Multi-dimensional finite strain consolidation theory: modeling study

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ABSTRACT
A multi-dimensional finite strain consolidation theory for self-weight quiescent conditions was developed. The theory assumes that pore fluid can flow in any direction and pore pressure dissipation only results in vertical deformation. A model study of the theory was developed for both two and three dimensional conditions. A two-dimensional simulation of a pond beach shows that the steeper the beach slope the faster the rate of consolidation near the beach while further away to the middle of the pond a one-dimensional condition prevails. Results also show that a realistic surface slope only has a small influence on the simulated rate of consolidation and bottom drainage is the most effective drainage boundary. For a three-dimensional simulation, a cylinder-cone shape containment as in a thickener vessel is simulated and results indicate that the initial rate of compression is faster than that for the one and two dimensional cases.

RÉSUMÉ
Une théorie finie multidimensionnelle de consolidation de contrainte pour des états tranquilles de art de l'auto-portrait-weight a été développée. La théorie suppose que le fluide de pore peut entrer dans tous les résultats de direction et de dissipation de pression de pore seulement dans la déformation verticale. Une étude modèle de la théorie a été développée pour les conditions bidimensionnelle et tridimensionnelles. Une simulation bidimensionnelle d'une plage d'étang prouve que plus la pente de plage est raide plus la vitesse de la consolidation près de la plage tandis qu'est rapide autre loin au milieu de l'étang un état un dimensionnel règne. Les résultats prouvent également qu'une pente extérieure réaliste a seulement une petite influence sur le taux simulé de consolidation et drainage du fond est la frontière de drainage la plus efficace. Pour une simulation tridimensionnelle, une retenue de forme de cylindre-cône comme dans un navire d'épaississant est simulée et les résultats indiquent que le taux initial de compression est plus rapide que celui pour celle et des cas bidimensionnels.

1 INTRODUCTION

The consolidation analysis of a slurry deposit is generally performed by the one-dimensional finite strain consolidation theory developed by Gibson, England and Hussey (1967). The governing equation of this theory in terms of void ratio is expressed as Equation 1.

$$\pm\left(\frac{\rho_s}{\rho_f}-1\right)\frac{d}{de}\left[\frac{k(e)}{1+e}\right]\frac{\partial e}{\partial z} + \frac{\partial}{\partial z}\left[\frac{k(e)}{\rho_f(1+e)}\frac{d\sigma'}{de}\frac{\partial e}{\partial z}\right]+\frac{\partial e}{\partial t}=0 \qquad [1]$$

Where ρ_s is solids density, ρ_f is fluid density, z is a material coordinate, $k(e)$ is vertical hydraulic conductivity, e is void ratio and σ' is vertical effective stress.

The one-dimensional approach is usually valid because most containment ponds have small depth compared to the width and length of the pond. The pore fluid flow direction and the settlement are thus primarily vertical and the one-dimensional assumption is suitable. However, in special cases, deposition of slurry in a containment pond with different dimensions and drainage conditions can result in a more complex multi-dimensional flow. This has an implication on the dewatering time of the slurry therefore a multi-dimensional consolidation model becomes necessary.

Departing from a one-dimensional finite strain consolidation theory, Somogyi et al. (1984) derived a quasi-two-dimensional finite strain consolidation model parallel to the one-dimensional derivation presented by Koppula (1970). One case history was presented by Somogyi et al. (1984) who indicated that quasi-two-dimensional finite strain consolidation can provide accurate estimates of full-scale behavior. In their analysis, the one-dimensional finite strain consolidation was also used and it was reported that the one-dimensional solution greatly underestimates the actual performance. The two-dimensional simulation also underestimated initially before inclusion of seepage stress. The influence of two-dimensional fluid flow only became significant when the width to height ratio was in the order to 5 or less.

Huerta and Rodriguez (1992) presented a pseudo bi-dimensional extension of the one-dimensional finite strain consolidation theory. They used the extended model to simulate the influence of vertical drains and showed that the drain influence is obviously more important during the first stages of consolidation because it induces large pore pressure gradients localized close to the wells.

Recently, a two-dimensional analysis of sedimentation and consolidation in various shapes of a thickener was presented by Bürger et al. (2004). An important assumption that the volumetric solids concentration is constant across each horizontal cross section was drawn. A result of this model thus gives a flat interface meaning that self-leveling was implemented through the above assumption. The simulation of a cone shape containment vessel indicates that settling velocity is decreased, different final settlements are yielded and a faster rate of sediment growth is obtained when compared to a cylinder shape containment. This approach however does not explicitly consider horizontal pore water flow.

In this paper, a quasi-multi-dimensional finite strain consolidation equation was derived by following the one-dimensional derivation by Schiffman (2001) and the equations were numerically implemented in both two and three dimensional problems. The models were then used in several deposition schemes to evaluate the initial rate of slurry consolidation which was affected by containment geometry and drainage conditions.

