Reliability of sandy soil liquefaction susceptibility using cone penetration testing

Yasser A. Hegazy
D'Appolonia Engineering, Monroeville, PA, USA

ABSTRACT
Empirical charts have been developed by researchers to assess sandy soil liquefaction susceptibility using cone penetration testing. Available methods depend on comparing field observed soil-response during an earthquake event and corrected cone tip resistance measurements. Developed charts are based on highly scattered data and the border lines between liquefiable and non-liquefiable soils were subjectively developed for practical use. In this paper, the reliability of assessing sandy soil liquefaction susceptibility is objectively evaluated using Monte Carlo simulation. A reliability index and the corresponding probability of exceeding the liquefaction soil resistance are determined using the field earthquake loads and soil resistance as function of in-situ cone penetration test measurements.

RÉSUMÉ
Les graphiques empiriques ont été développés par les chercheurs pour évaluer la susceptibilité de liquéfaction de sols sablonneuse en utilisant la mise à l'essai de pénétration de cône. Les méthodes disponibles sont basées a comparer les champs de reponse de sol observés lors d'un événement séismique avec les measures corrigés de la résistance de bout de cône de penetration. Les graphiques développés sont fondés sur des données hautement dispersées et sur les frontières entre les sols liquéfiables et non-liquéfiables et ils ont été subjectivement développées pour l'utilisation pratique. Dans l'article present, la fiabilité d'évaluer la susceptibilité de liquéfaction de sols sablonneuse est objectivement évaluée en utilisant la simulation de Monte-Carlo. L'index de la fiabilité et la probabilité correspondante d'excéder la résistance de sol de liquéfaction sont déterminés en utilisant les charges de séisme de terrain et la résistance du sol en fonction des mesures d'épreuve de pénétration de cone dans le milieu du travail.

1 INTRODUCTION

Liquefaction susceptibility of sandy soil can be determined using laboratory and/or in-situ testing methods. Practical methods have been developed to evaluate soil liquefaction potential using Standard Penetration Test (SPT), Cone Penetration Test (CPT) and shear wave velocity (V_s). CPT has become more popular in-situ test for site investigation and geosystems design because it provides continuous, reliable and repeatable records (Robertson and Campanella 1985).

Researchers have developed empirical charts to assess sandy soils liquefaction susceptibility using CPT (Shibata and Teparaksa 1988; Stark and Olson 1995; Suzuki et al. 1995; Robertson and Fear 1998; Youd et al. 2001; Moss et al. 2006). Liquefaction assessment charts were developed using highly scattered data and the border lines between liquefiable and non-liquefiable soils were subjectively developed for practical use. In this study, a Monte Carlo simulation technique is used to objectively determine the reliability and probability of occurrence of soil liquefaction in the case of an earthquake using CPT-based charts.

2 CPT-BASED LIQUEFACTION DATA

There are several publications on CPT and seismic data for sites where sandy soil layers were liquefied and not liquefied based on field observations. Shibata and Teparaksa (1988) published a CPT-based liquefaction database where separating liquefiable and non-liquefiable soils was based on the cyclic stress ratio (CSR), normalized cone tip resistance (q_{c1}) and the mean grain size (D_{50}). Stark and Olson (1995) expanded Shibata and Teparaksa database and added more sites. Recently, Moss et al. (2006) published a comprehensive liquefaction database documenting 18 different earthquakes over 4 decades. Liquefiable and non-liquefiable soils were differentiated based on CSR, q_{c1} and normalized cone friction ratio (F) similar to the approach adopted by Robertson and Wride (1998) and Youd et al. (2001). In this paper, the databases summarized by Stark and Olson and Moss et al. were used in the reliability analyses.

