization of 3-D displacement field under a model footing using transparent soil



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ABSTRACT

A 3-D displacement field under a model footing is presented in this paper, which was visualized by using transparent soil and an image processing technique. It has explored the opportunities to measure more advanced 3-D displacement fields. Transparent soils are made of either amorphous silica gels or powders with a pore fluid having the same refractive index. Geotechnical laboratory tests showed that transparent soils exhibited macro-geotechnical properties similar to those of natural soils. An optical system consisting of a laser light, a CCD camera, a frame grabber, and a PC was developed to optically slice the transparent soil model. A distinctive laser speckle pattern is generated by the interaction of laser light and transparent soil. A digital image cross-correlation algorithm named adaptive cross-correlation was used to obtain a 2-D displacement field between two consecutive images captured during footing's settlement. A 3-D displacement field was obtained in Matlab[®] by combining several cross-sections of 2-D displacement fields. Test results showed that the developed optical system and transparent soil are suitable for studying 3-D soil-structural interaction problems.

RÉSUMÉ

Un 3-D champ de déplacement sous un modèle footing est présenté dans ce papier, qui a été visualisé en utilisant du sol transparent et une technique de traitement d'image. Il a exploré les opportunités de mesurer de 3-D champs de déplacement plus avancés. Les sols transparents sont faits des gels de silice amorphes ou des poudres avec un liquide étudie soigneusement ayant le même index réfringent. Les essais de laboratoire de Geotechnical ont montré que les sols transparents ont exposé des propriétés macro-geotechnical semblables à ceux de sols naturels. Un système optique se composant d'une lumière à laser, un appareil de photo CCD, un capteur de page-écran et un ordinateur a été développé à coupent optiquement le modèle de sol transparent. Un dessin de moucheture à laser distinctif est produit par l'action réciproque de sol clair et transparent à laser. La trans-corrélation adaptable appelée d'un algorithme de trans-corrélation d'image numérique a été utilisée pour obtenir un 2-D champ de déplacement entre deux images consécutives capturées pendant le règlement de footing. Un 3ème champ de déplacement a été obtenu dans Matlab[®] en combinant plusieurs coupes transversales de 2-D champs de déplacement. Les résultats d'essai ont montré que le système optique développé et le sol transparent sont convenables pour étudier de 3-D problèmes d'action réciproque structurels de sol.

1 INTRODUCTION

Visualization of three-dimensional (3-D) deformation or strain field can significantly improve the interpretation and understanding of geotechnical engineering problems and help in solving these problems. Many techniques and methods have been utilized in geotechnical engineering research to obtain soil deformation and strains inside. Radiography was implemented to visualize information inside soils for many years (Bergfelt 1956 and Roscoe et al 1963) and microscopic behaviour of soil under shear by Nemat-Nasser and Okada (2001). However, all the investigations mentioned above are intrusive measurement by embedding lead shots inside soil sample. These lead shots cannot follow soil movement under excessive movement condition. Computerized axial tomography (CAT scan) and magnetic resonance imaging (MRI) were utilized for experimental modeling of geotechnical problems (Wong 1999). Routine applications of these techniques are limited by the high cost and technical difficulties.

Transparent materials have also been used in model studies in investigating stress and flow field inside photoelastic material or opaque material in model tests (Konagai et al 1992). However, these studies are limited by the fact that these materials cannot model soil behaviour and have poor quality of transparency. The development of transparent materials, which can be customized to meet model test requirements in terms of strength and deformation, is the necessary step to model natural soil in deformation and strength studies.

Transparent soil used in this research is made of transparent amorphous silica powder or gel with a pore fluid having the same refractive index. This paper presents a 3-D displacement field under a model footing obtained non-intrusively using transparent soil. The results show that transparent soil can be used to model soil-structural interaction problems.

2 TRANSPARENT SOIL

Transparent soil, developed by Mannheimer and Oswald (1993), was tested to exhibit macroscopic geotechnical properties similar to natural clay. This material was made of transparent silica powder and a pore fluid with a matching refractive index. Another family of transparent soil was also developed to model sand, which is made of silica gel. Both materials have the same refractive index

using the same fluid, which permits in modeling stratum conditions, for example, sand over clay condition. The developed materials are truly transparent, not merely translucent, thereby permitting the use of simple optical techniques to visualize 3-D flow paths and spatial deformation patterns within the soil mass, shown in Figure 1. Transparent soil mentioned in this paper has been used by other researchers to measure the response of transparent soils in model tests.