2 QUASI-THREE-DIMENSIONAL FINITE STRAIN CONSOLIDATION THEORY

The quasi-three-dimensional finite strain consolidation theory derived in this study is based on two major assumptions. The theory assumes that pore water can flow in any direction and pore pressure dissipation causes only vertical deformation. This section provides a short derivation of the quasi-three-dimensional finite strain consolidation theory.

In a three-dimensional condition, a small element in the soil can be illustrated in Figure 1 for time of t = 0 and t > 0. It is noted that a, b and c are Lagrangian coordinate and ξ is convective coordinate.

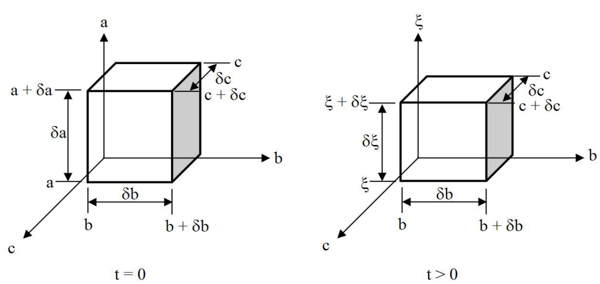

Figure 1. Three-dimensional configuration of soil under consolidation

Another important coordinate for finite strain consolidation modeling is a material coordinate, z. The material coordinate is used to describe consolidating soil layer at anytime in term of volume of solids. Mathematical expressions for vertical coordinate transformation between Lagrangian, convective and material coordinates are expressed as Equations 2, 3 and 4. Details of these coordinate systems are given by Schiffman (2001).

$$\frac{\partial a}{\partial \xi} = \frac{1+e_0}{1+e} \qquad [2]$$

$$\frac{\partial z}{\partial a} = \frac{1}{1+e_0} \qquad [3]$$

$$\frac{\partial \xi}{\partial z} = 1+e \qquad [4]$$

Where e is a void ratio at time t and e_0 is an initial void ratio.

2.1 The Conservation of Mass of Pore Water

Pore water flow in a soil element is illustrated in Figure 2 showing three-dimensional water flows into an element (*L'M'N'O'P'Q'R'S'*) and anytime t. In a vertical direction, the pore water flows into the element with an influx $J_{wv}(\xi,t)$ across the face (*N'O'R'S'*) and flows out of the element, across (*L'M'P'Q'*) with an efflux $J_{wv}(\xi+\delta\xi,t)$. Horizontal influxes and effluxes are similarly expressed in Figure 2.

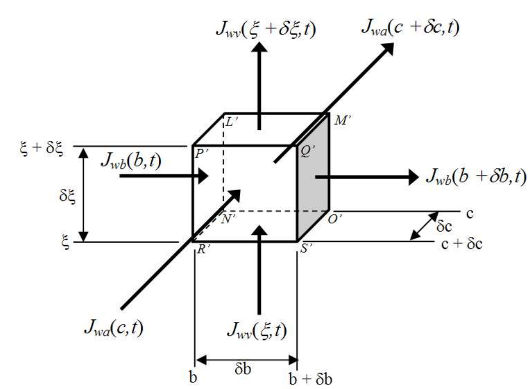

Figure 2. Flux of pore water through the representative elementary volume

By considering a conservation of mass of water and a manipulation of coordinate systems, an equation of continuity for the pore water is expressed as Equation 5.

$$\frac{\partial}{\partial a}\left(n\left(v_{wv}-v_{sv}\right)\right)+\frac{\partial}{\partial b}\left(nv_{wb}\right)\left(\frac{1+e}{1+e_0}\right) + \frac{\partial}{\partial c}\left(nv_{wc}\right)\left(\frac{1+e}{1+e_0}\right) = -\frac{\partial}{\partial t}\left(n\frac{\partial \xi}{\partial a}\right) \qquad [5]$$

Where n is porosity, v_{sv} is vertical solids velocity, and v_{wv}, v_{wb} and v_{wc} are vertical, horizontal b and horizontal c direction fluid velocities respectively.

2.2 The Conservation of Mass of Solids

The continuity of the flow of solids can be developed in the same manner as for the pore fluid by replacing n with ($1 - n$) and v_w by v_s and γ_w by γ_s in Equation 5. By considering solids velocity in the horizontal directions to be zero, the equation of continuity for the solids can be expressed as Equation 6.

$$\frac{\partial}{\partial t}\left[(1-n)\frac{\partial \xi}{\partial a}\right]=0 \qquad [6]$$

2.3 The Continuity of Mixtures

By combining Equations 5 and 6, considering Darcy-Gersevanov flow relationship in all directions, and transforming vertical Lagrangian coordinate to material coordinate, z, Equation 7 is obtained.

$$\begin{aligned}&\frac{1}{(1+e_0)}\frac{\partial}{\partial z}\left[\frac{-k_v}{\gamma_w(1+e)}\frac{\partial u}{\partial z}\right]\\&+\left(\frac{1+e}{1+e_0}\right)\left\{\frac{\partial}{\partial x}\left[\frac{-k_h}{\gamma_w}\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{-k_h}{\gamma_w}\frac{\partial u}{\partial y}\right]\right\}\\&+\frac{\partial}{\partial t}\left[\frac{1+e}{1+e_0}\right]=0\end{aligned} \qquad [7]$$

Where u is excess pore pressure and k_v and k_h are vertical and horizontal hydraulic conductivity.

