A new elastic visco-plastic model for time dependent behaviour of normally consolidated and lightly over-consolidated clays

Rajibul Karim and Carthigesu T. Gnanendran
ACME, UNSW@ADFA, ACT, Australia

ABSTRACT
A new elastic-viscoplastic model is proposed in this paper to describe the rate sensitive behaviour of soft clay soils. The model is based on Bjerrum's (1967) concept of delayed compression and Perzyna's (1963, 1966) formulation of visco-plasticity and adopting Modified Cam-Clay (Roscoe and Burland, 1968) framework. The proposed model is similar to that proposed by Kutter and Sathialingham (1992) but with a modified shape for the flow surface and critical state cone in the octahedral plane. The validity of the model was varified by simulating some laboratory undrained tests and the predictions were found to be in good agreement with the laboratory (undrained triaxial) test results as discussed in the paper.

RÉSUMÉ
Une nouvelle élastique-viscoplastique modèle est proposé dans le présent document pour décrire le comportement sensibles aux taux de sols d'argile molle. Le modèle est fondé sur la Bjerrum (1967) notion de retard de compression et de Perzyna's (1963, 1966) formulation de visco-plasticité et l'adoption Mis à jour le Cam-Clay (Roscoe et Burland, 1968) cadre. Le modèle proposé est similaire à celui proposé par Kutter et Sathialingham (1992) mais avec une modification de forme pour le flux de surface et état critique dans le cône octaédral plan. La validité du modèle a été varified en simulant certains undrained tests de laboratoire et les prévisions ont été jugées en bon accord avec le laboratoire (undrained triaxial) les résultats des tests comme indiqué dans le document.

1 INTRODUCTION

The stress-strain behaviour of clayey soils is non-linear, irreversible and time dependent. The design of structures directly and indirectly on or in the clayey soils need good understanding and modeling of the time-dependent stress-strain behaviour of the soils (Yin 2001).

The time dependency of clayey soils is too significant to be ignored. Bjerrum (1967) suggested a conceptual time line model for modeling the delayed compression in 1-D straining condition. He suggested that the volumetric strain in soil is of two types: *delayed* and *instant*. The proposed model is based on the hypothesis that there is no *instant* inelastic strain and consequently all inelastic strains require time to occur.

It is observed that in the traditional consolidation tests, the time dependent deformation becomes obvious only after the completion of primary consolidation at which point the creep becomes noticeable. This does not mean that *delayed* compression does not occur during primary consolidation. A number of studies have revealed that creep occurs at higher rate initially and becomes slower with time (Kutter and Sathialingham 1992; Yin et al. 2002) that is, it becomes evident only after the pore water pressure is dissipated that is, when the hydrodynamic lag does not control the process anymore. Based on these concepts and Perzyna's (1963, 1966) formulation of viscoplasticity, Kutter and Sathialingham (1992) proposed an elastic-viscoplastic model to describe the time dependent behaviour of soils. However the formulation had the following limitations: (a) the formulation was for triaxial stress conditions only and (b) the shape of the critical state surface (failure surface) and the flow surface in the octahedral-plane (the plane in principal stress space orthogonal to the mean normal stress axis, also known as π-plane) was considered as a circle. The critical state line forms a conical shape (Drucker-Prager (1952) yield surface) in 3D stress space. The shape appears in Cam-clay or Modified Cam-clay (Schofield and Wroth 1968), not as a yield surface but as a critical state cone (Britto and Gunn 1987). However Drucker-Prager (1952) yield surface (i.e. the critical state cone here) gives a worse fit to the data of soil failure (Britto and Gunn 1987). It is noted that the failure of geo-materials follows better to the shape of Mohr-Coulomb's failure criterion (Britto and Gunn 1987; Yin 2001) In the model proposed in this paper, the shape of the failure surface and flow surface in the π-plane is modified with an approximation of hexagon (ABAQUS theory Manual version 6.6; Yin 2001) which incorporates the third invariant of deviatoric stress in the formulation. The model has been generalized in 3D stress space as well.

It is generally believed that Mohr-Coulomb failure criterion can better describe the failure of soil (Yin et al. 2002). However the trace of the failure surface on the π-plane in the principle stress space is an irregular hexagon. The surface exhibits singularities in corners. To remove this singularity the Mohr-coulomb hexagon is approximated by a convex surface as shown in figure 2 and is discussed later.