2.1 Amorphous Silica Powder for Modeling Clay

Amorphous silica powder is a commercial product of PPG.Inc., used without further processing. It has a twopore system inside. It consists of ultrafine particles with individual diameters on the order of 0.02 µm. These particles combine to form larger porous aggregates. Four different silicas ranging in aggregate size from 1.4 to 175 um were used for modeling clay. Amorphous silica powders exhibited typical stress-strain behaviour in conventional triaxial compression tests. The measured behaviours of amorphous silica powder appear to represent a generic clay whose properties depend on the size of silica and consolidation history. The hydraulic conductivities ranged between $2.3*10^{-7}$ to $2.5*10^{-5}$ cm/sec. The compression index, Cc, and recompression index, Cr, were found to be 2.35 and 0.23 respectively. The angles of friction ranged between 21° and 36°. The deformation mechanism and magnitudes were consistent with those of natural clays at strains less than 15%, which permits modeling deformational problems, since the strength corresponding to the strain of interest is comparable to that of natural soils. More details can be found in Iskander et al (2002a) and Liu et al (2003).



Figure 1. Target viewed through 2 in. thick transparent soil

2.2 Amorphous Silica Gel for Modeling Sand

Silica gel is a colloidal form of silica. It is inert and porous, and is available in sizes ranging from 0.5 mm to 5 mm. Silica gel shares the same two-pore system like amorphous silica powder. The most common shapes are rounded bead and angular particle. Typical stress-strain behaviours for both fine and coarse silica gel are consistent with those of natural sands. Dense specimens of fine silica gel exhibited typical shear dilatancy behaviour, particularly at low confining pressures. The residual strength of dense samples approaches the peak strength of loose samples at the same normal pressure. The angles of friction from triaxial tests were 30°–36° for fine silica gel and 31°–34° for coarse silica gel. Young's Modulus ranged between 24-84 MPa (3–12 ksi) depending on density and size. These values are similar to the values reported for natural sands. More details can be found in Iskander et al (2002b).

2.3 Pore Fluids

Two liquids resulted in sufficiently clear samples. The first is a blend of mineral oil and a normal paraffinic solvent by one-to-one weight ratio, which has a refractive index of 1.447 at 25°C. The second is a brine mixture blended from calcium bromide and water to have a refractive index of 1.448 at 25°C. These two fluids are not miscible which permits studying multiphase flow problems, such as contamination of aquifer with petroleum products.

3 DIGITAL IMAGE PROCESSING TECHNIQUE

3.1 Digital Image Cross-Correlation

Digital image cross-correlation (DIC) is a classic pattern recognition technique where two images are compared to obtain the relative displacement between them. DIC is widely used in many engineering fields to obtain spatial deformation patterns, albeit with several names. The discrete form of standard cross-correlation function, C, given by Huang and Tsai (1981), is as follows:

$$C(\Delta x, \Delta y) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n) g(m + \Delta x, n + \Delta y)$$
[1]

where M and N are the dimensions of the interrogated images, f and g are the intensities of two images being interrogated. The correlation function given above is sensitive to the average intensity of f(m, n) and g(m, n). Therefore, the zero normalized cross-correlation function (ZNCC) is normally used in the analysis, which is given by Giachetti (2000) as

$$ZNCC = \frac{\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} [f(m,n) - \bar{f}][g(x+m, y+n) - \bar{g}]}{\sqrt{\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} [f(m,n) - \bar{f}]^2 \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} [g(m,n) - \bar{g}]^2}}$$
[2]

where f and g are the average intensities of the two images, f(m, n) and g(m, n).

3.2 Adaptive Cross-Correlation (ACC)

Adaptive cross-correlation (ACC) is an iterative procedure that is used to increase the accuracy of DIC (Scarano and Riethmuller 1999). In ACC, the position and size of interrogation window is continuously refined to obtain the optimal prediction. At the beginning, the correlation starts with larger interrogation windows. First, conventional DIC is performed on these windows and the deformation is obtained for each window. Second, before correlation analysis each window is divided into four smaller windows that are shifted from their original positions by deformation calculated from the first step. This procedure is repeated for each of the smaller interrogation windows, until the final window size is reached.

ACC is implemented in Flow Manager software (Dantec 2000), which is intended for particle image velocimetry (PIV) to measure the flow distribution by tracing the flow of embedded markers. Flow Manager was adapted for this research to study soil deformation inside a transparent soil model. Detailed comparison between DIC and ACC in rotational and translational predictions can be found in Liu and Iskander (2003). Please be noted here the rotation of individual particles cannot be calculated here, since DIC is a region matching technique. At the same time the laser speckles from transparent soil do not delineate the individual particles.