Applying the principle of effective stress into the last term of Equation 7 and rearranging the resulting equation, the governing equation of quasi-three-dimensional finite strain consolidation in terms of excess pore pressure is obtained in Equation 8.

$$\begin{aligned}&\frac{\partial}{\partial z}\left[\frac{-k_v}{\gamma_w(1+e)}\frac{\partial u}{\partial z}\right]\\&+(1+e)\left\{\frac{\partial}{\partial x}\left[\frac{-k_h}{\gamma_w}\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{-k_h}{\gamma_w}\frac{\partial u}{\partial y}\right]\right\}\\&+\frac{de}{d\sigma'}\left[(Gs-1).\gamma_w\frac{\partial \Delta z}{\partial t}-\frac{\partial u}{\partial t}\right]=0\end{aligned} \qquad [8]$$

3 QUASI-TWO-DIMENSIONAL FINITE STRAIN CONSOLIDATION THEORY

A quasi-two-dimensional finite strain consolidation theory can be derived similarly to the above derivation. The two-dimensional theory is a reduced form of the three-dimensional theory by neglecting one horizontal coordinate. Thus the governing equation of a quasi-two-dimensional finite strain consolidation in terms of excess pore pressure can be expressed as Equation 9.

$$\begin{aligned}&\frac{\partial}{\partial z}\left[\frac{-k_v}{\gamma_w(1+e)}\frac{\partial u}{\partial z}\right]+(1+e)\frac{\partial}{\partial x}\left[\frac{-k_x}{\gamma_w}\frac{\partial u}{\partial x}\right]\\&+\frac{de}{d\sigma'}\left[(Gs-1).\gamma_w\frac{\partial \Delta z}{\partial t}-\frac{\partial u}{\partial t}\right]=0\end{aligned} \qquad [9]$$

4 CONVENTIONAL 3D THEORY

To prove that the quasi-three-dimensional theory is a generalized form of the conventional Terzaghi three-dimensional consolidation theory (small strain), this section reduces the three-dimensional finite strain consolidation as following.

By transforming material coordinates to the Lagrangian coordinate system (with an initial height of Z), Equation 8 changes to Equation 10.

$$\begin{aligned}&(1+e_0)\frac{\partial}{\partial Z}\left[\frac{-k_v(1+e_0)}{\gamma_w(1+e)}\frac{\partial u}{\partial Z}\right]\\&+(1+e)\left\{\frac{\partial}{\partial x}\left[\frac{-k_h}{\gamma_w}\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{-k_h}{\gamma_w}\frac{\partial u}{\partial y}\right]\right\}\\&+\frac{de}{d\sigma'}\left[(Gs-1).\gamma_w\frac{\partial \Delta z}{\partial t}-\frac{\partial u}{\partial t}\right]=0\end{aligned} \qquad [10]$$

For a small strain theory e_0 is approximately e and hydraulic conductivity is constant in every direction. By neglecting changes in total stress and rearranging Equation 10, the Terzaghi three-dimensional consolidation theory is obtained in Equation 11.

$$\frac{\partial u}{\partial t}=C_v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial Z^2}\right) \qquad [11]$$

5 NUMERICAL MODELING

In order to solve the governing equation, two constitutive relationships which are effective stress-void ratio and void ratio-hydraulic conductivity are required. Conventionally, a power law is assigned and the expression for both compressibility and hydraulic conductivity relationships are shown as Equations 12 and 13 respectively.

$$e = A\sigma'^{B} \qquad [12]$$

$$k = Ce^{D} \qquad [13]$$

Where A, B, C and D are laboratory determined parameters referred to as finite strain parameters.

Equations 12 and 13 are used for the quasi-multi-dimensional finite strain consolidation modeling in this article. It is noted that other constitutive relationships can also be applied in a similar manner for both compressibility and hydraulic conductivity relationships and the choices of the relationships are largely dependent on the compressibility and hydraulic conductivity behavior of the subject slurry.

Models of the quasi-multi-dimensional theory were coded in FlexPDE version 4.2.16 finite element program. The derivation of the governing equations utilized the material coordinate. This provides convenience in solving the equations in the FlexPDE program as the coordinate stays unchanged during the consolidation process. In order to transform material coordinate back to Lagrangian or convective coordinate systems, Equations 2 and 3 are used.

Due to the fact that FlexPDE performs many operations to setup a problem and solve differential equation systems, a numerical experiment was performed by comparing a one-dimensional finite strain consolidation model by finite difference methods (programmed in Visual Basic) and a quasi-multi-dimensional finite strain consolidation model (one-dimensional equivalent) in the FlexPDE program. Results indicated that both models are identical. Therefore, the modeling study of the multi-dimensional consolidation theory was performed with the FlexPDE code for the remaining of this paper.