This model is then used to describe some laboratory test results - undrained monotonic triaxial tests at different strain rates and at different over-consolidation ratios.

2 MATHEMETICAL FORMULATION

A similar approach as Kutter and Sathialingham (1992) has been followed while deriving the constitutive equations. The strain rate is additively decomposed into elastic and viscoplastic parts.

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{vp}_{ij} \quad [1]$$

Here $\dot{\varepsilon}_{ij}$ is the total strain rate, $\dot{\varepsilon}^{e}_{ij}$ is the elastic strain rate and $\dot{\varepsilon}^{vp}_{ij}$ is the viscoplastic strain rate. The elastic strain rate is given by,

$$\dot{\varepsilon}^{e}_{ij} = E_{ijkl}\,\dot{\sigma}_{kl} \quad [2]$$

Here E_{ijkl} is the fourth order compliance tensor with sub-indices k = 1, 2, 3 and l = 1, 2, 3. Summation is implied if two sub-indices of two items are the same. The elastic deformation of the soil is assumed to be isotropic and there are only two constants (for example a shear modulus G^e and a bulk modulus K^e). σ_{kl} is the effective stress tensor. The superimposed dots indicates time derivative. Note that all the quantities in the formulation are in terms of effective stress and the prime symbol is omitted.

The formulation of $\dot{\varepsilon}^{vp}_{ij}$ is based on Perzyna's (1963) formulation of viscoplasticity. He assumed the existence of the so called excess stress function F which is represented by the difference between the dynamic loading function $f_d\,(\sigma_{ij}, \dot{\varepsilon}^{vp}_{ij}) = k_d$ and static yield function given by $f_s(\sigma_{ij}, \dot{\varepsilon}^{p}_{ij}) = k_s$.

The excess stress function F was defined as follows

$$F = \frac{f_d}{f_s} - 1 \quad [3]$$

and the viscoplastic strain rate tensor for the simple case of an infinitesimal strain field was given by,

$$\dot{\varepsilon}_{ij}^{vp} = <\Phi(F)>\frac{\partial f_d}{\partial \sigma_{ij}} \quad [4]$$

It is noted that in the original formulation of Perzyna (1963) $<\Phi(F)> = 0$ for $F \le 0$ and $<\Phi(F)> = \Phi(F)$ for $F > 0$

In the present formulation of stress strain relation of the viscoplastic strain rate, $<\Phi(F)>$ is replaced by excess stress function φ and the equations takes the form as,

$$\dot{\varepsilon}^{vp}_{ij} = \varphi\frac{\partial \bar{f}}{\partial \bar{\sigma}_{ij}} \quad [5]$$

The functional form of φ can be determined theoretically or experimentally. Here $\bar{f}$ represents the reference surface (to be discussed in the next section) and $\bar{\sigma}_{ij}$ is the image stress on the reference surface.

Figure 1 shows the two yield surfaces considered in the present formulation namely the loading surface and the reference surface. Dependency of strain or strain rate on q (deviatoric stress) is replaced by another stress function y. Here y is defined as,

$$y = \frac{1}{2}q\left[1+\frac{1}{k}-\left(1-\frac{1}{k}\right)\left(\frac{r}{q}\right)^3\right] \quad [6a]$$

where

$$r = \left(\frac{9}{2}s_{ij}\,s_{jk}\,s_{ki}\right)^{\frac{1}{3}} \quad [6b]$$

k is considered to be the ratio of the slope of Critical State Line (CSL) in extension (M_E) to the slope of CSL in compression (M_C) (Yin 2001) and can be expressed as $k = (3-\sin\varphi')/(3+\sin\varphi')$.

The hardening and softening is assumed to be dependent only on the volumetric strain. The material is assumed to fail when $y = pM_C$ (ABAQUS theory manual, version 6.6)

The loading surface ($f = 0$) is analogues to the dynamic yield surface of Perzyna (1963) and is a surface of constant φ. The reference surface ($\bar{f} = 0$) which is also a surface of constant φ, is similar to the static yield surface of Perzyna (1963). In the original formulation of

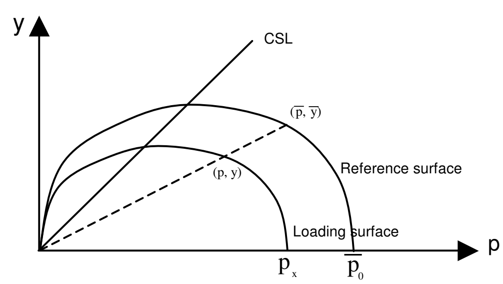

Figure1. Reference surface and Loading surfaces adopted in the present formulation

Kutter and Sathialingham (1992), another surface of constant φ namely potential surface was considered. As associative flow rule will be adopted, the plastic flow is considered to be normal to the reference surface and the potential surface is not necessary in this formulation.