2.1 Liquefaction Assessment Methods

Soil liquefaction susceptibility has been evaluated in the literature using empirical charts and regressed formulas. Shibata and Teparaksa (1988) recommended the following equation to estimate critical normalized cone tip resistance $(q_{c1})_{cr}$:

$$(q_{c1})_{cr} = C\left[5 + 20 * \frac{CSR - 0.1}{CSR + 0.1}\right]; \quad (MPa) \qquad [1]$$

$$If \quad D_{50} \geq 0.25\,mm: \quad C = 1 \qquad [2a]$$

$$If \quad D_{50} < 0.25\,mm: \quad C = \frac{D_{50}\,(mm)}{0.25} \qquad [2b]$$

$$q_{c1} = C_Q q_c = \left(\frac{0.17}{\sigma_{vo}' + 0.07}\right) q_c \qquad [3]$$

in which D_{50} is the mean grain size, C_Q is the effective overburden stress correction factor and σ_{vo}' is the vertical effective stress. Equation 1 indicates no liquefaction for a soil with $q_{c1} > (q_{c1})_{cr}$.

Robertson and Wride (1998) and Youd et al. (2001) recommended estimating the cyclic resistance ratio (CRR) for an earthquake magnitude (M) = 7.5 using the following equation:

$$If \quad (q_{c1N})_{cs} < 50:$$
$$CRR_{M=7.5} = 0.833[(q_{c1N})_{cs}/1000] + 0.05 \qquad [4a]$$

$$If \quad 50 \leq (q_{c1N})_{cs} < 160:$$
$$CRR_{M=7.5} = 93[(q_{c1N})_{cs}/1000]^3 + 0.08 \qquad [4b]$$

in which $(q_{c1N})_{cs}$ is the equivalent clean sand normalized cone tip resistance, as follows:

$$(q_{c1N})_{cs} = K_c \, q_{c1N} \qquad [5a]$$

$$q_{c1N} = \frac{q_{c1}}{P_a} = \left(\frac{q_c}{P_a}\right)\left(\frac{P_a}{\sigma_{vo}'}\right)^n \qquad [5b]$$

in which q_c is the cone tip resistance and P_a is a reference pressure = 100 kPa. K_c is a correction factor that is a function of the soil grain characteristics. K_c is calculated using the normalized cone tip resistance (Q) and F as explained in Robertson and Fear (1998). A sandy soil layer will not liquefy If $CRR_{M=7.5}$ exceeds $CSR_{M=7.5}$.

Moss et al. (2006) developed a correlation to estimate critical CRR as a function of cone data, vertical effective stress, earthquake magnitude and probability of liquefaction, as follows:

$$CRR = \exp\left[\left\{\begin{matrix} q_{c1}^{1.045} + q_{c1}(0.110^*R_f) + (0.001^*R_f) + \\ n(1+0.850^*R_f) - 0.848^*\ln(M_w) - 0.002^*\ln(\sigma_{vo}') \\ -20.923 + 1.632^*\Phi^{-1}(P_L) \end{matrix}\right\}\frac{1}{7.177}\right] \qquad [6]$$

in which q_{c1} is calculated as in Equation 5, R_f is the ratio of the sleeve friction (f_s) to q_c in percent, P_L is the probability of liquefaction and $\Phi^{-1}(P_L)$ is the inverse cumulative normal distribution function. P_L was taken = 0.15 to obtain equivalent deterministic critical CRR as recommended by Moss et al. (2006).

3 RELIABILITY ANALYSIS

A reliability index and the corresponding probability of exceeding the liquefaction soil resistance are determined using the field earthquake loads and soil resistance using in-situ cone penetration test measurements. To accomplish this, a limit state equation is developed that incorporates and relates together the variables that affect the potential for sandy soil liquefaction. The parameters of load and resistance are considered as random variables. The limit state equation is:

$$g = R - Q \qquad [7]$$

in which g = the random variable representing the safety margin; R = the random variable representing resistance; and Q = the random variable representing load. Figure 1a illustrates a schematic of load and resistance function distributions. Figure 1b shows g (the difference between R and Q) and the probability of exceedance, P_e. Figures 1a and 1b illustrate lognormal distributions for load and resistance. In Figure 1a, failure is represented by the zone where the load and resistance distributions overlap, and the area under the curve equals the probability of exceedance (P_e). Figure 1b is an alternative representation of the Q and R distributions in which distributions are combined to represent the limit state function, R – Q. P_e is typically represented by the reliability index, β, which represents the number of standard deviations of the mean of R – Q, or $\overline{R} - \overline{Q}$, to the right of the origin.