6 TWO-DIMENSION MODEL STUDY

The analysis of the geometry of a pond in this section was performed by using oil sands fine tailings as a study material. The material parameters used are solids content = 30.6%, specific gravity = 2.28, depth = 1.0m and the finite strain parameters are A = 28.71, B = -0.3097, C = 7.538×10^{-11}, and D = 3.824 (units in Pa and m/s). Two factors were investigated: containment shapes and drainage conditions. The consolidation period was chosen to be 5 years at which time the excess pore pressure was considered negligible for this particular material.

6.1 Containment Shape

Two cross sectional geometries were examined. One is a rectangular shape and the other is a trapezoidal shape. Only surface drainage was allowed.

Investigation of the width to height ratio (W:D) of a rectangular pond (Figure 3) was performed by using a W:D ratio of 0.5, 1, 2, 3 and 4. The simulated pond was assumed to have a very large length to width therefore two-dimensional performance can be assumed.

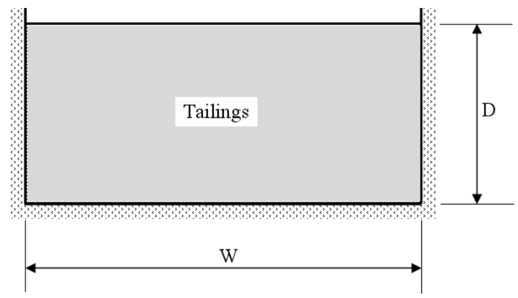

Figure 3 Rectangular cross section containment

Simulations of averaged void ratio at the middle of the pond with time for different W:D ratios show identical results (Figure 4). As expected, changing of the W:D ratio does not affect the predicted rate of consolidation for a rectangular cross sectional shaped pond.

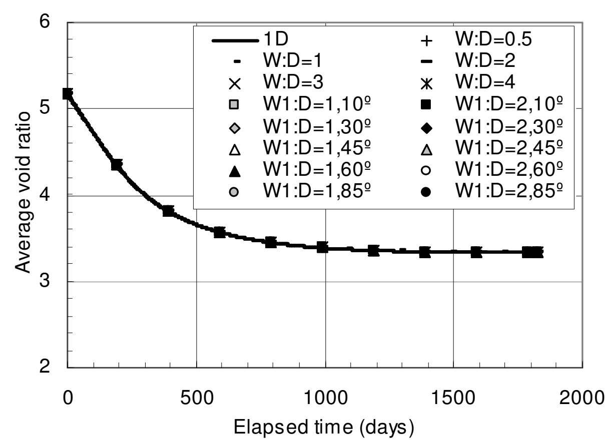

Figure 4 Average void ratios vs. time at the middle of a rectangular and trapezoidal shaped pond

A trapezoidal cross section pond is shown in Figure 5. In Figure 5, the beach angle, α, is defined as the angular slope of the beach. In order to examine the effect of beach angle to rate of change of void ratio at the center, the depth of the pond was set to be constant and the beach angle was varied from 10° to 85°. For each of the beach angles, the W_1:D ratio was also varied from 0 to

2 to study the effect of the base width related to the drainage length from the center to the beach.

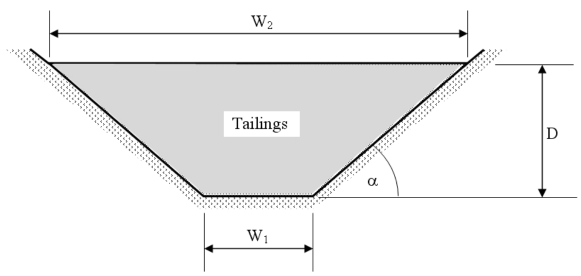

Figure 5 Trapezoidal cross section containment

Figure 4 compares the average void ratio change with time for W_1:D = 1 and 2 and the beach angle varying from 10° to 85°. The results in Figure 4 show that at the middle of the pond, the rate of reduction of void ratio is not affected by the slopes for W_1:D≥1. However, if the base is reduced to zero (Figure 6), the average void ratio change with time is affected by the beach angle.

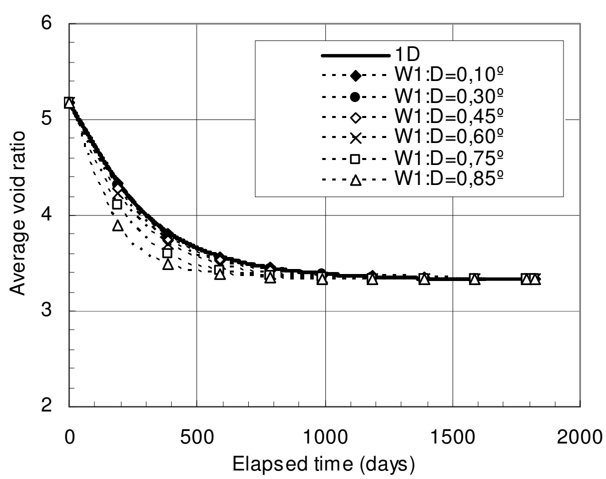

Figure 6 Average void ratio changes vs. time (base width of 0)

To compare the rates of consolidation of the trapezoidal shape in Figures 4 and 6, the percent increase of rate of change of initial compression rates were plotted with beach angle in Figure 7. The base consolidation rate in the comparison is the rate for the one-dimensional condition.