A radial mapping rule as used by Dafalias and Herrman (1982) have been used to map the current stress state σ_{ij} to the reference surface. The image stress on the reference surface is denoted by $\bar{\sigma}_{ij}$. As proposed by Kutter and Sathialingham (1992) the stress difference function φ represents the difference between the current stress state σ_{ij} on the loading surface and the corresponding stress state $\bar{\sigma}_{ij}$ on the reference surface.

Similar to Kutter and Sathialingham (1992), the current stress could be larger or smaller than the corresponding stress state on the reference surface depending on the rate of loading. It is different from Perzyna's (1963) overstress function which is only defined for cases where $f_d > f_s$. φ is defined for all values of f and $\bar{f}$ in the present formulation.

The reference surface is smooth and consists of two ellipse with the following equations,

$$\bar{f} = (\bar{p} - \bar{p}_0)\left[\bar{p} + \left(\frac{R\text{-}2}{R}\right)\bar{p}_0\right] + (R\text{-}1)^2\left(\frac{\bar{y}}{M}\right)^2 = 0 \qquad [7a]$$

$$\bar{f} = \left(\bar{p} - 2\,\frac{\bar{p}_0}{R}\right) + \left(\frac{\bar{y}}{M}\right)^2 = 0 \qquad [7b]$$

The loading surface has the same functional form and the equation of loading surface can be obtained by replacing the symbols $\bar{f}, \bar{p}, \bar{y}$ and $\bar{p}_0$ in the equation 7a and 7b by f, p, y and p_x. Incorporation of y instead of q in the equation of yield surfaces changes the shape of the flow surface and critical state surface in the π-plane. The circular shape critical state cone and flow surface on the π-plane turns into an approximation of Mohr-Coulomb hexagon. Though the value of k has been considered as the ratio of slope of CSL in extension to the slope of CSL the value of k should be remain within the range of $0.778 \le k \le 1$ to avoid numerical difficulties evolved because of the surface being concave (ABAQUS theory manual version 6.6). When k is unity the hexagon turns into a circle and at that point y becomes equal to q. Any value beyond the lower limit may cause the approximated hexagon to be concave in shape which will cause numerical difficulties. The effect of k on the shape of critical state surface and flow surface on π–plane is shown in figure 2.

The constant R in equation 6 controls the shape of the yield surface in the p - y plane especially on the wet side of the critical state line. In p - q plane for $R = 2$ the two yield surfaces makes an ellipse which is of the same shape as of Modified Cam-clay yield surface.

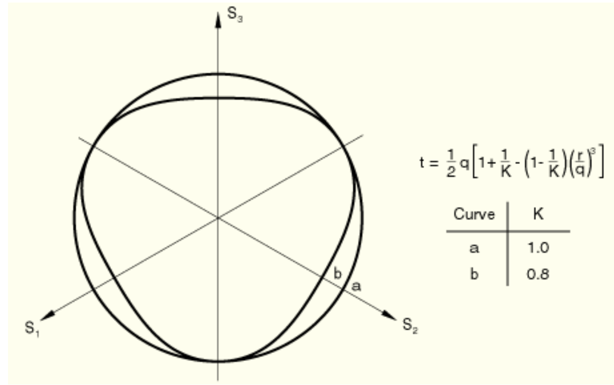

Figure 2. Shape of critical state surface and flow surface on octahedral plane (from ABAQUS theory manual ver-6.6)

2.1 Derivation of the Mapping Parameter

The equation of reference surface as in equation 7

$$\bar{f} = (\bar{p} - \bar{p}_0)\left[\bar{p} + \left(\frac{R\text{-}2}{R}\right)\bar{p}_0\right] + (R\text{-}1)^2\left(\frac{\bar{y}}{M}\right)^2 = 0$$