For each Monte Carlo run using Lumenaut (2007) software, Version 3.4.21, which is compatible with Microsoft Excel, 10,000 iterations (i.e., g's) are generated. The iteration results are summarized in the form of a histogram and corresponding intervals are summarized in a table. A spread sheet, including the actual 10,000 iterations, is also generated as an output. The g values are then sorted and ranked in an ascending order. The cumulative probability at each g, P(g), is calculated as (g rank/(10,000+1)). Then the standardized normal value (z) of a P(g) is calculated using the NORMSINV function in Microsoft Excel where z = NORMSINV [P(g)]. The reliability index, β, is equal to (-z) at g = 0.

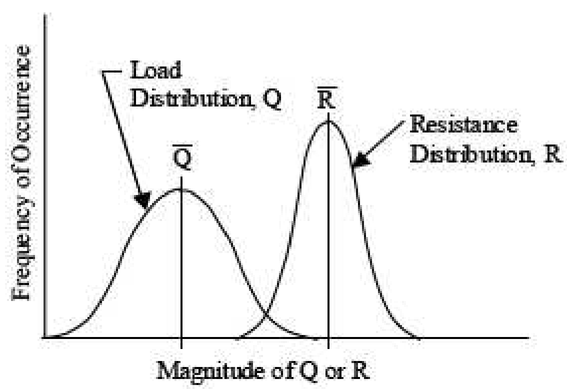

Figure 1a. Schematic of normally distributed load (Q) and resistance (R)

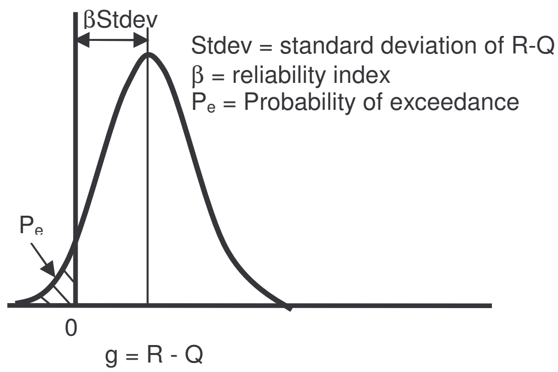

Figure 1b. Probability of exceedance and reliability index

3.1 Data Statistical Properties

The variable statistical parameters needed to perform the Monte Carlo analysis include the mean, μ, and standard deviation, Stdev (Allen et al. 2005). Statistical parameters of R and Q are summarized in Table 1.

Normal and lognormal statistical distributions are typically used in a reliability analysis of geotechnical data. Using a Monte Carlo analysis requires predetermination of the statistical distribution of the data. The Shapiro-Wilk test for normality (W test) was applied on the statistical data of Q and R to check the goodness of fit of each data set to a normal distribution (Shapiro and Wilk, 1965). The test statistic W ranges from $0 < W \leq 1$, where 0 indicates no normality and 1 indicates normality. The W tests were performed using Lumenaut (2007) software. The normality W test results are shown in Table 2. The data in Table 2 indicates that a normal distribution well represents R = q_{c1}, and Q, = $(q_{c1})_{cr}$, of Stark and Olson (1995) database. Also, it well represents R = CRRM, and Q = CSRM of Moss et al. (2006) database. However, a normal distribution did not well fit R = CRRM =7.5 of liquefiable soils where the goodness of fit of the W test was only 0.46. Therefore, a lognormal distribution was used to represent R = CRRM =7.5 and Q = CSRM =7.5 of Moss et al. database where W test resulted in a goodness of fit more than 0.9.

3.2 Reliability of Liquefaction Assessment Methods

A reliability analysis using Monte Carlo simulation was performed for the Shibata and Teparaksa liquefaction assessment method. A limit state function, g was formed using q_{c1} and $(q_{c1})_{cr}$ of Stark and Olson (1995) database.