Except as noted otherwise all used parameters of ACC in this paper were the following: The interrogation window sizes 128×128, 64×64, 32×32, 16×16 pixels (final window size).

4 OPTICAL SETUP FOR DEFORMATION MEASUREMENT

4.1 Optical Test Set-up

The optical test set-up, shown in Figure 2, consisted of a Cohu 2622 black & white CCD camera, a laser, a loading frame, a test table, a linear stage, and a PC for image processing. The camera, which was set 20 cm away from the model, has a resolution of 640×480 pixels. It is controlled by the PC through a Matrox Meteor II/4 frame grabber. The control software was customized under Visual C++ environment using the Matrox MilLite software commands. The used laser was 35 mW Melles Griot HeNe with a wavelength of 632.8 nm. A Plexiglas mode with dimensions of 50×150×300 mm was used in the tests. A model footing with a plan dimension of 50×25 mm was used to simulate a continuous footing with a width of 25mm. Load was applied vertically through a screw mechanism. A load cell and an LVDT were connected to the loadframe to measure load and deformation during the test. The linear stage was used to slice the sample for 3-D displacement measurements.

4.2 Laser Speckle Image in Transparent Soil

A laser light sheet was used to target an interested crosssection inside a transparent soil model. The laser light sheet was generated by passing a laser beam through a line generator lens. The interaction between transparent soil and laser light sheet produced a distinctive speckle pattern, shown in Figure 3 from a model of "sand" overlying soft "clay" used for 3-D analyses. The speckle pattern created by a coherent light beam, such as a laser, scattered from a rough surface or from particles in a liquid, has a well-defined spatial structure (Goodman 1975). The speckle properties typically depend on both the roughness and reflectance of the surface. Laser speckle techniques have been used intensively in many engineering fields, especially in surface deformation (Erf 1978).



Figure 2. Optical test set-up



Figure 3. Speckles generated in transparent soil

5 2-D DISPLACEMENT FIELD COMPARISON WITH NATURAL SOIL

5.1 Test Program

Silica gel is opaque under dry condition and becomes transparent under saturated condition. Only a comparative study under saturated dense condition is discussed in this paper because it is of more interest to geotechnical engineers for measuring soil deformation inside the transparent model. The pore fluid used to saturate silica gel was a blend of mineral oil and a paraffinic solvent, while water was used to saturate sand. In this comparative study, all deformations were measured at model surface for both sand and silica gel samples. More comparative studies under dry condition can be found in Liu (2003).

Silica gel (SG1) from *Multisorb Technologies Inc.* was used in this study. SG1 is an angular fine silica gel with an aggregate diameter of 0.5-1.5 mm. Sand sample was selected to have the same grain size distribution in order to obtain more comparable test results.

The minimum relative density according to ASTM Standard D4254 was γ_{min} =14.93 kN/m³ for sand and 6.89 kN/m³ for silica gel. Because of the two-pore system inside silica gel, a vibrator compaction method was used to determine the maximum relative density. It was γ_{max} =7.51 kN/m³ for silica gel. For sand sample, the maximum relative density was 16.67 kN/m³, which was slightly higher than 16.29 kN/m³ determined from ASTM D698. The silica gel sample used in the study had a density of γ =7.34 kN/m³ corresponding to a relative density of 15.87 kN/m³ and a relative density of 71.3% in sand. The friction angles derived from dry direct shear tests according to ASTM D3080 were approximately 38° for sand and 33° for silica gel.

5.2 Two-Dimensional Displacement Field Comparison

The load settlement p-z curves of the model footing on dense silica gel and sand samples are shown in Figure 4. The images captured at the corresponding time were labelled in the figure. The failures in both sand and silica gel appear more like general shear failure, which is consistent with sample density used in this study (Vesic 1975). The bearing capacity of footing on sand was higher than that in silica gel because of higher values in density and shear strength in sand sample. Two typical cases were used in the analyses: before failure corresponding to displacement between image 1 and image 2, and after failure for images 5 and 6. The footing width, B, was used for normalizing distance or depth in the comparative study.

The displacement fields are found to be similar to each other in general. The influence from wall friction was ignored due to the fact it is very difficult to quantify and the influence was assumed to be the same for both materials. At the beginning, the movement of the footing caused a larger influence zone, which shrank as loading increased. At the failure stage, the depth of influence was reduced to approximately a one-time footing width below the footing base. Typical predicted displacement fields are presented in Figure 5 at failure stage.

