Probabilistic slope stability analysis of the Red River Floodway channel

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ABSTRACT
The Red River Floodway is the primary line of flood protection infrastructure for the City of Winnipeg. A multi-million dollar project to expand the Floodway to provide increased levels of flood protection is currently underway. The current study is part of an ongoing research project to conduct a probabilistic stability assessment of the expanded floodway channel slopes. The current paper describes a Monte-Carlo simulation program that was developed for use with commercially available geotechnical software. The probabilistic results were used in conjunction with traditional contouring methods to select a probabilistically critical failure slip surface from a grid of potential candidates. The calculated annual probability of failure at each section can be used to relate the reliability of the channel side slopes to that of the overall flood protection system.

RÉSUMÉ

Le canal de dérivation de la rivière Rouge est la source primaire de protection contre les inondations pour la ville de Winnipeg. Un projet, de plusieurs millions de dollars, pour s'étendre le canal de dérivation est maintenant en marche; ceci fournira une meilleure protection contre les plus hauts niveaux d'inondations. Cette étude courante fait partie d'un projet de recherche qui mènera une évaluation probabiliste de stabilité des pentes du canal étendu. Ce papier décrit un programme de la méthode Monte-Carlo développer pour l'utilisation avec un logiciel géotechnique. Les résultats probabilistes ont été utilisés avec les méthodes de contournage traditionnelles pour sélectionner une surface d'éboulement critique d'un réseau de candidats potentiels. La probabilité d'éboulement annuel de chaque section peut être utilisé pour relier la confiance de talus du canal et le système de protection contre des inondations.

1 INTRODUCTION

Uncertainties in geotechnical design have traditionally been safeguarded against by maintaining a prescriptive allowance in the factor of safety above unity. In continuing to doing so, we fail to systematically assess and account for the various sources of uncertainty. The deterministic factor of safety alone contains no information regarding the level of conservatism, uncertainty or reliability of a designed structure. Structures with varying degrees of soil variability, groundwater fluctuations or differing methods of measurement, monitoring or construction may be designed with the same factor of safety, resulting in differing levels of long term reliability.

The Red River Floodway is an excellent example of a large scale geotechnical project that has been designed using traditional deterministic analyses. The Floodway was originally constructed in the 1950's and is currently undergoing expansion works to provide additional flood relief. The assumed material properties and boundary conditions for design were selected based on subjective analysis of available laboratory testing and monitoring data. As a result, the level of conservatism inherent in the selected values for each of the input parameters is both unknown and likely inconsistent between the parameters. Furthermore, since the deterministic slope stability model yields no information regarding the reliability of the channel side slopes, the reliability of the flood diversion system to convey the design flood event cannot be fully quantified.

The objective of the current study was to provide an estimate of the probability of failure for the channel side slopes in order to relate the reliability of the channel to the reliability of the flood protection system as a whole. The study involved the development of a deterministic slope stability model as well as statistical distributions for its input parameters. Together, the stability model and the statistical distributions form the inputs to a probabilistic simulation. The focus of this paper is on the development of a probabilistic simulation technique and its application to probabilistic slope stability modeling of the Red River Floodway channel.

2 SITE DESCRIPTION

The Red River Floodway is approximately 48 km long and extends east around the City of Winnipeg from its upstream inlet from the Red River located south of the city to its downstream outlet to the north. The majority of the existing channel is excavated at a 6H:1V side slope in surficial glaciolacustrine clays typical of the Red River basin, as shown in Figure 1 for cross-sections at various locations along the channel. The cross-sectional geometry shown was selected based on a prescriptive design allowance in the factor of safety for the channel. The same side slopes were selected for the expanded channel based on deterministic modeling conducted during the Floodway Expansion Project pre-design phase.

The glaciolacustrine clays are typically grey, soft to firm, highly plastic and normally to lightly over-consolidated in their unweathered state. The upper few metres of the deposit are typically weathered due to moisture and temperature variations that occur seasonally near the surface. The weathered crust is typically brown, firm to stiff, medium to high plastic, lightly to highly over-consolidated with some presence of silt lensing and stratification nearest the ground surface. Underlying the glaciolacustrine clay is a layer of glacial till, followed by a highly-fractured and weathered upper Carbonate bedrock aquifer.