Figure 2 shows a comparison between q_{c1} and $(q_{c1})_{cr}$. Except for few points, there is a distinction between non-liquefiable and liquefiable soil data. The parameters qc1 and $(q_{c1})_{cr}$ were calculated for the liquefiable soils using Equations 1, 2 and 3. The analysis results are summarized in Figure 3 which shows the standardized normal variable (z) versus randomly generated g. The reliability index, β is equal to 2.0. The corresponding probability of exceedance, P_e = 2.5% which means that 97.5% of liquefiable soils are identified as liquefiable soils and only 2.5% of them are unconservatively identified as non-liquefiable soils. Also, Monte Carlo analysis was performed using the non-liquefiable soil data of Stark and Olson database. The results are summarized in Figure 4 indicating β = 1.73 and a corresponding P_e = 4.2%. This means that 95.8% of non-liquefiable soils are designated as non-liquefiable and conservatively, 4.2% of them are designated as liquefiable.

Table 1a. Statistical properties of R and Q for liquefiable soils.

Liquefaction assessment method	R or Q	Liquefiable soils statistical properties	
		μ	Stdev
Shibata and Teparaksa (1988)[1]	R = q_{c1} (MPa)	4.859	1.241
	Q =$(q_{c1})_{cr}$ (MPa)	7.734	0.787
Robertson and Wride (1998)[2]	R = $CRR_{M=7.5}$	0.226	0.058
	Q = $CSR_{M=7.5}$	0.303	0.089
Moss et al. (2006)[3]	R = CRR_M	0.103	0.049
	Q = CSR_M	0.282	0.083

Table 1b. Statistical properties of R and Q for non-liquefiable soils.

Liquefaction assessment method	R or Q	Non liquefiable soils statistical properties	
		μ	Stdev
Shibata and Teparaksa (1988)[1]	R = q_{c1} (MPa)	6.339	0.645
	Q =$(q_{c1})_{cr}$ (MPa)	11.709	2.990
Robertson and Wride (1998)[2]	R = $CRR_{7.5}$	0.067	0.072
	Q = $CSR_{7.5}$	0.228	0.282
Moss et al. (2006)[3]	R = CRR	0.440	0.502
	Q = CSR	0.205	0.062

Notes:
[1] Stark and Olson (1995) database was used to evaluate the reliability of Shibata and Teparaksa liquefaction method.
[2] Moss et al. database was used to evaluate the reliability of Robertson and Wride liquefaction method.
[3] The database summarized by Moss et al. (2006) was used to evaluate their proposed liquefaction method.

Table 2. Normality (W) test results.

Database	R or Q	W test results	
		Liquefiable soils	Non-liquefiable soils
Stark and Olson (1995)	R = q_{c1} (MPa)	0.95	0.95
	Q =$(q_{c1})_{cr}$ (MPa)	0.97	0.94
Moss et al. (2006)	R = $CRR_{M=7.5}$	0.46	0.74
	Q = $CSR_{M=7.5}$	0.93	0.85
Moss et al. (2006)	R = CRR_M	0.85	0.72
	Q = CSR_M	0.92	0.83

Similarly, Monte Carlo reliability analyses were performed for the Robertson and Wride (1998) liquefaction assessment method. A limit state function, g was formed using R = $CRR_{M = 7.5}$ and Q = $CSR_{M = 7.5}$ of Moss et al. (2006) database. Values of $CRR_{M = 7.5}$ were calculated using Equations 4 and 5. Figure 5 shows a comparison between $CRR_{M = 7.5}$ and Q = $CSR_{M = 7.5}$. Liquefiable and non-liquefiable soil data are mixed and uneasy to delineate in two separate clusters. Figure 6 shows the reliability analysis results of liquefiable soils. Figure 6 depicts β of 0.73. The corresponding P_e is 23% which means that 23% of the liquefiable soils are misinterpreted as non-liquefiable soils. Figure 7 shows the reliability analysis results of non-liquefiable soils with β of 0.57. The corresponding probability indicates that 28% of non-liquefiable soils are conservatively interpreted as liquefiable soils.

Finally, Monte Carlo reliability analyses were performed for Moss et al. (2006) liquefaction assessment method. The limit state function, g was a function of CRR calculated using Equations 5 and 6 and CSR from Moss et al. collected database. A comparison between CRR and CSR is shown on Figure 8 indicating distinguished separation between liquefiable and non-liquefiable soil data. Some estimated CRR values were greater than 0.6 and not shown on Figure 8 which has a vertical scale of limited to CRR = 0.6. Figure 9 depicts the reliability analysis results of liquefiable soils. The reliability index, β is 1.82 with a corresponding P_e of 3.5%. Figure 10 shows the reliability analysis results of non-liquefiable soils. The resulting β and P_e are 0.46 and 32%, respectively. Moss et al. liquefaction assessment method underestimates the liquefiablity of 3.5% of the liquefiable soils and conservatively overestimates the liquefaction potential of 32% of the non-liquefiable soils.