In Figure 7, the 2*D* consolidation model for the trapezoidal shaped pond shows that the higher the beach angle the faster the initial rate of consolidation. The case of W:D_1=0 provides a faster initial rate of consolidation than that for W:D_1>0. The reason for the faster consolidation rate in the W:D_1=0 case is because the shorter drainage path elements (elements on the beach) are located closer together and the consolidation of these elements are relatively faster compared to the center because of the length of the drainage path. Due to lower excess pore pressures in these elements compared to the center, they allow excess pressure at the center to dissipate horizontally and cause faster consolidation close to the beach. In Figure 7, the effect of horizontal dissipation can also be seen for cases other than the base width of zero. At the toe of the beach the rate of initial consolidation is increased as the beach angle increases for the W:D_1 ratio of 1 and 2.

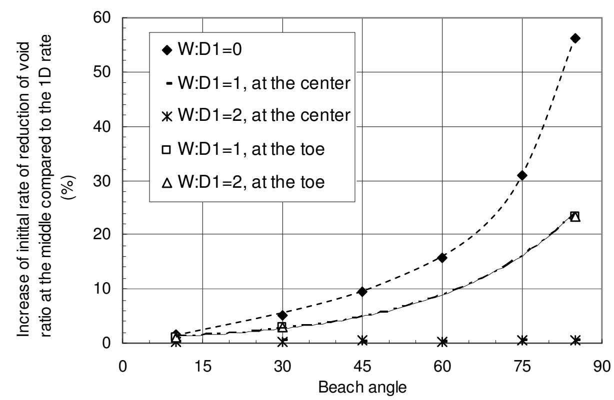

Figure 7 Percent increase of initial rate of reduction in void ratio vs. beach angle

To illustrate the behavior of the quasi-2*D* finite strain consolidation theory, the void ratio and excess pore pressure distributions for the trapezoidal shaped pond are presented in Figures 8 and 9 respectively for four elapsed times. The simulations shown in Figures 8 and 9 were plotted in a convective coordinate system allowing the void ratio profiles to be viewed in a real sense. It is noted that the pond height is 1m, base length is 1m and beach slope is 45° at both sides.

Figures 8a) and 9a) show the beginning of consolidation. The dissipation occurs at the edges at the top where the drainage path is the shortest and the effective stress is the smallest (Figure 8a)). During the consolidation process, the dissipation occurs faster in the triangular sections compared to the middle section (Figures 9b) 9c) and 9d)). This is because self-weight consolidation always propagates from bottom up due to the triangular stress distribution in the soil mass. Therefore the model predicts that at the same elevation or the same initial self-weight stress, the soil in the triangular area will be able to finish consolidating quicker because the dissipation front reaches the specific soil element faster. Consolidation was nearly completed in Figures 8d) and 9d). The void ratio profiles indicate a bottom void ratio of about 2.5 and a surface void ratio of about 5.17 (the initial surface void ratio).

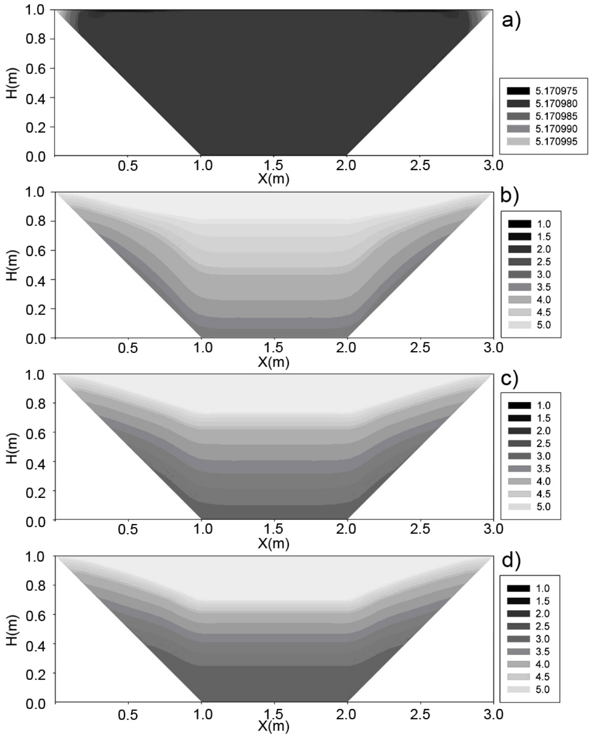

Figure 8 Void ratio contours with time in trapezoidal shape pond a) 0.1 days, b) 240days, c) 590 days, and d) 1825 days.