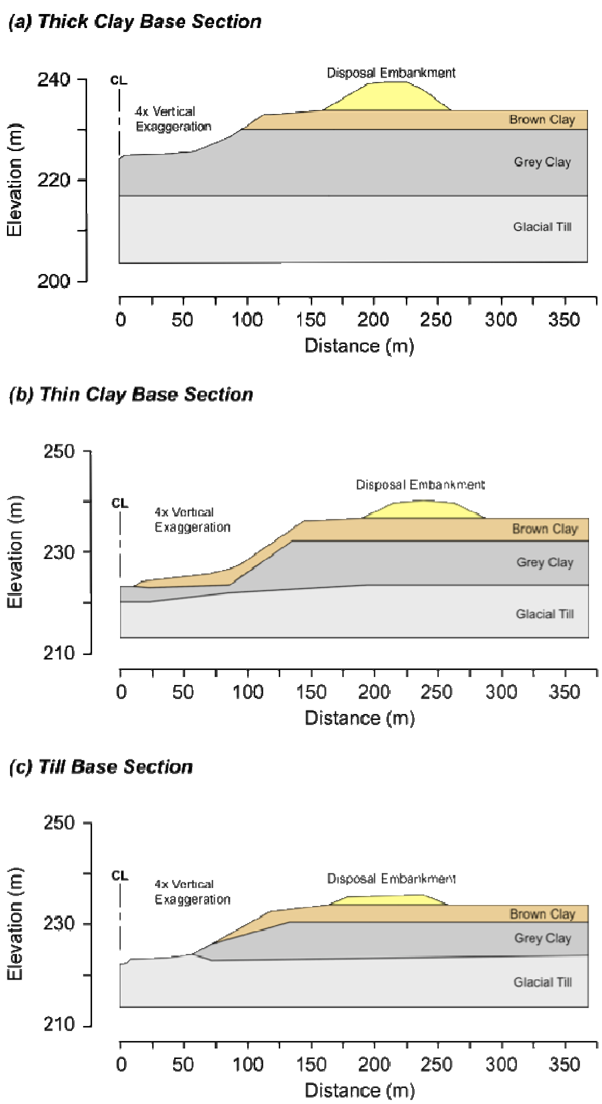

Figure 1. Cross-sectional Geometry and Stratigraphy

2.1 Site Investigation and Monitoring Program

The site investigation program conducted as part of the Floodway expansion project consisted of drilling, sampling, laboratory testing and groundwater monitoring components, which were primarily focused on characterizing the properties of the glaciolacustrine clays and the groundwater conditions within the clay, till and bedrock layers. Laboratory testing included 84 triaxial tests and 21 oedometer tests conducted on undisturbed specimens obtained from the glaciolacustrine clays using thin-walled Shelby tube cylinders.

Pneumatic and standpipe piezometers were installed to monitor the pore-water pressures in the clay and till units. Piezometric head in the underlying Carbonate aquifer was monitored using provincial monitoring wells that were installed prior to the original construction of the Floodway. Seasonal increases of the piezometric pressures in the Carbonate aquifer are transferred up through the permeable till layer and result in upward seepage gradients that may destabilize slopes in the lacustrine clay layer (Tutkaluk, 2000).

3 PROBABILISTIC MODEL DEVELOPMENT

In traditional slope stability analysis and design, input parameter uncertainty is accounted for by subjectively selecting acceptably conservative values based on available data and on experiential judgment. In probabilistic slope stability analysis, the input parameter uncertainty is described explicitly using probability distributions and methods such as Monte-Carlo simulation are used to infer the distribution of the output variable, the factor of safety.

3.1 Monte-Carlo Simulation Technique

Monte-Carlo simulation is a technique used in many fields to estimate statistical distributions of potential outcomes based on a statistical description of input parameter uncertainty. For example, meteorologists use Monte-Carlo simulation to estimate the uncertainty in trajectory of major storms and hurricanes based on statistical uncertainty in the moisture, temperature and atmospheric pressure conditions. Geotechnical engineers can use Monte-Carlo simulation to quantify the uncertainty in the factor of safety for slopes, given statistical distributions for input parameters such as shear strength, unit weight or pore-water pressure distributions.

Monte-Carlo simulation is a cyclical process where input variables are randomly sampled from their respective probability distributions to generate random outcomes from a numerical model. For slope stability, random combinations of each of the input variables are input to a deterministic slope stability model to compute the factor of safety. After thousands of iterations of the simulation, a distribution for the factor of safety can be obtained.