Using a radial mapping rule as in the bounding surface formulation of Dafalias and Herrmann (1982) we can write

$$\bar{p} = \beta p \qquad [8a]$$

$$\bar{q} = \beta q \qquad [8b]$$

$$\bar{\sigma}_{ij} = \beta \sigma_{ij} \qquad [8c]$$

$$\bar{y} = \beta y \qquad [8d]$$

Putting them in the equation of flow surface we can get

$$\beta = \frac{\bar{p}_0}{p_x} \qquad [9a]$$

where

$$p_x = p + \frac{y^2}{pM_C^2} \qquad [9b]$$

2.2 Derivation of φ function

Bjerrum (1967) used his model to explain the mechanism and effect of quasi-preconsolidation of normally consolidated clays. In figure 3 below the void ratio-effective stress relationship for normally consolidated clay is presented. In the present formulation Bjerrum's (1967) concept is used to obtain the overstress function following the approach of Borja and Kavazanjian (1985) and Kutter and Sathialingham (1992).

Creep at a constant effective stress is usually defined using the secondary compression index (C_α) as follows,

$$C_\alpha = \frac{\Delta e}{\Delta(\log t)} \qquad [10a]$$

and

$$\alpha = \frac{C_\alpha}{\ln 10} \qquad [10b]$$

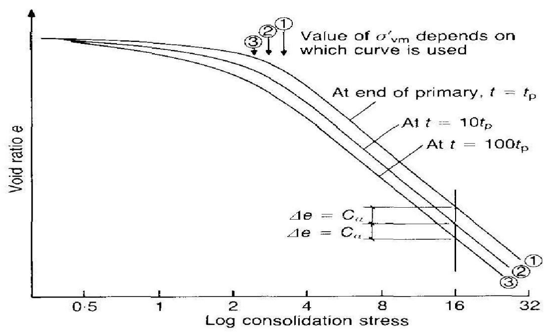

Figure 3. Effect of secondarily consolidation on the location of the compression curve (Bjerrum 1967; Kutter and Sathialingham 1992)

Here t is the time and Δe is the change in void ratio during creep compression. This is illustrated in the following figure (Figure 4).
The soil sample at point "a" sitting for and exact time $t = t$ with respect to reference time $\bar{t}$, undergoes creep as time passes and the reference void ratio $\bar{e}$ decreases from "a" to "b" whereas the apparent preconsolidation pressure increases from "c" to "d". If the final void ratio after creep is e, then the equation 10a becomes

$$C_\alpha = -\left(\frac{\bar{e} - e}{\log \bar{t} - \log t}\right) \qquad [11a]$$

or

$$\frac{t}{\bar{t}} = 10^{(\bar{e}-e/C\alpha)} \qquad [11b]$$

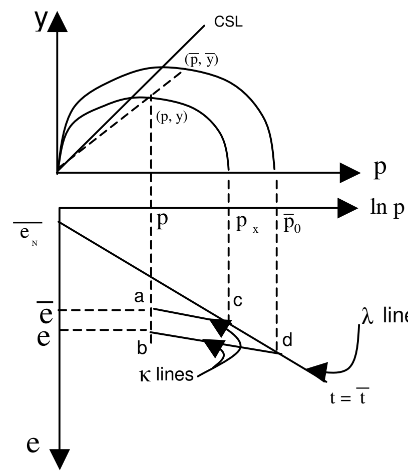

Figure 4: Relative locations of p_x and $\bar{p}_0$ in $e - \ln p$ space and $p - y$ space

The reference time is generally taken as one day. The reference void ratio will be obtained if the soil is normally consolidated for the same stress for reference time $\bar{t}$. Differentiating equation 11b with respect to time gives,

$$\frac{de}{dt} = -\frac{\alpha \exp\left[\frac{(e-\bar{e})}{\alpha}\right]}{\bar{t}} \qquad [12]$$

again,

$$\dot{\varepsilon}_v^{vp} = -\frac{de}{dt}\frac{1}{(1+\bar{e})} \qquad [13]$$

where $\dot{\varepsilon}_v^{vp}$ is the volumetric viscoplastic strain rate. Comparing equation 12 and equation 13 yields

$$\dot{\varepsilon}_v^{vp} = \frac{\alpha}{\bar{t}(1+\bar{e})}\exp\left[\frac{(e-\bar{e})}{\alpha}\right] \qquad [14]$$