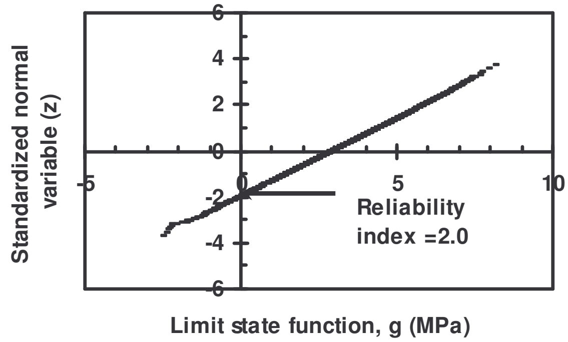

Figure 3. Reliability analysis results of liquefiable data using Shibata and Teparaksa assessment method (CPT-liquefaction based data from Stark and Olson 2005)

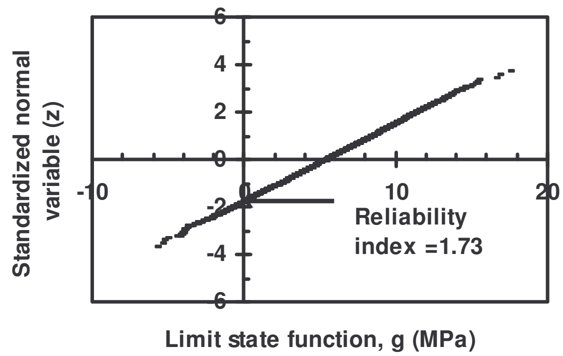

Figure 4. Reliability analysis results of non liquefiable data using Shibata and Teparaksa assessment method (CPT-liquefaction based data from Stark and Olson 2005)

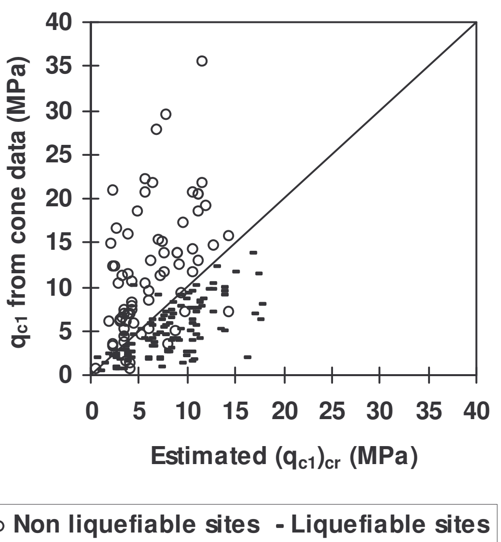

Figure 2. Comparison between q_{c1} from cone data and estimated $(q_{c1})_{cr}$ using Shibata and Teparaksa assessment method (CPT-liquefaction based data from Stark and Olson 2005)

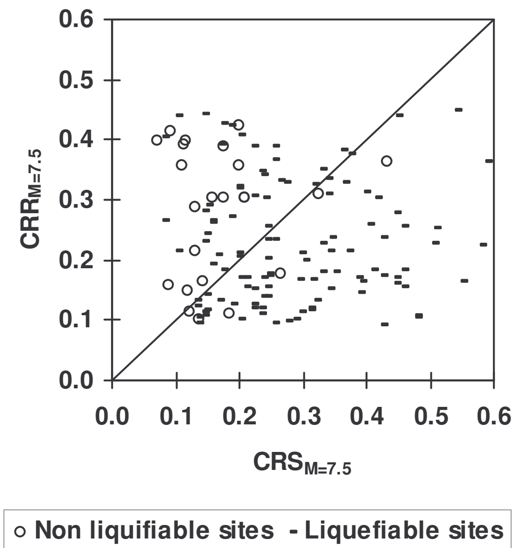

Figure 5. Comparison between $CSR_{M=7.5}$ and estimated $CRR_{M=7.5}$ using Robertson and Wride (1998) assessment method (CPT-liquefaction based data from Moss et al. 2006)

