Dependency of slope stability on strength anisotropy and spatial variability of soils

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ABSTRACT
This paper examines the sensitivity of slope stability with respect to anisotropy and spatial variability of soil strength parameters. Following a description of the treatment of anisotropy, the kinematic element method is extended to take into account the directional dependency of soil strength. Using the fast Largrangian analysis of continua, the influence of the spatial variability of soil strength on slope stability is performed by Monte-Carlo simulations and the subsequent finite difference analyses using FLAC. Numerical examples show that both the strength anisotropy and the spatial variability have significant impact on slope stability, with the factor of safety being more sensitive to anisotropy.

RÉSUMÉ
Cet article examine la sensibilité de la stabilité d'une pente en fonction de l'anisotropie et de la variabilité spatiale des paramètres de résistance des sols. Après une description du traitement de l'anisotropie, la méthode a base de cinématique des éléments est developée pour prendre en compte la dépendance directionnelle de la résistance du sol. Utilisant la methode dite "FLAC" (pour Fast Lagrangian Analysis of Continua), l'influence de la variabilité spatiale de la résistance du sol sur la stabilité d'une pente de talus est estimée par simulations Monte-Carlo, et ensuite a l'aide de l'analyse par la méthode des différences finies en utilisant la technique FLAC. Les exemples numériques montrent que l'anisotropie de la résistance et la variabilité spatiale ont un impact significatif sur la stabilité de la pente, le facteur de sécurité étant le plus sensible à l'anisotropie..

1 INTRODUCTION

Considerable research has been carried out on slope stability analysis, given its practical importance, for example for the design and construction of highway embankments, excavations and earth dams. For several decades, the limit equilibrium method based on simplified failure mechanisms such as slip circles has dominated the geotechnical engineering practice. The method, originating from a basically empirical background (Fellenius, 1936), has been significantly improved by, e.g., Morgenstern and Price (1965), Spencer (1967), Janbu (1973), Fredlund and Krahn (1977) and Duncan (1992). In spite of the popularity, both the kinematic and static admissibility may be violated potentially leading to significant errors in factor of safety estimates (Yu et al., 1998).

A more rigorous approach for analyzing stability makes use of the limit theorems of plasticity to provide lower and upper bound solutions that take into account equilibrium and collapse mechanisms, respectively, as described by Yu et al. (1998) and Kim et al. (2002). A compromise between limit equilibrium and limit analysis procedures is the kinematic element method (KEM). Developed by Gussmann (1982, 2000), the KEM considers rigid elements and failure conditions rigorously taking into account both the kinematics and static equilibrium.

Unfortunately, the analysis of actual geotechnical problems is difficult due to complex soil properties including history-dependent behaviour and anisotropy of the materials involved. Moreover, the uncertainty of measured material properties and their *in-situ* spatial variability make the analysis more complicated. As a result, improvements due to more realistic modelling may be undermined by complex soil properties and the uncertainties associated with the actual boundary-valued problems. Furthermore, improvements in the numerical and analytical tools for slope stability analysis do not necessarily improve the confidence in the estimation of factor of safety.

The effect of soil anisotropy and the spatial variability of soil strength on slope stability has been studied in the past. A probabilistic approach for slope stability analysis using finite element method can be used to accommodate the randomly spatial variability of soil properties; see, e.g., Griffiths and Fenton (2000, 2004). However, this method does not appear to have received acceptance in practice and hence may still be considered as a research tool. Using the conventional limit equilibrium method, Lo (1965) and Ohta et al. (1975) investigated the effect of cohesion anisotropy on the stability of slopes in cohesive soils. Using the upper bound method of limit analysis, Chen et al. (1975) studied the influence of cohesion anisotropy on the stability of slopes in $c-\phi$ soils. It should be noted that either the kinematic or the static admissibility is violated potentially in the work of Lo (1965) and Chen et al. (1985) due to the assumptions about failure mechanisms.