The following equations can be obtained from figure 4

$$e = \bar{e}_N - \bar{\lambda}\ln\bar{p}_0 + \kappa\ln\left(\frac{\bar{p}_0}{p}\right) \qquad [15a]$$

$$\bar{e} = \bar{e}_N - \bar{\lambda}\ln\bar{p}_x + \kappa\ln\left(\frac{\bar{p}_0}{p}\right) \qquad [15b]$$

Here $\bar{e}_N$ is the void ratio for the reference mean time at unit mean normal pressure on the isotropic normal consolidation line and can be related to the Γ parameter of MCC formulation (Schofield and Wroth 1968) by the following equation:

$$\bar{e}_N = \Gamma + \left(\bar{\lambda} - \kappa\right)\ln 2 \qquad [16]$$

From equation 15a and equation 15b we can get

$$\left(e-\bar{e}\right) = \left(\bar{\lambda}-\kappa\right)\ln\left(\frac{p_x}{\bar{p}_0}\right) \qquad [17]$$

Substituting equation 17 in equation 14 we can write,

$$\dot{\varepsilon}_v^{vp} = \frac{\alpha}{\bar{t}\left(1+\bar{e}\right)}\left(\frac{p_x}{\bar{p}_0}\right)^{\left(\frac{\bar{\lambda}-\kappa}{\alpha}\right)} \qquad [18]$$

The volumetric viscoplastic strain can be expressed as

$$\dot{\varepsilon}_v^{vp} = \frac{\partial \bar{f}}{\partial \bar{p}} \qquad [19]$$

The viscoplastic flow function φ therefore can be determined by combining equation 18 and equation 19

$$\varphi = \frac{\alpha}{\bar{t}\left(1+\bar{e}\right)}\frac{1}{\frac{\partial \bar{f}}{\partial \bar{p}}}\left(\frac{p_x}{\bar{p}_0}\right)^{\left(\frac{\bar{\lambda}-\kappa}{\alpha}\right)} \qquad [20]$$

All the functions in the equation have been defined previously except the partial derivatives. The normal at any point on $\bar{f}$ is given by

$$\frac{\partial \bar{f}}{\partial \bar{\sigma}_{ij}} = \frac{\partial \bar{f}}{\partial \bar{p}}\frac{\partial \bar{p}}{\partial \bar{\sigma}_{ij}} + \frac{\partial \bar{f}}{\partial \bar{y}}\frac{\partial \bar{y}}{\partial \bar{\sigma}_{ij}} \qquad [21]$$

2.3 The time dependent hardening rule

The time dependent evaluation law of the $\bar{f}$ surface is dependent on the current viscoplastic increment of volumetric strain, $\partial\varepsilon_V^{vp}$, and is accounted via equation 17. Taking derivatives with respect to time of equation 17 and substituting $de = -d\,\varepsilon_v^{vp}(1+\bar{e})$ and $d\bar{e}/dt = -(1+\bar{e})\,\dot{e}_V^{vp}$, the time dependent hardening rule can be obtained as follows:

$$\dot{\bar{p}}_0 = \frac{d\bar{p}_0}{dt} = \bar{p}_0 \exp\left[\frac{(1+\bar{e})\,\partial\varepsilon_v^{vp}}{(\kappa-\bar{\lambda})}\right]\left[\frac{(1+\bar{e})\,\partial\varepsilon_v^{vp}}{(\bar{\lambda}-\kappa)}\right] \qquad [22]$$

All the terms in the equation is defined previously and it is to be noted that from this equation $\bar{p}_0$ changes with time dependent viscoplastic deformation and time is implicitly represented by $\partial\varepsilon_V^{vp}$.

3 DETERMINATION OF MODEL PARAMETERS

The way of determining the parameters are as described by Kutter and Sathialingham (1992). For the sake of completeness a brief description is added here.
The constitutive equations need seven material parameters for the full description of soil behaviour. Material constants includes traditional critical state parameters λ, κ, M_c and Poisons ratio μ and the void ratio at unit mean normal pressure after $\bar{t}$ days of normal consolidation, e_N which is related to MCC (Schofield and Wroth 1968) Γ parameter. λ, κ and e_N can be determined in conventional way from 1D compression or isotropic compression tests. The reference time is an arbitrary quantity and can be chosen from convenience to match the load increment duration used in the laboratory testing. The coefficient of secondary compression C_α or α can be determined from long term 1D compression test on a normally consolidated soil in conventional way.
The shape parameter is R adopted from Dafalias and Herrmann (1986) and represents the ratio of mean normal stresses for the surface at y = 0 and at q = pM_C. A good estimation of R can be found form undrained stress paths (Yin and Zhu 1999). In MCC formulation Roscoe and Burland (1968) adopted R to be equal to 2 irrespective of the clay type. R can take any value from 1 to infinity (Kutter and Sathialingham 1992) although values in the range of 2 to 3 have been used in the bounding surface formulation (Kaliakin 1985). R was taken as 2.5 by Hermann et al. (1981) and 2.8 by Adachi et al. (1985). The values were kept the same in this paper while simulating their tests.