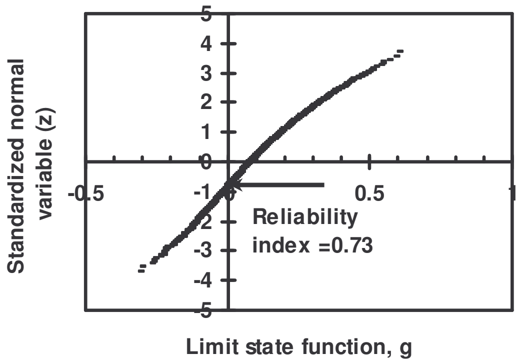

Figure 6. Reliability analysis results of liquefiable data using Youd et al. assessment method (CPT-liquefaction based data from Moss et al. 2006)

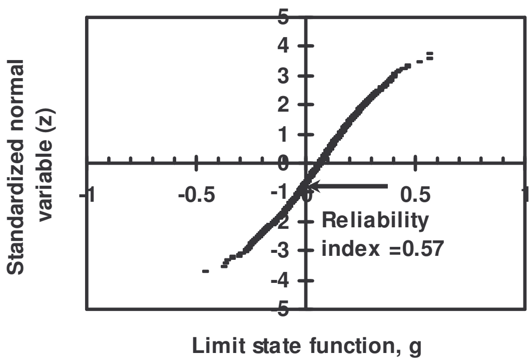

Figure 7. Reliability analysis results of non-liquefiable data using Youd et al. assessment method (CPT-liquefaction based data from Moss et al. 2006)

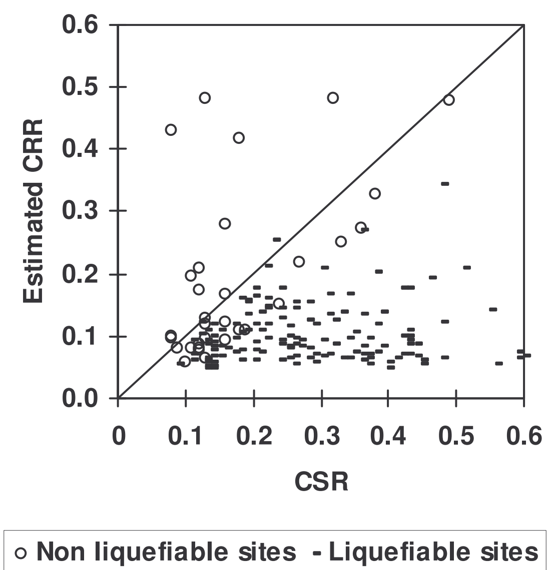

Figure 8. Comparison between CSR and estimated CRR using Moss et al. (2006) assessment method and database

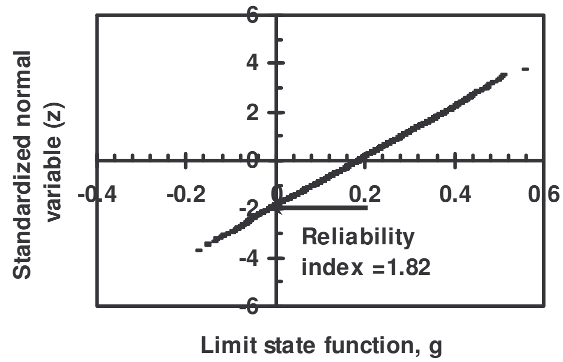

Figure 9. Reliability analysis results of liquefiable soils using data and assessment method from Moss et al. 2006