It has been known that various possibilities exist in slope stability analysis regarding geometry, material properties and failure modes. This paper mainly focuses on the effect of anisotropy (i.e., the directional dependency of soil strength) relative to the significance of random variations of soil properties. Three approaches, namely, the kinematic element method, the Bishop's method and continuum modelling based on fast Lagrangian analysis, are used to evaluate the factor of safety, involving different assumptions regarding equilibrium and failure modes. The influences of ground water and cyclic loading are not addressed in this study.

Following a brief review of the mathematical description of strength parameters characterising anisotropic soil strength, the kinematic element method developed by Gussmann (1982, 2000) is extended to perform stability analysis for slopes in anisotropic soils. The resulting factor of safety is compared with that calculated using a modified Bishop's method that takes into account anisotropic soil strength. A finite difference program FLAC, which performs fast Lagrangian analysis of continua, together with the Mohr-Coulomb model that accommodates random variation of the cohesion and the friction angle is then used to study the influence of the spatial variability of soil strength. It is shown that slope stability is more sensitive to the soil strength anisotropy than to its spatially random variation.

2 DIRECTIONAL DEPENDENCY OF SOIL STRENGTH

Numerous research indicates that both the cohesion c and the friction angle ϕ exhibit various levels of anisotropy and dependency on the loading direction; see, e.g., Casagrande and Carillo (1944), Duncan and Seed (1966), Oda (1972), Mayne (1985), Ohta et al. (1985), Tatsuoka et al. (1990) and Vaid and Saivathayalan (2000). For cohesive soils, Casagrande and Carillo (1944) assumed the cohesion of cohesive soil varied as a function of the major principal stress direction:

$$c = c_h + (c_v - c_h)\cos^2\delta \qquad [1]$$

where c is the cohesion when the major principal stress σ_1 is inclined at an angle δ to the normal of the bedding plane (i.e., the vertical in Figure 1), c_h and c_v are the cohesion for $\delta = 90^0$ (or σ_1 in the bedding plane) and 0^0 (σ_1 perpendicular to the bedding plane), respectively. By definition, c_h and c_v can be considered as the cohesion obtained in conventional triaxial compression and extension tests in laboratory, as shown in Figure 1b. Introducing the cohesion anisotropy ratio $K_c = c_h / c_v$, Eq. [1] becomes

$$c = c_v\left[K_c + (1-K_c)\cos^2\delta\right] \qquad [2]$$

The applicability of Eqs. [1] and [2] has been confirmed by many other researchers, e.g. Lo (1965) and Reddy and Srinivasan (1970).

Given a friction angle ϕ at failure, then according to the Mohr-Coulomb's criterion, the major principal stress direction makes an angle $\alpha = \pi/4 - \phi/2$ with the sliding

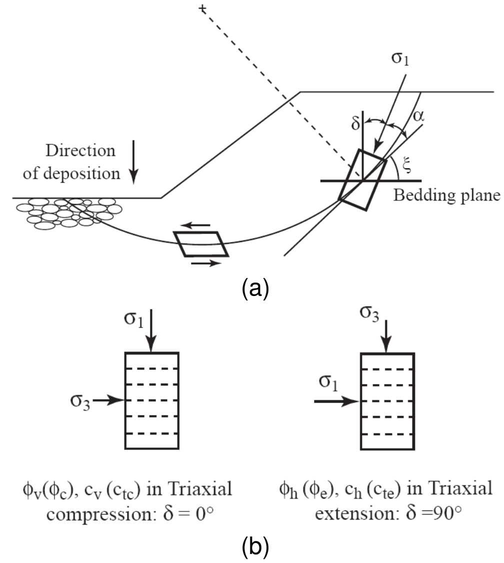

Figure 1. Directional dependency of soil strength in slope stability analysis

plane. This allows us to relate angle δ to the orientation of the sliding plane via

$$\delta = \frac{\pi}{2} - \xi - \alpha = \frac{\pi}{4} + \frac{\phi}{2} - \xi \qquad [3]$$

where ξ is the angle between the sliding plane and the bedding plane, as shown in Figure 1a.