The Monte-Carlo simulation technique developed here uses a custom-built program that conducts random sampling of input variables and solves a GeoStudio 2007 (Geo-Slope International 2007) limit-equilibrium model to calculate the factor of safety for each input parameter combination. The first step in the cyclical process is random number generation, as shown in Figure 2. Random numbers between 0 and 1 are generated and represent cumulative probability distribution values for each of the input variables. The cumulative probability values are related through the respective statistical distributions to a unique value of each input variable, to form a random combination of input variables (step 2). In step 3, the combination of input parameters is saved to the slope stability model, which is solved to calculate the factor of safety. The computed factor of safety is appended to a record of the calculated values for all previously simulated input parameter combinations and used to compute statistics on the distribution of values (step 4). Probabilistic outcome quantities such as the mean and variance of the factor of safety, the probability of failure and the reliability index are all calculated based on the distribution of possible factor of safety outcomes. The process is repeated until a stable solution for the probabilistic outcome quantities is achieved.

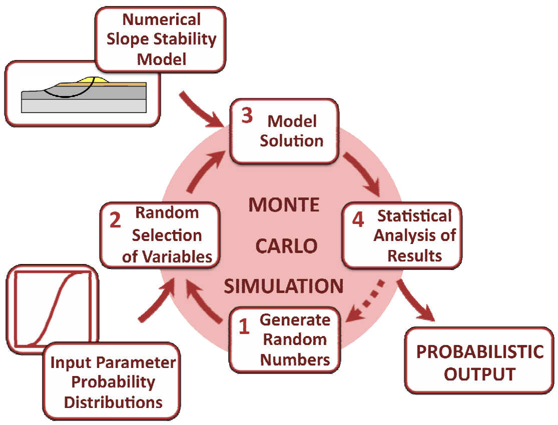

Figure 2. Flowchart of Monte-Carlo Simulation

3.2 Deterministic Slope Stability Model

The deterministic slope stability model component of the probabilistic simulation forms the mathematical relationship between the input parameters and the factor of safety. For the case of the Red River Floodway channel, finite element stress and seepage conditions were used in a limit-equilibrium model to calculate the factors of safety over a range of potential slip surfaces defined using the grid and radius method.

Assumed values for the input parameters from the pre-design phase of the expanded Floodway channel were used to calculate the factor of safety for three distinct cross-sections characterized primarily by the presence or thickness of clay beneath the base of the channel, as shown in Figure 1. The thin clay section had the lowest estimated factor of safety based on the assumptions from the pre-design phase, as summarized in Table 1. A long-term factor of safety criterion of FS = 1.5 was selected during pre-design.

Table 1. Deterministic Factors of Safety

Cross Section	Deterministic Factor of Safety
Thick Clay Base	1.858
Thin Clay Base	1.583
Till Base	2.180

3.3 Statistical Distributions for Input Parameters

The development of the statistical distributions for the input parameters will not be presented here, since it was previously reported by Van Helden and Blatz (2007). However, the resulting statistical distributions are of relevance to the current discussion on the probabilistic model. All variables were assumed to follow normal (Gaussian) distributions, which are symmetric about the mean value. The normal distribution is defined by the mean value and either the standard deviation or the variance about the mean.

In terms of large-strain shear strength parameters, the brown and grey clay are assumed to have identical cohesion and friction angle distributions, since stress history has no effect on residual strengths (Van Helden and Blatz 2007). The mean and standard deviation for the cohesion intercept and the friction angle for the clays are listed in Table 2. It should be noted that the distributions for both the cohesion and friction angle were truncated below zero, thus only positive values were permitted in the random sampling process. Despite measured correlation between cohesion and friction angles in the sample data set, it was not deemed to be physically realistic for the clay deposit. The much stronger and stiffer till layer was modeled as impenetrable in the slope stability model.

The unit weight of the clays was also selected as an uncertain input, which was defined in the finite element

Table 2. Statistical Distributions for Stability Analysis Input Variables

Parameter	Mean	Standard Deviation	Distribution Type*
Brown and Grey Clay Properties			
Cohesion Intercept (kPa)	15.30	11.41	Normal (Gaussian) Distribution
Friction Angle (degrees)	13.03	2.86	Normal (Gaussian) Distribution
Unit Weight (kN/m^3)	16.90	0.68	Normal (Gaussian) Distribution
Carbonate Aquifer Total Head			
Thick Clay Base Section (m)	225.63	0.71	Normal (Gaussian) Distribution
Thin Clay Base Section (m)	226.40	0.47	Normal (Gaussian) Distribution
Till Base Section (m)	224.15	0.50	Normal (Gaussian) Distribution

** All soil parameter distributions were restricted to positive values*

Table 3. Summary Statistics for Critical Slip Surfaces

Cross Section	Mean Factor of Safety $E[FS]$	Factor of Safety Variance $V[FS]$	Number of Simulated Failures N_f	Probability of Failure $P(FS<1)$	Reliability Index $\beta = \frac{E[FS]-1}{\sqrt{V[FS]}}$
Thick Clay Base	2.81	0.76	111	0.37%	2.08
Thin Clay Base	2.58	0.70	316	1.05%	1.88
Till Base	3.88	1.15	23	0.08%	2.69

stress component of the analysis. The mean and standard deviation of the clay unit weight are also listed in Table 2.

