Comparison of two damage tensor formulas for rock masses

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ABSTRACT
The main types of fractures in rock masses will be discussed. The geometric superposition formula and the energy equivalence formula of damage tensor for rock masses are also introduced. Based on a simple engineering case, these two formulas are analyzed comparatively. It is proposed that the geometric superposition formula of damage tensor is preferable in general engineering practices.

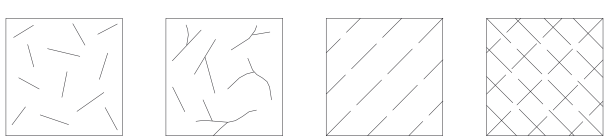
RÉSUMÉ
On a discuté les types principaux de la fracture dans le rocher. On a également introduit la formule de superposition géométrique et la formule d'équivalence d'énergie sur la tension de la destruction du rocher. Par une comparaison, on peut bien connaître ces deux formules. On propose la formule de superposition géométrique de la tension de la destruction dans l'exécution du projet.

1 INTRODUCTION

Geological body is a kind of natural damaged material with self-organizing structure and characteristic of stress memory. Material damage is the existence, emergence and expansion of microscopic defects. In macroscopic damage theory, materials or objects containing deficiencies of all kinds are generally regarded as a continuum with micro-damage field; accordingly the formation, growth, transmission and gathering of this micro-damage field are treated as damage evolution processes. At the same time, appropriate damage variables are introduced to characterize the physical nature of this damage continuum. From an engineering point of view, it is feasible to treat the fractured rock masses containing widespread joints as damaged materials (Tao Zhenyu et al, 1993).

2 TYPES OF CRACKS IN ROCK MASSES

First look into the main types of cracks in rock masses (Figure 1). Figure 1(a) shows rock structure is dense with minor damage. With a higher degree of damage, figure 1(b) is evolved from figure 1(a) when geo-stress changes. Figure 1(c) is formed under a single load, such as simple compression, with a comparatively larger main value of damage. Figure 1(d) is formed under multiple loads, such as double press shear loads, which could be regarded as the equivalent superposition of damages in different directions from figure 1(c).



(a) (b) (c) (d)

Figure 1. Main types of cracks in rock masses

3 DAMAGE TENSOR FORMULAS FOR ROCK MASSES

3.1 The Geometric Superposition Formula

T. Kawamoto, Y. Ichikawa and T. Kyoya (1988) applied the damage theory early in the research of jointed rock masses. They developed a total damage tensor to describe jointed rocks, which can be derived from the damage tensors for different joint sets by simple superposition. The equation can be written as

$$\Omega_{total} = \sum_{i=1}^{M} \Omega_i = \frac{1}{V} \sum_{i=1}^{M} \left[\bar{b}_i (n_i \otimes n_i) \sum_{k=1}^{N} S_{k,i} \right] \qquad [1]$$

where Ω_{total} is the total damage tensor for all joint sets; Ω_i is the damage tensor for the joint set *i*; *M* is the number of joint sets; *V* is the volume of rock masses for the study; $\bar{b}_i$ is the average aperture width of joint set *i*; n_i is the unit vertical vector for the surface of joint set *i*; $\otimes$ is the dyadic symbol for vectors; *N* is the total number of joints in set *i*; $S_{k,i}$ is the area of joint *k* in set *i*.

If there is no dominant direction in the distribution of rock joints, it can be viewed as similar to random, and rock damage can be considered as isotropic. Thus, we obtain the following equation:

$$\Omega = \frac{1}{V} \cdot \bar{S} \cdot \bar{b} \cdot \sum_{i=1}^{T} (n_i \otimes n_i) \qquad [2]$$

where T is the total number of joints in volume V; $\overline{S}$ is the total average surface area of joints in volume V; $\overline{b}$ is the average aperture of all joints in volume V.

3.2 The Energy Equivalence Formula

Considering the anisotropic property of rock masses and interaction among different joint sets, G. Swoboda, M. Stumvoll & Han Beichuan (1990) proposed that it was not accurate to calculate the global damage tensor by direct superposition of the single damage tensors. Based on the energy principle, they deduced the expression for global equivalent damage tensor which reflected the inter-influence of all cracks. The equation could be written as:

$$\Omega_g = I - \left[\sum_{i=1}^{M} \left(I - \Omega_i \right)^{-1} - \left(M - 1 \right) I \right]^{-1} \qquad [3]$$

where I is a unit matrix; M is the number of joint sets. At the same time they came to an important conclusion: If sets of cracks exist in a damaged body, the additional damage strain caused by the cracks under the applied load condition is equal to the sum of additional damage strains from every single set of cracks under the same applied load condition (Figure 2).

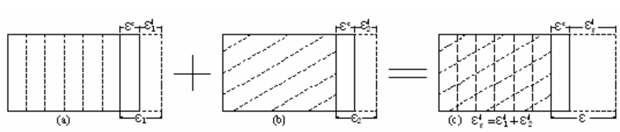


Figure 2. Additional damage strain (Swoboda et al, 1990)

4 CASE COMPARISON OF DAMAGE TENSOR

4.1 Damage Tensor of the Geometric Superposition Formula

Based on equation [1], the damage tensor of a single joint set can be evolved to the following expression:

$$\Omega_i = J_{vi} \cdot a_i \cdot b_i \cdot n_i \otimes n_i \qquad [4]$$

where J_{vi} is the volume density of the joint set i, a_i is the average area of joint set i, b_i is the average aperture of joint set i, n_i is the unit normal vector for joint set i.