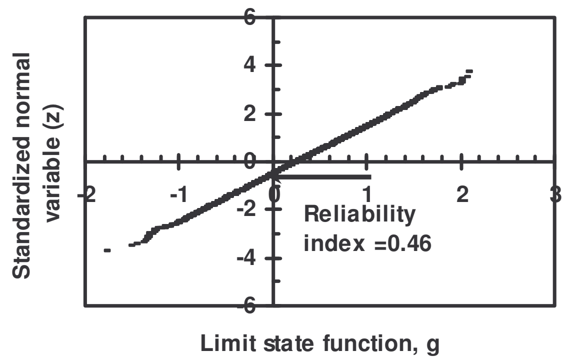

Figure 10. Reliability analysis results of non-liquefiable soils using data and assessment method from Moss et al. 2006

4 CONCLUSIONS

In this paper, the reliability of assessing sandy soils liquefaction susceptibility is objectively evaluated using Monte Carlo simulation. The analyses were performed for three assessment methods using CPT-based liquefaction data. The assessment method developed by Shibata and Teparaksa as a function of q_{c1} and D_{50} estimates soil liquefaction susceptibility with a relatively high reliability. The probability of underestimating or overestimating soil liquefaction response is less than 5%. The Robertson and Wride liquefaction assessment method as a function of $CRR_{M=7.5}$ and $Q = CSR_{M=7.5}$ was generally less reliable with a probability of approximately 25% of underestimating or overestimating soil liquefaction potential. Also, the reliability of the liquefaction assessment method developed by Moss et al. as a function of CCR and CSR was evaluated. The latter evaluation indicated reliable estimate of soil liquefaction susceptibility with a probability of underestimating soil liquefaction potential less than 5%. Moss et al. method conservatively overestimated soil liquefaction potential with an approximate probability of 30%.

The analyses indicated that using the normalized cone tip resistance and D_{50} to predict soil liquefaction susceptibility is more reliable than using the normalized cone tip resistance and sleeve friction. The developed analysis method can be applied to other in-situ measurements of soil resistance including standard penetration test data and shear wave velocity measurements.

REFERENCES

Allen, T.M., Nowak, A.S. and Bathurst, R.J. 2005. *Calibration to determine load and resistance factors for geotechnical and structural design*, Transportation Research Circular E-C079, Transportation Research Board, Washington, DC.

Lumenaut 2007. Software, *Manual*, Version 3.4.21, www.lumenaut.com.

Moss, R.E.S., Seed, R.B., Kayen, R.E., Stewart, J.P., Der Kiureghian, A. and Cetin, K.O. 2006. CPT-based probabilistic and deterministic assessment of in situ seismic soil liquefaction potential, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 132(8) : 1033-1051.

Robertson, P.K. and Campanella, R.G. 1985. Liquefaction potential of sands using the CPT, *Journal of Geotechnical Engineering*, ASCE, 111(3): 384-403.

Robertson, P.K. and Wride, C.E. 1998. Evaluating cyclic liquefaction potential using the cone penetration test, *Canadian Geotechnical Journal*, 35(3): 442-459.

Shapiro, S.S. and Wilk, M.B. 1965. An analysis of variance test for normality," *Biometrika*, 52(3): 591-599.

Shibata, T. and Teparaksa, W. 1998. Evaluation of liquefaction potentials of soils using cone penetration tests, *Soils and Foundations*, Japaneese Society of Soil Mechanics and Foundation Engineering, 28(2): 49-60.

Stark, T.D. and Olson, S.M. 1995. Liquefaction resistance using CPT and field case histories, *Journal of Geotechnical Engineering*, ASCE, 121(12) : 856-869.

Suzuki, Y., Tokimatsu, K., Koyamada, K., Taya, Y. and Kubota, Y. 1995. Field correlation of soil liquefaction based on CPT data, *Proceedings, International Symposium on Cone Penetration Testing*, 2: 583-588.

Youd, T.L., Idriss, I.M., Andrus, R.D., Arango, I., Castro, G., Christian, J.T., Dobry, R., Liam Finn, W.D., Harder, L. F., Hynes, M.E., Ishihara, K., Koester, J.P., Liao, S.SC., William F. Marcuson, III, W.F., Martin, G.R., Mitchell, J.K., Moriwaki, Y., Power, M. S., Robertson, P.K. Seed, R.B., and Stokoe, II, K.H. 2001. Liquefaction resistance of soils : summary report from the 1996 NCEER and 1998 NCEER/NSF workshops on evaluation of liquefaction resistance of soils, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 127(10) : 817-833.