The annual peak piezometric head in the underlying Carbonate aquifer was considered an uncertain parameter that may act as a triggering mechanism for slope movement. Annual peak piezometric conditions are normally attributed to spring recharge of the aquifer and to flooding in the Red River. The annual peak piezometric head in the aquifer is considered the most likely time-dependent triggering mechanism for slope failure. In advance or in recession of floodwaters from the Floodway, high piezometric head in the aquifer is transferred up through the till and result in destabilizing upward seepage gradients in the clay portion of the channel side slopes.

The piezometric head in the Carbonate aquifer was applied as a total head boundary condition at the base of the model. For each random sample of the aquifer piezometric head, the finite element seepage model was solved with a new boundary condition applied at the till-bedrock interface, which resulted in a new pore-water pressure distribution within the clay each time. A separate pressure head boundary condition was applied at the ground surface to simulate a groundwater table at 1 m below the base and side slopes of the channel and 2 m below the original ground surface away from the slope.

Since the piezometric conditions vary both in time and space, the statistical distribution of the annual peak piezometric head in the underling aquifer depends on the location along the floodway channel. As listed in Table 2, the variance in the peak piezometric head near the thick clay section was highest while monitoring wells near the other two sections showed significantly less variation, likely because the channel itself acts as a relief drain to the residual uplift pressures at these sections.

4 PROBABILISTIC MODEL RESULTS

The deterministic model developed previously was used to compute a distribution of the factor of safety for 30,000 iterations of the Monte-Carlo simulation. The probability of failure was computed for the critical failure mechanism at each section, based on the calculated factors of safety. The probability of failure was calculated as a relative frequency of factor of safety values that fall below unity amongst the total number of iterations:

$$P(FS<1.0) = \frac{\text{Number of Failures } (FS<1)}{\text{Number of Iterations}} \qquad [1]$$

A histogram of the computed factor of safety is shown in Figure 3 for the thin clay section, which had the highest computed probability of failure of the three cross-sections. As shown for the probabilistically critical slip surface, 316 out of 30,000 potential outcomes were simulated where the factor of safety fell below unity, resulting in an estimated probability of failure of 1.05%.

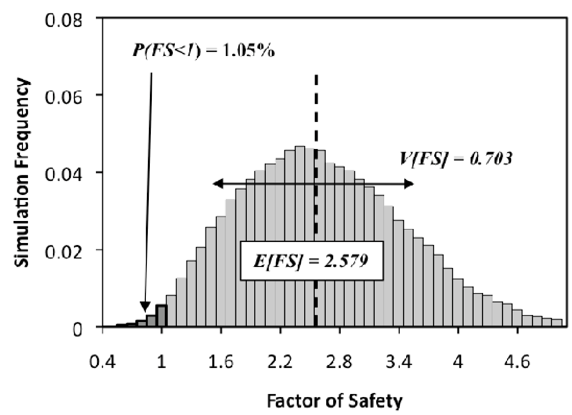

Figure 3. Thin Clay Base Factor of Safety Frequency Histogram for Probabilistically Critical Slip Surface.

The slip surface with the highest simulated probability of failure was selected as the probabilistically critical failure mechanism. Slip surfaces were defined using the Grid and Radius method available in GeoStudio 2007. Factor of safety distributions, such as the one shown in Figure 3, and summary statistics were recorded for each potential slip surface and updated throughout the Monte-Carlo simulation. Summary statistics such as the mean factor of safety, variance of the factor of safety, probability of failure and reliability index were plotted by manipulating built-in contouring functions in GeoStudio 2007. Figure 4 shows contours of the probability of failure for the grid of potential slip surfaces at the thin clay section. The contours can be used to select families of probabilistically critical failure mechanisms directly from the simulation results, resulting in a more accurate estimation of the