Both experimental and theoretical studies show that the friction angle of soils, particularly cohesionless soils, highly depends on the fabric of the material as well as the orientation of the failure plane or the major principal stress direction with respect to the bedding plane. Various relations have been proposed to describe the dependency of soil friction angle on the orientation of major principal stresses relative to the bedding plane. For example, Jamiolkowski et al. (1985) assume that ϕ increases from ϕ_c to ϕ_e according to

$$\phi = \phi_e + (\phi_c - \phi_e)\cos^2\delta \qquad [4]$$

or

$$\phi = \phi_c\left[K_\phi + (1-K_\phi)\cos^2\delta\right] \qquad [5]$$

with $K_\phi = \phi_e / \phi_c$ with ϕ_e and ϕ_c corresponding to triaxial extension and compression conditions. Meyerhof (1978) proposed an alternative relation for the orientation dependency of friction angle; i.e.

$$\phi = \phi_c - (\phi_c - \phi_e)\delta/90 = \phi_e + (\phi_c - \phi_e)(1-\delta/90) \qquad [6]$$

which may be considered as an approximation of Eq. [4]. However, as shown in Figure 2, experimental results of Tatsuoka et al. (1990) reveal that the friction angle does not vary monotonically with δ as described by Eq. [4] or [6].

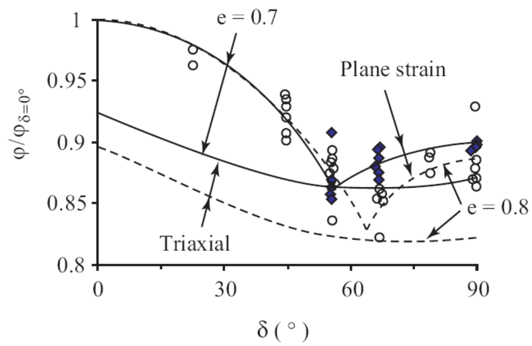

Figure 2. Dependency of friction angle on major principal stress direction: experimental evidence

Based on a micromechanical analysis, Guo and Stolle (2004) show that the friction angle of granular materials at failure varies with the angle δ, the degree of anisotropy ϖ as well as the angle between the sliding plane and the bedding plane ξ, which may be expressed as

$$\tan\phi_{local} \approx \mu_0\left(1-\varpi\cos 2\xi\right) \qquad [7]$$

where μ_0 is a material consistent, $\xi=\pi/2-(\alpha+\delta)$ with $\alpha=\pi/4-\phi/2$. By definition, ϖ is related to the spatial distribution of contact normal of particles. Since it is difficult to measure ϖ directly, as an alternative, ϖ may be calculated using the friction angle of soil specimens when sliding occurs along different directions. Specifically, when $\xi=0$ and $\pi/2$, which correspond to sliding along and cross the bedding plane respectively, one has

$$\begin{aligned}\mu_= &= \tan\phi_= = \mu_0\left(1-\varpi\right)\\ \mu_\perp &= \tan\phi_\perp = \mu_0\left(1+\varpi\right)\end{aligned} \qquad [8]$$

with subscripts "=" and "⊥" corresponding to values when sliding occurs along or cross the bedding plane, respectively. It follows that

$$\varpi=\frac{\mu_\perp-\mu_=}{\mu_\perp+\mu_=};\ \mu_0=\frac{1}{2}(\mu_=+\mu_\perp) \qquad [9]$$

Substituting Eq.(9) into (8) yields

$$\tan\phi_{local} \approx \mu_\perp-(\mu_\perp-\mu_=)\cos^2\xi=\mu_\perp\left[K_\phi+(1-K_\phi)\sin^2\xi\right] \qquad [10]$$

with $K_\phi=\mu_=/\mu_\perp$. According to Guo and Stolle (2004), the following relation may also be used for medium anisotropic granular soils with large friction angles

$$\sin\phi \approx m_0\left[1-\frac{\varpi}{2}(\cos 2\xi+\cos 2\delta)\right] \qquad [11]$$