5.3 Displacement Comparisons along Planes

Comparisons of displacements between different horizontal and vertical planes were preformed and general agreement was found between silica gel and sand, especially in distribution along the plane. However, there were some discrepancies in the magnitude. It is believed that the differences in material's properties, especially density contribute to these discrepancies.



Figure 4. P-z curves for footing on sand and silica gel



b) Sand Figure 5. Displacement field in saturated dense silica gel

5.4 Strain Field Comparison

Strains can be calculated using the predicted displacement field per Roscoe *et al.* (1963) and Yamamoto and Kusuda (2001). The displacements within the small region were considered to be the same as the displacement in the centre point. This assumption can be proved to be reasonable if the movement is relatively small. Green's strains were used regardless of the magnitude of the displacement, as follows.

$$\epsilon_{x} = \frac{\partial u}{\partial x} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial x} \right)^{2} + \left(\frac{\partial v}{\partial x} \right)^{2} \right]$$

$$\epsilon_{y} = \frac{\partial v}{\partial y} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial y} \right)^{2} + \left(\frac{\partial v}{\partial y} \right)^{2} \right]$$
[3]
[4]

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left[\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\right]$$
[5]

where u and v are the displacement components in the x and y directions. ϵ_x and ϵ_y are the axial strains in the x and y directions. γ_{xy} is the shear strain. The maximum shear strain, γ_{max} , can be calculated using the strains above.

$$\gamma_{\text{max}} = \sqrt{(\varepsilon_{x} - \varepsilon_{y})^{2} + \gamma_{xy}^{2}}$$
 [6]

The failure plane can be studied by locating the peak of the maximum shear strain field, which was assumed to coincide with the failure plane. The maximum shear strain appeared at the same region on both sides of the footing. The failure envelope in sand was deeper than that in silica gel. It is believed that the differences in material's properties contributed to this difference.

The maximum shear strain was concentrated mainly at the footing edge area during the whole test. As loading increased, the shear strain converged from both sides toward one failure plane, which was more appreciable in sand than in silica gel. There were some inconsistencies close to the boundaries due to lack of data beyond the boundaries, which can be minimized using a larger area in the future.

The triangular zone immediately under the footing proposed by Terzaghi (1943) was not obvious in both cases. The model footing with a smooth base is believed to cause this variance, which is consistent with the failure mode proposed by Hencky (1923) for a smooth footing (Vesic 1975). Two passive zones were clearly shown even at the beginning in the case of sand case and appeared only after failure in silica gel.

6 3-D DEFORMATION ANALYSES

6.1 Displacement Field Analyses

A 3-D displacement field was generated by combining several 2-D displacement fields which were calculated by cross-correlating two consecutive images at each crosssection. The displacement pattern along the footing was consistent. Similar displacement fields were obtained for different loading stages. A typical 3-D displacement field is shown in Figure 6, where the shade intensity is proportional to the magnitude of total displacement.



Figure 6. Slicing of a 3-D displacement magnitude field

6.2 Strain Field Analyses

The maximum shear strain on the section 10 mm inside the transparent soil model is shown in Figure 7 for the first loading stage, where the peak shear strain regions were shaded. A punching shear failure mode occurred in the top "sand" layer and extended downward to the surface of the lower clay layer. In the bottom "clay" layer, the shear strain developed along the envelope assumed by Terzaghi (1943) and Vesic (1975). The failure model was similar to that reported by Meyerhof (1974) for a footing on thin sand overlying clay, which is the problem being simulated in this study.

A typical 3-D view of maximum shear strain field is shown in Figure 8, where the shear strain is the strain increment occurred at a specific loading step. No continuous failure plane was developed even at the last loading stage. However, the failure envelopes developed on the last loading stage were clearer than the first loading stage, which was consistent to their loading conditions. The last loading stage exceeded the bearing capacity of the model footing.

7 CONCLUSIONS

In this paper, a 3-D displacement field under a model footing was measured non-intrusively using transparent soil and a digital image processing technique. A distinctive laser speckle pattern generated by the interaction between the laser light and transparent soil was used in this study. An advanced digital image crosscorrelation algorithm named adaptive cross-correlation was used to obtain the relative displacement between two consecutive images captured before and after the deformation. A comparative study found that transparent soil generates similar displacement fields with natural soil, though some discrepancies exist in magnitude due to the differences in material's properties. A soil profile consisting of "sand" overlying soft "clay" was simulated in the model test. The deformation and strain field under the model footing were similar to those published. The results show that transparent soil and developed optical system is suitable for studying 3-D deformation measurements in geotechnical engineering.

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Figure 7. Typical maximum shear strain field



Figure 8. A 3-D maximum shear strain field