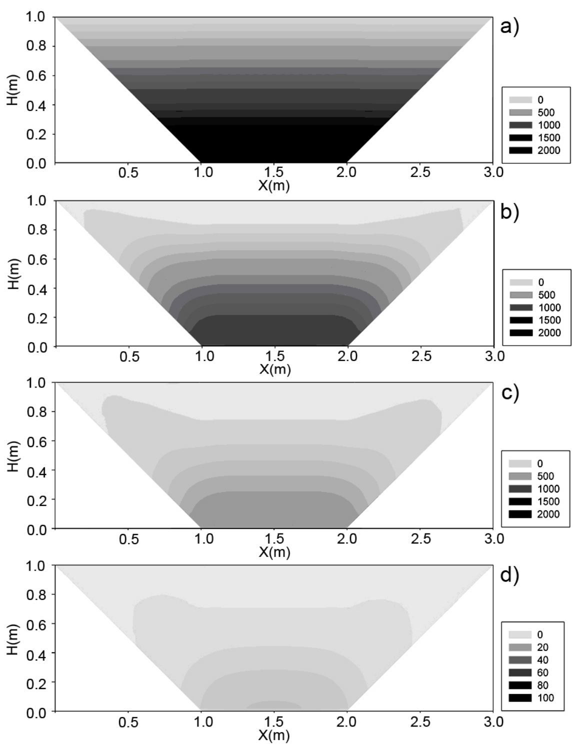

Figure 9 Excess pore pressure contours with time in trapezoidal shape pond a) 0.1 days, b) 240days, c) 590 days, and d) 1825 days.

6.2 Drainage Conditions

To study the effect of drainage through dykes and foundation materials, different boundary conditions were setup. To eliminate other shape factors, a 2*D* rectangular cross section pond was selected. The simulation results are shown in Figures 10 and 11 at the center of a pond and at the middle between the wall and the center of a pond respectively. In Figures 10 and 11, DD, FD and AD stand for dyke drainage, foundation drainage and all around drainage boundary conditions.

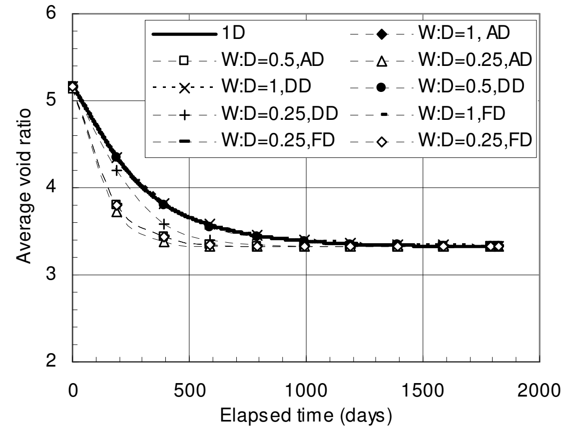

Figure 10. Average void ratios vs. time for three drainage conditions at the center of a pond

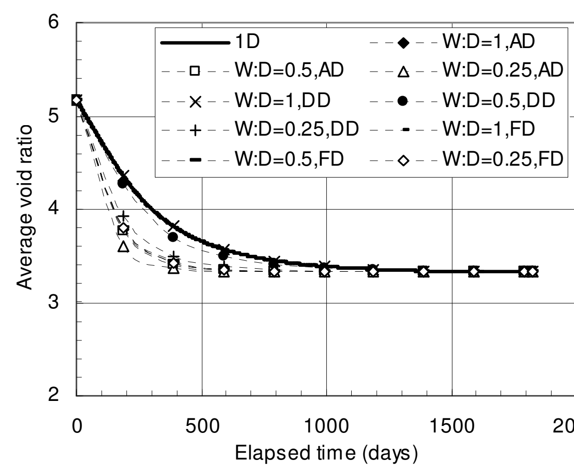

Figure 11. Average void ratios vs. time for three drainage conditions at the middle between the wall and the center of the pond

The influence of the drainage boundaries in Figures 10 and 11 is summarized in Figure 12 showing the percent increase of initial rate of reduction in void ratio for all three drainage conditions. The model behavior shows that the all around drainage boundary condition gives the highest initial rate of consolidation and that foundation drainage is a more influential boundary condition compared to dyke drainage. This is due to the distribution of the self-weight stress. At the center of the pond, the drainage through the foundation can increase the initial rate of reduction in void ratio more than 65% for all base widths while the drainage through the dyke increases the rate slightly under 20%. The effect of the drainage through the dyke can become more pronounced as the W:D ratio decreases (Figure 12).

It is concluded that the simulation shows that drainage through the foundation is more influential than the dyke drainage and is unaffected by the W:D ratio. An increase in the rate of consolidation by dyke drainage can be obtained by decreasing the width of the pond.

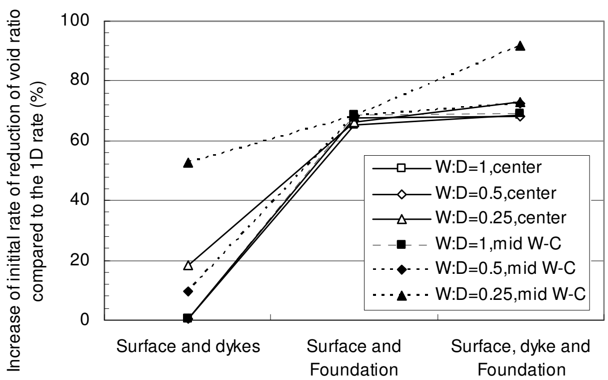

Figure 12. Percent increase of initial rate of reduction of void ratio at the center of the pond and at the middle between the wall and the center of pond (mid W-C) for different drainage conditions

6.3 Surface Slope

The pond surface slope can be controlled by changing the tailings properties and is a viable method to enhance lateral drainage. In this section the pond surface slope, β, is varied between 0° to 10°. The pond section used in this exercise is also a rectangular shape to eliminate other shape factors (Figure 13).