4 VERIFICATION OF THE MODEL

To verify the predicting capability of the model some experimental published laboratory test results have been predicted and compared. The tests include undrained triaxial compression with different strain rate and undrained stressing with the same strain rate with different OCR. Figures below present the test results of (after Yin et al. 2002, Herrmann et al. 1981) and simulated results (along with the Kutter and Sathialingham (1992) model prediction) for CIU tests on the mixture of Kaolin and bentonite with different degree of over consolidation. The bold lines are the prediction of the current model and the thinner lines are the prediction of the original Kutter and Sathialingham (1992) model.

The specimens (mixture of Kaolin and Bentonite) were overconsolidated and had the OCRs of 1, 1.3, 2 and 6. The preconsolidation pressure of each specimen was 392 kpa and the axial strain rate used for the simulation was during shear was 0.6% per hour. The parameters used are listed in table 1. The predicted

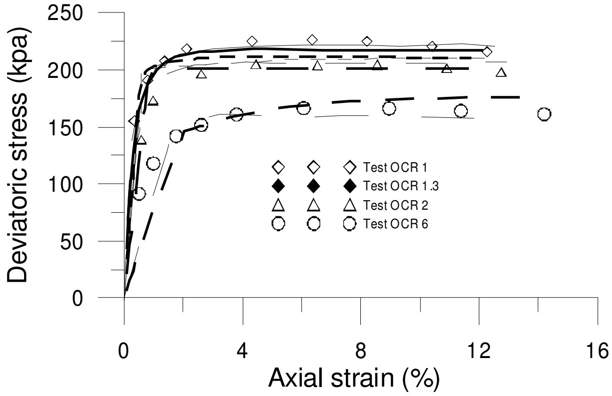

Figure 5: Comparison between measured and predicted axial strain vs. deviatoric stress response of CIU tests on mixture of Kaolin and bentonite (data from Dafalias and Herrmann 1986)

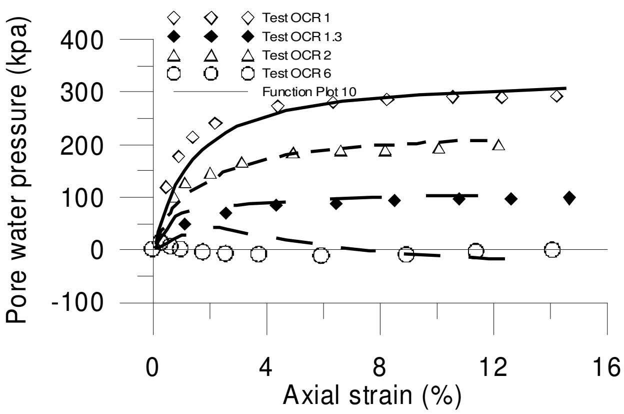

Figure 6: Comparison between measured and predicted axial strain vs. pore water pressure response in CIU tests on mixture of Kaolin and bentonite (experimental data from Dafalias and Herrmann, 1986)

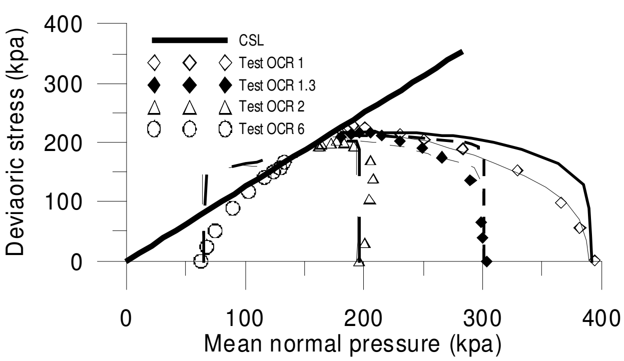

Figure 7. Comparison between measured and predicted mean normal stress verses deviatoric stress response in CIU tests on mixture of Kaolin and bentonite (data from Dafalias and Herrmann, 1986)