where

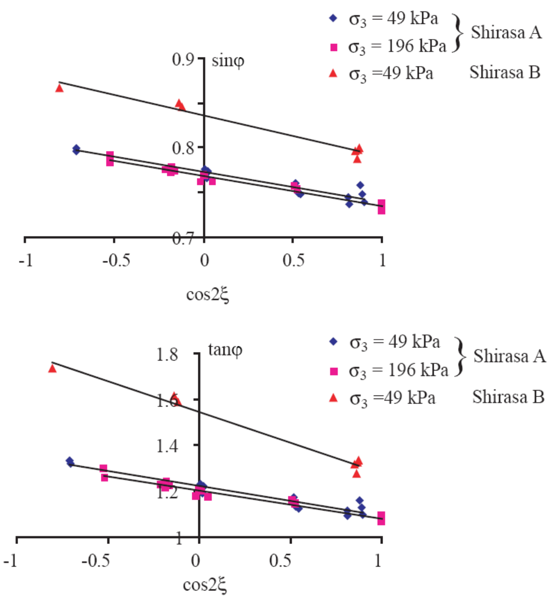

Figure 3. Dependency of friction angle on the orientation of sliding planes

$$\varpi=2\frac{m_\perp-m_=}{m_\perp+m_=};\quad m_0=\frac{m_=+m_\perp}{2+\varpi(m_=+m_\perp)/2}\approx\frac{1}{2}(m_=+m_\perp) \qquad [12]$$

in which $m_==\sin\phi_=$ and $m_\perp=\sin\phi_\perp$. Eqs. [7] and [11] agree with the experimental results of Miura et al. (1986), and Haruyama and Kitamura (1984), as illustrated in Figure 3. It has also been found that Eq. [12] provides better description on the orientation dependency of ϕ with respect to angle δ (Guo and Stolle, 2004).

It should be noted that Eqs. [5], [10] and [11] yield different trends in the variation of friction angle with respect to major principal stress directions, which may in turn have an impact on the results of slope stability analysis. Consequently, care must be exercised when selecting the most appropriate mathematical description for the orientation dependency of soil strength.

3 KEM FOR SLOPES IN ANISOTROPIC SOILS

3.1 Outline of KEM for isotropic soils

The stability analysis of slopes based on the kinematic element method (KEM), first presented by Gussmann (1982), takes into account both kinematics and statics. Referring to Figure 4, the equations for both statics and kinematics are expressed in terms of the forces and velocities at the boundaries, which are identified by the normal, of rigid elements. More specifically, the kinematic analysis determines the relevant kinematical compatibility via the analysis of the relative movement between any two adjacent elements, while the equilibrium of the elements is achieved via the statics analysis. In this study, it is assumed that sliding of elements can only occur along the boundaries, with no dilatancy, penetration, separation or rotation of elements being allowed. As a result, the normal component of the relative displacement between any two adjacent elements vanishes in the equations for the virtual displacements. The kinematics of the global system can then be expressed as

$$\mathbf{K}_v \mathbf{v} + \hat{\mathbf{v}}_n = 0 \qquad [13]$$

where $\mathbf{v}$ is the vector of the absolute element displacement, $\hat{\mathbf{v}}_n$ is the vector of the normal components of the virtual displacements of the surrounding elements, and $\mathbf{K}_v$ denotes the non-symmetric "geometry" matrix, which contains the direction cosines of the outward normals of the element boundaries. Referring to Figure 4, $l_{c/f}$ and $n_{c/f}$ are the direction cosines of boundary P_iP_j for Element f. The sign convention of the relative displacement between two adjacent elements is also shown in Figure 4.

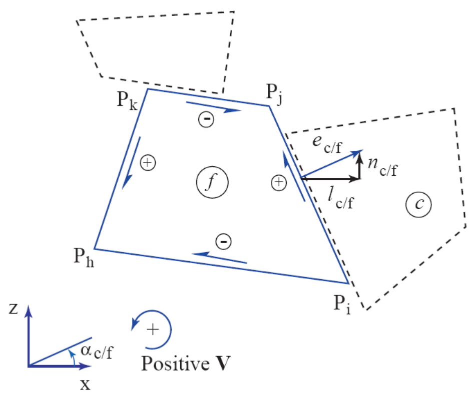

Figure 4. An element used for the KEM and the sign conventions

The static analysis, which takes into account all forces applied on the elements, provides a set of force equilibrium equations in terms of the effective normal forces on element boundaries, expressed as

$$\bar{\mathbf{K}}\mathbf{N}' + \bar{\mathbf{F}} = \mathbf{0} \qquad [14]$$

where $\mathbf{N}'$ is the yet unknown effective normal forces on element boundaries, $\bar{\mathbf{F}}$ refers to the known resultant force of cohesion c and pore pressure force (when applicable), and $\bar{\mathbf{K}}$ is the "direction" or "friction" matrix that depends on friction angle ϕ for the normal force $\mathbf{N}'$. The reader is referred to Gussmann (1982) for details.