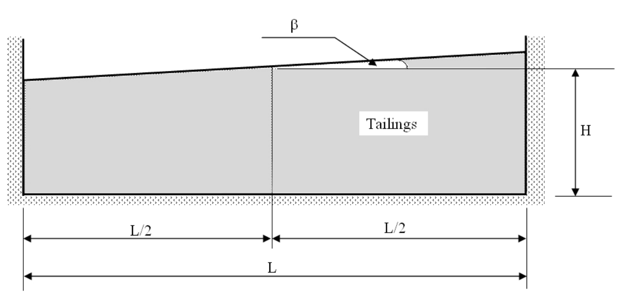

Figure 13. Surface sloping pond

Numerical experiments were performed by varying the surface slope and L with a constant H. It was found that all simulations gave negligible changes (<0.5%) compared to the 1*D* case. This is possibly because the combination of the width and angle creates high and low pressure areas at the upstream and downstream areas of the pond. The steeper the angle, even though it gives shorter horizontal drainage paths, it also creates higher excess pore pressure in the upstream side of the center. Therefore the evaluation of the surface slope should be evaluated at the same controlled stress and therefore the investigation was changed to fix the upstream depth and vary the surface angle and the length of the pond. Results are shown in Figure 14 and the percent increase of the rate of consolidation with the surface slope angle is shown in Figure 15.

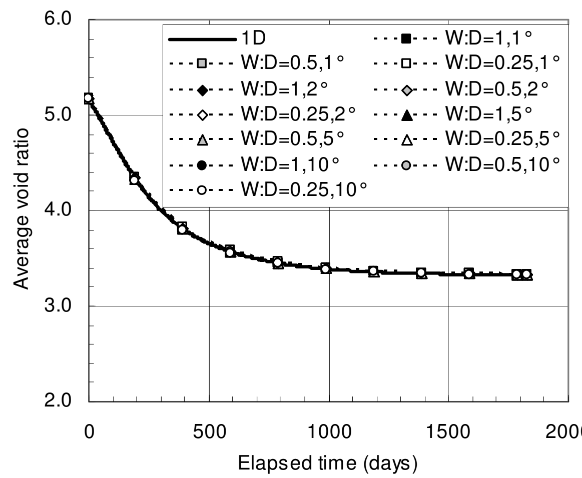

Figure 14. Average void ratios vs. time for various W:D ratios and surface slope angle at the upstream bound

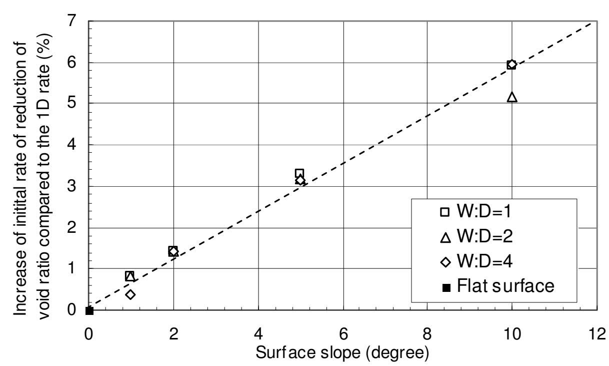

Figure 15. Percent increase of initial rate of reduction in void ratio vs. surface slope for different W:D ratios at the upstream bound

Figure 15 indicates that the steeper the slope the faster the rate of consolidation. This is because the pore pressure is controlled to be the same but the pore water can flow both vertically and horizontally along the length of the pond. With the steeper slope surface, a shorter horizontal drainage path will allow excess pore pressure to dissipate more quickly. This has a slight advantage over a flat surface geometry which shows the same rate as the 1*D* condition. It is noted that the effect of horizontal drainage is also affected by the horizontal hydraulic conductivity. Generally horizontal hydraulic conductivity is higher than vertical hydraulic conductivity but in this example, the hydraulic conductivity is assumed to be equal in both directions. The influence of the surface drainage could be higher in a real case when the horizontal hydraulic conductivity is specified. It is concluded that the rate of consolidation is slightly increased by the increase of a tailings surface slope.

7 THREE-DIMENSION MODEL STUDY

This section further examines the earlier finding that the beach angle has an influence on the simulated consolidation rate in a three-dimensional analysis. This is done by comparing a cylinder-cone shape containment (Figure 16) with the 2*D* trapezoidal results. The cone has an angle with the vertical of 30° and is 0.8m high and the cylinder on the top is 0.2m high.