Table 1. Values of parameters used in the prediction

Parameter	Herrmann et al. (1982)	Adachi et. al. (1985)
λ	0.151	0.372
k	0.018	0.054
M	1.25	1.28
μ	0.30	0.30
e_N (1 day)	1.515	3.653
$C_{\alpha 0}$	0.0139	0.341
R	2.5	2.8

deviatoric stress responses with change in axial strain are in good agreement with the measured values as shown in figure 5. Shown in figure 6 and figure 7 are the predicted axial strain vs. pore water pressure response and mean normal stress vs. deviatoric stress curves and also are in good agreement with the test results. Prediction of both the models are very close to each other except for the deviatoric strain vs. deviatoric stress plot, the new model shows a bit better prediction at lower strain level than that of the original Kutter and Sathialingham (1992) model.

Adachi et al. (1985) presented undrained constant strain rate compression tests on undisturbed samples of Osaka alluvial clay at strain rates of 1.0, 2.1E-2 and 7.8E-4 %/min. The sample was initially consolidated at 588 kpa before straining. The figures below show the comparison of the simulated behaviour and the test data. The value of R was taken as same as that adopted by Kutter and Sathialingham (1992). The other material parameters are as used by Adachi et al. (1985) and are listed in Table 1.

In the following figures also the thicker lines represent the current model and the thinner lines represent the Kutter and Sathialingham (1992) model.

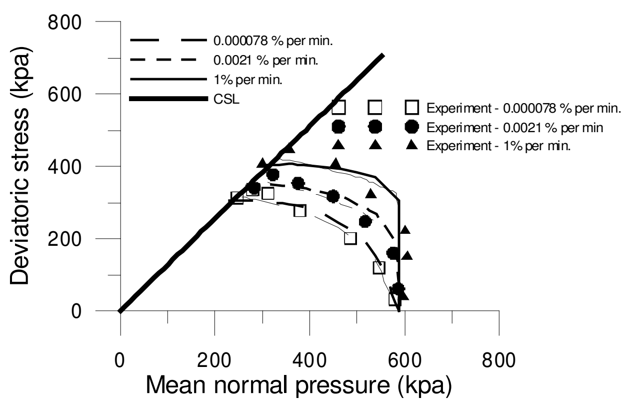

Figure 8: Predicted and experimental undrained stress paths for the various strain rates (Data from Adachi et al. 1985)

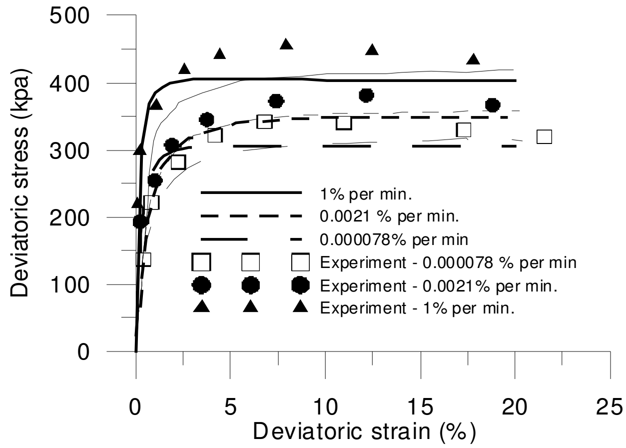

Figure 9: Predicted and experimental stress-strain response for various strain rates (Data from Adachi et al. 1985)

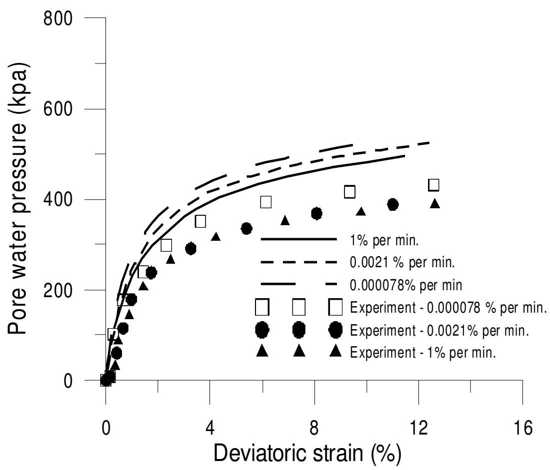

Figure 10: Predicted and experimental strain-pore water pressure relationship (Data from Adachi et al. 1985)

The mean normal pressure vs. deviatoric stress graphs and the variation of deviatoric stress with deviatoric strain were in good agreement with the test results for both the models and in this case also deviatoric stress-strain response at lower strain level was a bit better predicted by the new model. The excess pore water pressure response against the deviatoric strain was over predicted to some extent.