3.2 Extended KEM for Anisotropic Soils

As discussed in the previous sections, the anisotropy of c and ϕ may be expressed as a function of angle δ or the orientation of the sliding plane with respect to the bedding plane. According to the Mohr-Coulomb failure criterion, the major principal stress σ_1 makes an angle $\alpha = \pi/4 - \phi/2$ with the sliding plane. Referring to Figure 5, it can be shown that angle δ, which describes the major principal stress direction, is related to the direction

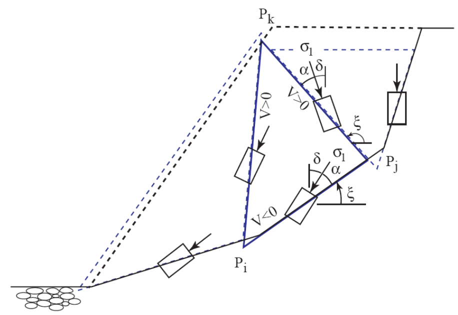

Figure 5: Failure mechanism in the KEM

of the relative displacement on the boundary of an element via

$$\cos^2\delta = \cos^2\left(\xi - \langle \mathbf{v} \rangle \alpha\right); \quad \alpha = \frac{\pi}{4} - \frac{\phi}{2} \qquad [15]$$

with $\langle \mathbf{v} \rangle = 1$ for $\mathbf{v} > 0$ and $\langle \mathbf{v} \rangle = -1$ for $\mathbf{v} < 0$.

4 BISHOP'S METHOD FOR SLOPES IN ANISOTROPIC SOILS

When the soil strength parameters are made direction dependent, the method of slices can be easily extended to include orientation dependent soil properties; see, e.g., Lo (1965). Without considering ground water, the factor of safety for Bishop's method is determined by

$$F_s = \frac{1}{W_j \sin\xi_j} \sum \frac{c_j b_j + \left[W_j - (X_{j+1} - X_j)\right]\tan\phi_j}{m_{\alpha(j)}} \qquad [16]$$

with

$$m_{\alpha(j)} = \cos\xi_j + \frac{\tan\phi_j \sin\xi_j}{F_s} \qquad [17]$$

where W_j is the total weight of the j th slice, which has width of b_j, X_{j+1} and X_j are the horizontal interaction forces on the inter-slice boundaries of slice j, while c_j and ϕ_j are the cohesion and friction angle on the slice base that makes an angle ξ_j with the horizontal. For slopes in anisotropic soils, the variation of c_j and ϕ_j with the orientation of the sliding plane may be described by the relations discussed in the previous sections.

5 EFFECTS OF SOIL ANISOTROPY ON FACTOR OF SAFETY

In order to demonstrate the effect of soil anisotropy, Eqs. [2] and [5] are used to describe the directional dependency of soil strength parameters, together with the assumption $K_c = K_\phi$, in the following numerical examples. In all simulations, the strength parameters corresponding to horizontal bedding planes are kept unchanged (i.e. c_v and ϕ_v are constant), while the influence of soil anisotropy is investigated by assuming different values of c_h and ϕ_h via changing the anisotropic ratios K_c and K_ϕ.