Assume each joint is disk-shaped, and the mean diameter of each disk is equal to its trace length, then:

$$a_i = 0.25 \cdot \pi \cdot d_i^2 \qquad [5]$$

Table 1 gives the statistical parameters, required for damage tensor calculation, of rock masses from a certain dam site located in China.

Table 1. Parameters of joint sets for damage tensor calculation

Set no.	J_{vi} (number/m^3)	a_i (m^2)	b_i (mm)	Mean occurrence (°)
1	0.1353	20.5887	0.127	117.0∠27.68
2	0.1364	9.8980	0.127	117.8∠50.17
3-1	0.1932	2.8953	0.290	180.0∠82.78
3-2	0.1933	2.8953	0.290	346.5∠81.84
4	0.2663	8.2448	0.127	279.0∠63.16
5	0.1245	5.3913	0.213	69.1∠76.39

The damage tensor based on the geometric superposition formula in the dam coordinate system is:

$$\Omega = \sum_{i=1}^{5} \Omega_i = \begin{bmatrix} 5.2286 & -0.5103 & -2.2120 \\ -0.5103 & 3.2576 & 0.1656 \\ -2.2120 & 0.1656 & 4.1841 \end{bmatrix} \times 10^{-4} \qquad [6]$$

4.2 Damage Tensor of the Energy Equivalence Formula

Based on the formula [3] and data in table 1, the corresponding damage tensor in the dam coordinate system is:

$$\Omega_g = \begin{bmatrix} 5.2265 & -0.5095 & -2.2105 \\ -0.5095 & 3.2571 & 0.1655 \\ -2.2105 & 0.1655 & 4.1830 \end{bmatrix} \times 10^{-4} \qquad [7]$$

Comparing the two results of damage tensor shown in expressions [6] and [7], we can deduce the following two conclusions:

(1) The corresponding components of Ω and that of Ω_g are very close, and the largest relative deviation is less than 1‰.

(2) The absolute value of each component of Ω_g is slightly smaller than that of the corresponding component of Ω, and modulus $|\Omega_g|$ (=8.1098×10^{-4}) is slightly smaller than modulus $|\Omega|$ (=8.1129×10^{-4}), the relative deviation is less than 1‰ as well.

Through further calculation on the damage tensor result [6], the principle values and their corresponding principle directions in the dam coordinate system are obtained as follows:

$\Omega_1=7.0461\times10^{-4}$, $\boldsymbol{p}_1=(-0.7806\ 0.1319\ 0.6109)^T$ namely $170.4°\angle37.7°$;

$\Omega_2=3.2336\times10^{-4}$, $\boldsymbol{p}_2=(0.03826\ \ -0.9656\ \ 0.2573)^T$ namely $-87.7°\angle14.9°$;

$\Omega_3=2.3906\times10^{-4}$, $\boldsymbol{p}_3=(0.6239\ 0.2239\ 0.7488)^T$ namely $19.7°\angle48.5°$.

The results of damage tensor for the studied case show that the principle direction corresponding to the largest principle value points to the upper reaches; the principle direction corresponding to the middle principle value points to the inner of left dam shoulder; and the principle direction corresponding to the minimum principle value points to the lower reaches, as are favourable to the stability of rock masses at dam foundation and left shoulder slope.

5 CONCLUSION

The geometric superposition formula and the energy equivalence formula of damage tensor for rock masses are not contradictory, because when i=1, formula [3] is equivalent to formula [1]. But these two formulas are substantially different, because when i≠1, formula [3] can’t be transferred into formula [1].

Ω is the volumetric damage tensor with a pure geometric sense of rock fractures, yet Ω_g is the volumetric damage tensor with consideration on the additional damage strain caused by the interaction of rock fractures.

By example, we can see that the interaction of fractures will consume a very small portion of energy, so from the viewpoint of engineering this tiny energy can be ignored. In addition, the energy equivalence formula of damage tensor is rather complicated involving computing between large values and small values of which the order of magnitude exceeds ten as may lead to an abnormal computer result.

Therefore, this paper suggests that the geometric superposition formula of damage tensor is preferable, of which the calculated results are slightly conservative, in general engineering practices. As well, the energy equivalence formula of damage tensor is available for further research on damage property of rock masses.

REFERENCES

Tao Zhenyu, Zhao Zhenying, Yu Qihua et al, 1993. *Properties of fractured rock masses and mechanical problems in construction of tunnel group (in Chinese)*, 1st ed., China University of Geosciences Press, Wuhan, Hubei, P. R. China.

Kawamoto, T., Ichikawa Y. and Kyoya, T. 1988. Deformation and fracturing behaviour of discontinuous rock mass and damage mechanics theory. *International Journal for Numerical and Analytical Methods in Geomechanics*, 12: 1-30.

Swoboda, G., Stumvoll, M. and Han Beichuan. 1990. Damage tensor theory and its application to tunnelling. *Proceedings of the International Conference on Mechanics of Jointed and Faulted Rock*, Balkema, Rotterdam, Holland, 51-58.