It can be concluded from the above figures that the effect of strain rate on soil stress-strain behaviour has been assessed reasonably well by the proposed theory with a little discrepancy in the prediction of excess pore water pressure response.

5 SUMMARY AND CONCLUDING REMARKS

This paper presents a new constitutive model based on the elastic viscoplasticity and Perzyna's theory. The model assumes that all inelastic strain rates are viscoplastic. It requires seven material parameters for the full description of soil behaviour and these parameters can be easily determined from conventional laboratory tests. The model is a modified form of Kutter and Sathialingham (1992) model. In particular the shape of flow surface in octahedral plane is modified with the inclusion of the third invariant of deviatoric stress and the ratio of slope of CSL in tension and slope of CSL in compression. The circular shape of flow surface on octahedral plane has been changed into an approximate hexagon. The predicted results were in good agreement with the test results and the deviatoric stress-strain response was found to be better predicted by the new model at lower strain levels. Better predictions are expected while this model will be implemented numerically to predict the multiple behaviour characteristics of some real structures such as embankments on soft foundation soil or other structures.

REFERENCES

Adachi, T., Mimura, M., and Oka, F. 1985. Descriptive accuracy of several existing constitutive models for normally consolidated clays. *In* Fifth International Conference on Numerical Methods in Geomechanics. Nagoya, Japan.

Bjerrum, L. 1967. Engineering geology of Norwegian normally consolidated marine clays as related to settlements of buildings Geotechnique, **17**(2): 81-118.

Borja, R.I., and Kavazanjian, E. 1985. A constitutive model for the stress-strain-time behaviour of 'wet' clays. Geotechnique, **35**(3): 283-298.

Britto, A.M., and Gunn, M.J. 1987. Critical State Soil Mechanics via Finite Elements. Ellis Horwood Limited, New York.

Dafalias, Y.F., and Herrmann, L.R. 1982. Bounding surface formulation of soil plasticity *In* In soil mechanics- transient and cyclic loads. Wiley, Chichester. pp. 253-282.

Dafalias, Y.F., and Herrmann, L.R. 1986. Bounding surface plasticity. II: application to isotropic cohesive soils. Journal of the Engineering Mechanics Division, ASCE, **112**(12): 1263-1291.

Drucker, D.C., and Prager, W. 1952. Soil mechanics and plastic analysis or limit design. Quartly of Applied Mathematics, **10**: 157-165.

Hermann, L.R., Shen, C.K., Jafroudi, S., DeNatale, J.S., and Dafalias, Y.F. 1981. A verification study for bounding surface plasticity model for cohessive soils., Naval construction battalion center, , Port Huenenme, Calif.

Kaliakin, V.N. 1985. Bounding surface elastoplasticity viscoplasticity for clays, University of California, California.

Kutter, B.L., and Sathialingam, N. 1992. Elastoviscoplastic modeling of the rate dependent behaviour of clays Geotechnique, **42**: 142-164.

Perzyna, P. 1963. Constitutive equations for rate-sensitive plastic materials. Quarterly of applied mathematics, **20**: 321-331.

Perzyna, P. 1966. Fundamental Problems in Viscoplasticity. Advances in Applied Mechanics, **9**: 244-368.

Roscoe, K.H., and Burland, J.B. 1968. On the Generalized Stress-Strain Behavior of Wet Clay. Engineering Plasticity: 535-609.
Schofield, A., and Wroth, P. 1968. Critical state soil mechanics.
Yin, J.-H., and Zhu, J.-G. 1999. Elastic viscoplastic consolidation modeling and interpretation of pore-water pressure responses in clay underneath Tarsiut Island. Canadian Geotechnical Journal, **36**(4): 708-717.
Yin, J.-H., Zhu, J.-G., and Graham, J. 2002. A new elastic viscoplastic model for time dependent behaviour of normally and overconsolidated clays: theory and verification. Canadian Geotechnical Journal, **39**(1): 157-173.
Yin, J.H. 2001. A refined elastic visco-plastic model for clayey soils. Journal of the Southeast Asian geotechnical society(April): 23-31.