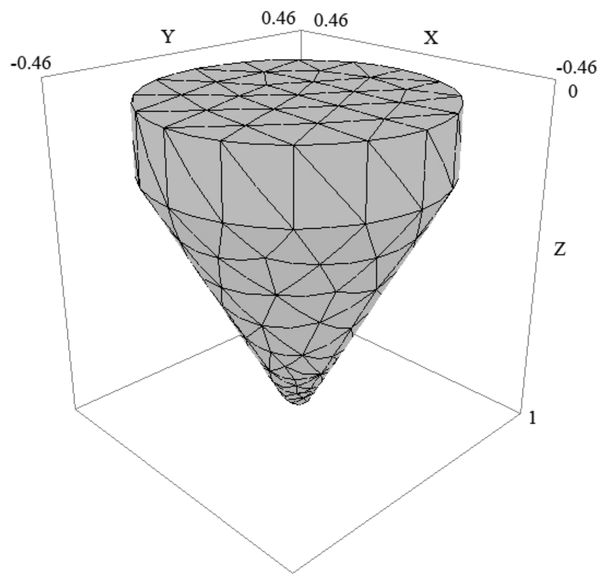

Figure 16. A cylinder-cone shape containment

The consolidation behavior in the containment vessel shows fast consolidation at the tip of the cone. The rate of compression at the center is compared with the 1*D* and 2*D* ponds in Figure 17. Simulation of the 3*D* cylinder cone shape containment gives an increase in consolidation rate of 20% compared to the 1*D* condition while the 2*D* V-shape pond with the same beach angle gives a 15% increase. The cause of the increase in the rate of the 3*D* model is the same as indicated in the 2*D* pond in which excess pore pressure dissipated horizontally near the angled beach. However, for the cone shape containment, the water can flow in all three-dimensional directions contributing to the further increase in the rate of reduction of void ratio.

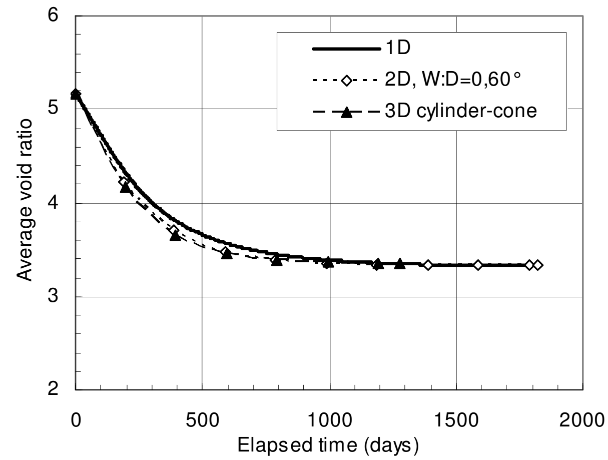

Figure 17. Comparison of rate of reduction in average void ratio for a 3*D* cylinder-cone containment

8 DISCUSSION AND CONCLUSIONS

The quasi-three-dimensional finite strain consolidation theory was derived in a form of excess pore pressure. The major assumptions used in the theory are that pore fluid is permitted to flow in any direction and deformation is strictly vertical. Mixing, flowing, sliding and shearing are neglected. The equations were coded in a FlexPDE finite element program which was implemented for modeling in 2*D* and 3*D* consolidation problems.

Based on the derived theory, a comparative study was performed to investigate the consolidation model's behavior of slurry for ponds with various shape and boundary conditions for quiescent condition. Only one driving mechanism, self-weight stress, was examined in all simulations.

The 2*D* consolidation model studies show that there is no shape effect for a rectangular shaped containment pond and is no different than 1*D* consolidation behavior. For V-shaped cross sectional ponds, the steeper the beach slope, the better the rate of consolidation. For trapezoidal shaped ponds, the beach slope does not show a significant effect at the center of the pond. However at the toe of the beach, it can be observed that the steeper the beach the better the rate of consolidation. The surface slope of the pond is found to give a small increase in the rate of consolidation due to the shorter horizontal drainage path. The larger the angle of the surface slope of the pond, the faster the initial rate of consolidation.

Drainage from all boundaries is the ideal case for slurry dewatering. Drainage through the foundation has a pronounced influence on the initial rate of consolidation while drainage through a dyke has negligible effect at the middle of a pond. In an area between the dyke and the center of a pond, the dyke drainage, however, helps to increase the rate of consolidation. The smaller the base width the more the effect of the drainage through dykes. It is noted that blinding of the foundation drainage could occur in a longer term and was not studied here.

The shape of a cylinder-cone containment vessel provides an increase in the initial rate of reduction in void ratio. The increased rate is faster than the 2*D* V-shaped pond with the same slope. This is due to three-dimensional flow.

Several aspects of the theory were investigated and the importance of containment shapes and drainage conditions on the simulated rate of consolidation were shown. The theory precludes several aspects of slurry consolidation that can be significant in some cases. Future extension of the quasi-multi-dimensional finite strain theory should be aimed to include 3*D* self-weight stress analysis, horizontal deformation and, flowing and sliding mechanisms.

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