We begin by comparing the solutions obtained via the Bishop's method, the KEM, and a fast Lagrangian procedure FLAC, assuming isotropic response of the slope shown in Figure 6a, but varying c and ϕ. As shown in Figure 6a, the failure patterns predicted by the three procedures are similar. Figure 6b further confirms that fairly good agreement between the computed factors of safety exist for varying strength parameters. From an engineering practice point of view, the failure patterns and factors of safety may be considered to be the same. One may then conclude that for the particular slope shown in Figure 6a, all three methods can be used to determine the factor of safety with the same level of confidence. It is, however, important to recognize here that similarity exists even though the assumptions in terms of the kinematics are not the same; i.e., the Bishop's method considers only the rotational slip; the KEM assumes rigid translational slip and the fast Lagrangian procedure allows for the compatible deformation field of a continuum.

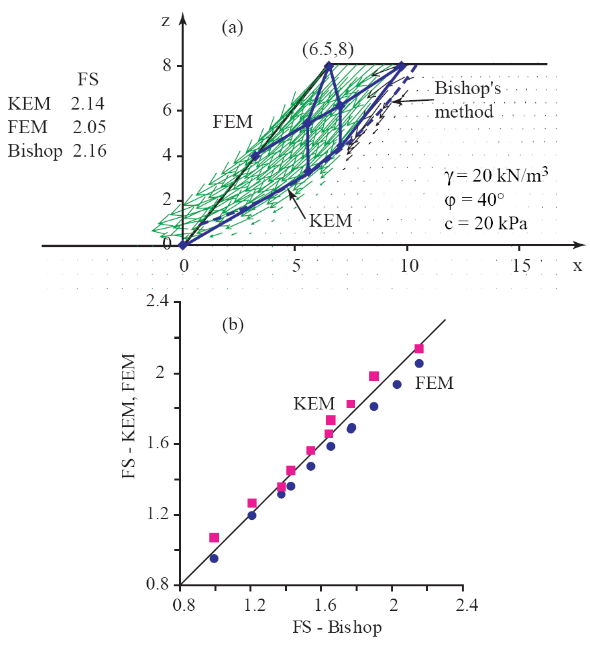

Figure 6. The stability analysis of a slope in isotropic soils

Figure 7 presents the effect of strength anisotropy on slope stability when the anisotropic ratio K varies in the range of 1.0 to 0.5 for $c_v = 20kPa$ and $\phi_v = 40^0$. One observes that the sliding surface gradually moves into the slope and the volume of sliding soil mass increases with a decrease of K (i.e., the decrease in c_h and ϕ_h). When $K \geq 0.75$, which corresponds to a mediumly anisotropic soil, one finds that the locations of the sliding surface are similar for both the KEM and Bishop's method predictions. On the other hand, for a highly anisotropic soil with $K = 0.5$, the Bishop's method predicts a larger volume of sliding soil mass; see Figure 7a. For this particular example, the factor of safety calculated from the KEM is slightly smaller than that from the Bishop's method. As shown in Figure 7b, when K varies from 1.0 to 0.5, the factor of safety F_s decreases from approximately 2.2 to 1.2 and 1.4 according to the KEM and Bishop's method, respectively. The results presented in Figure 7b show that using c_v and ϕ_v, which correspond to triaxial compression, and neglecting the potential anisotropy of soil strength tends to overestimate the F_s of a slope. On the other hand, if the soil is assumed to be isotropic and c_h and ϕ_h obtained from triaxial extension are used for a stability analysis, the F_s will then be under estimated. The comparison of the F_s calculated by the KEM and Bishop's method using different soil strength parameters are shown Figure 8.