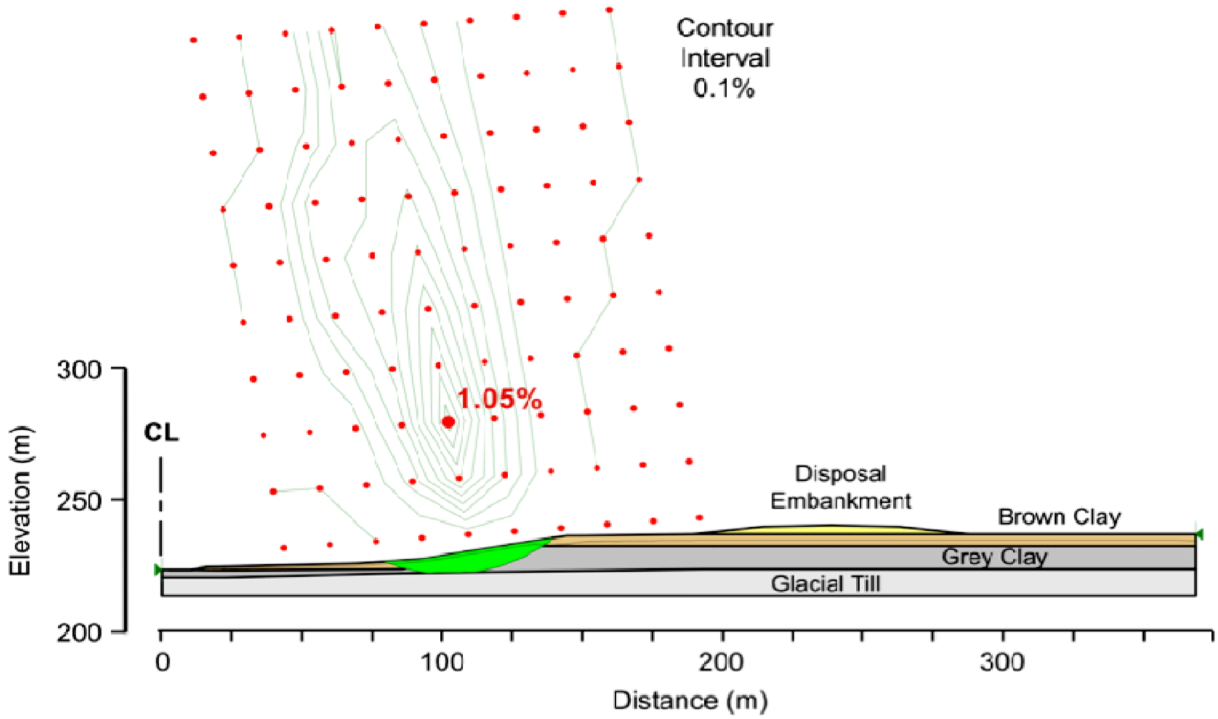

Figure 4. Contours of Probability of Failure for Thin Clay Section.

probability of failure and a more useful probabilistic modeling tool for design purposes.

Summary statistics for the critical slip surfaces from each cross-section are summarized in Table 3. The thin clay section was estimated to have the highest probability of failure amongst the three cross-sections. Thus, both the deterministic and probabilistic analyses predict that the thin clay section is critical. Although the variability in the Carbonate aquifer piezometric head was lowest at the thin clay base section, the estimated performance at this location was highly sensitive to upward seepage gradients, resulting in significantly higher uncertainty in the factor of safety.

It should be noted that the slip surface of minimum deterministic factor of safety based on design assumptions did not correspond to the maximum probability of failure. Thus, the probability of failure would be underestimated if the deterministically critical slip surface were the sole candidate selected for probabilistic analysis.

CONCLUSIONS

The current study involved the development of a Monte-Carlo probabilistic simulation program to randomly sample from statistical input parameter distributions and repeatedly solve a GeoStudio 2007 slope stability model. The probabilistic simulation program was applied at three cross-sections of the Red River Floodway to estimate the probability of failure based on variability in soil parameters and piezometric conditions.

The probabilistically critical failure mechanism was selected for each cross-section directly from a grid of slip surfaces using contours of probability of failure. The annual probability of failure for the channel side slopes can now be related to the reliability of the overall flood protection system and to the return period of the design flood event.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of the Manitoba Floodway Authority and the Natural Sciences and Engineering Research Council for this project. The technical guidance and support from Geo-Slope International was valuable in developing the Monte-Carlo simulation program for use with GeoStudio 2007.

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