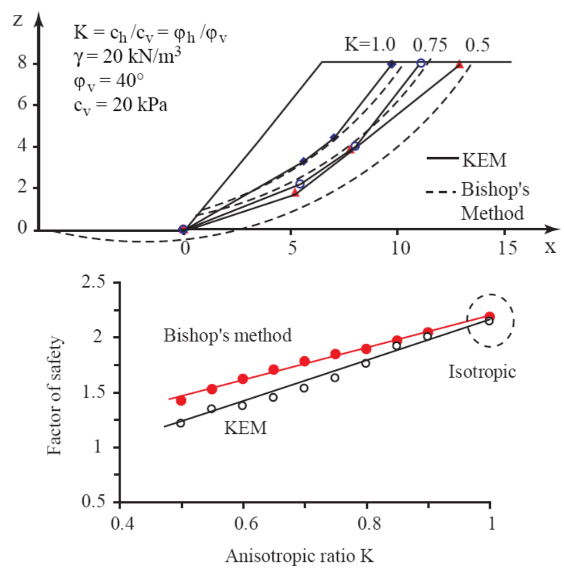

Figure 7. Stability analyses of a slope in anisotropic soils

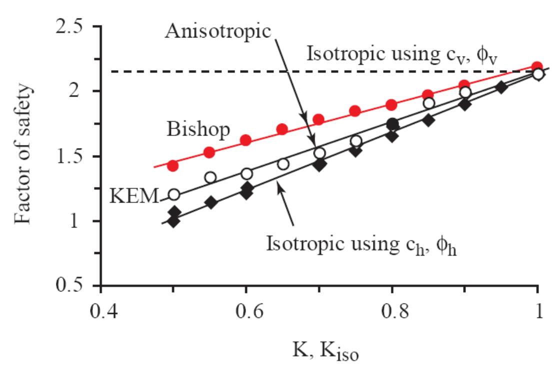

Figure 8. Factor of safety for slopes in anisotropic soils using different strength parameters

6 PROBABILISTIC ANALYSIS OF SLOPE STABILITY

6.1 Brief Description of the Method Used

In order to provide a deeper insight into the relative sensitivity of the factor of safety with respect to strength anisotropy and the random variability of soil properties in slope stability analyses, the slope shown in Figure 6 is re-analyzed taking into account the spatially random variability of soil properties. The variability of c and ϕ is assumed to be characterized by a normal distribution with the respective means being μ_c, μ_ϕ and the standard deviation σ_c, σ_ϕ. The corresponding coefficients of variation for c and ϕ are defined as

$$C.O.V_c = \frac{\sigma_c}{\mu_c}, \quad C.O.V_\phi = \frac{\sigma_\phi}{\mu_\phi} \qquad [18]$$

Moreover, c and ϕ are assumed to be independent random variables. The spatial correlation lengths for both c and ϕ are considered as large enough so that the single random variable (SRV) approach (e.g. Harr, 1987) can be used. It should be noted that the lognormal distribution may be more appropriate to represent non-negative soil properties.

The analyses in this section were carried out using FLAC, incorporating an elastic-perfectly plastic stress-strain law with the Mohr-Coulomb failure criterion. Material parameters for the soil are presented in Table 1.

Table 1: Soil parameters used in probabilistic analysis of slope stability

	Mean	Standard deviation	C.O.V
Unit weight γ	$20kN/m^3$	0	0
Cohesion c	$\mu_c = 20kPa$	$\sigma_c = 5kPa$	0.25
Friction angle ϕ	$\mu_\phi = 40^0$	$\sigma_\phi = 10kPa$	0.25

6.2 Dependency of c and ϕ on Spatial Variability of Soil Strength

For the assumed statistical properties, Monto-Carlo simulations were performed, involving 100 repetitions of the shear strength random field. The patterns of the spatial distribution of c and ϕ within the slope are varied for each repetition according to the normal probability density function. The mean and the standard deviation of the factor of safety from Monto-Carlo simulations were found to be $\mu_{Fs} = 2.05$, $\sigma_{Fs} = 0.10$, which correspond to a coefficient of variation of $C.O.V_{Fs} = 0.05$. As one might expect, the mean of the factor of safety μ_{Fs} is close to that of a homogeneous slope with c and ϕ being represented by μ_c and μ_ϕ, respectively. The variation of c and ϕ induced by the spatially random variation of soil strength, however, is much smaller than σ_c and σ_ϕ. This result should perhaps not be surprising given that the slope contains elements that are weaker and stronger than the mean, thus having the strong elements compensate for the weaker ones. As a result, the variation of F_s tends to be smaller than that of soil strength parameters.

Since it is assumed that c and ϕ are independent random variables characterized by normal distributions, one may expect that the factor of safety can also be described by a normal distribution. This has been confirmed by the results from the Monto-Carlo simulations, as shown in Figure 9. Given $\mu_{Fs} = 2.04$, $\sigma_{Fs} = 0.10$ corresponding to $\mu_\phi = 40^0, \mu_c = 20kPa$ and $C.O.V_c = C.O.V_\phi = 0.25$, one can calculate the probability of $F_s < 1.85$ and $F_s < 1.95$ as

$$p(F_s < 1.85) = \Phi\left(\frac{1.85 - \mu_{Fs}}{\sigma_{FS}}\right) = 0.023; \; p(F_s < 1.95) = \Phi\left(\frac{1.95 - \mu_{Fs}}{\sigma_{Fs}}\right) = 0.159$$

where Φ is the cumulative distribution function for a random variable with standard normal distribution.

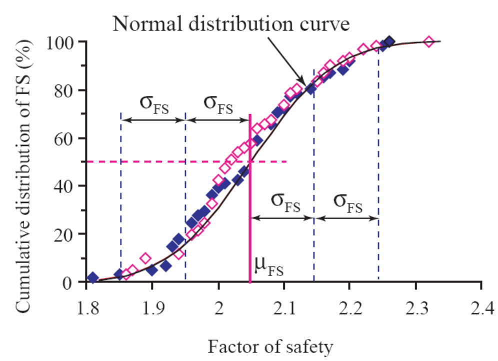

Figure 9: Cumulative distribution of the factor of safety

7 SENSITIVITY OF F_S WITH RESPECT TO ANISOTROPY AND SPATIAL VARIABILITY OF SOIL STRENGTH

The sensitivity of slope stability with respect to anisotropy and spatial variability of soil strength has been examined for the specific slope shown in Figure 6a. Referring to Figure 7b, one observes that for a slope in anisotropic soil, when c and ϕ of the soil in the horizontal direction is 25% less than those in the vertical direction (i.e., the anisotropic ratio K=0.75, the factor of safety based on the KEM and the Bishop's method are 1.66 and 1.82 respectively, which correspond to a decrease of 15% and 22% from the factor of safety for the slope in isotropic soils. On the other hand, for a slope in isotropic soil with random spatial variability of c and ϕ characterized by $C.O.V_c = C.O.V_\phi = 0.25$, the probability of $F_s < 1.85$ is 0.023. The observation suggests that one may conclude that the anisotropy of soil strength tends to have more significant effect on the stability of slopes. More specifically, for a slope in isotropic soils with the geometry given in Figure 7a, if the spatial variability of soil strength parameters is neglected, the probability of $F_s < 1.85$ is small. If the slope is in anisotropic soil with $\phi_h / \phi_v = c_h / c_v = 0.75,$ neglecting the strength anisotropy and using ϕ_v and c_v to calculate F_s would over estimate the F_s significantly.

Even though anisotropy of soil strength appears to have more significant influence of slope stability, one should not neglect the impact of spatial variability of soil properties, particularly when the factor of safety is close to unity. For example, when the mean of c and ϕ for the slope shown in Figure 6a are $\mu_\phi = 25^0$, $\mu_c = 15kPa$ respectively, it is found that the mean of the F_s is $\mu_{Fs} = 1.3$. If the standard deviation of F_s due to random variation of c and ϕ is $\sigma_{Fs} = 0.1,$ the probability of failure is then

$$p(F_s < 1.0) = \Phi\left(\frac{1.00 - \mu_{Fs}}{\sigma_{FS}}\right) = 1.35 \times 10^{-3}$$

However, when the soil has a larger random variation in its strength parameters and yields $\sigma_{Fs} = 0.15,$ the corresponding probability of slope failure will be increased to $p(F_s < 1.0) = 0.023,$ which is obviously very high for most engineering structures.

8 CONCLUDING REMARKS

This paper examined the impact of anisotropy and spatial variability of soil strength parameters on slope stability. Both the KEM and Bishop's method reveal that the factor of safety is highly influenced by the strength anisotropy of soil. This suggests that care should be exercised when evaluating the stability of slopes, in which anisotropic material behaviour is suspected. The spatial variation of soil strength also has some influence of the stability of slopes by increasing the probability. On the other hand, its influence does not appear to be as important as that of anisotropy.

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