Comparison of 2D and 3D distinct element analyses in stability of convex jointed rock slopes



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ABSTRACT

In this paper, using distinct element method, a comparison is made between 2 dimensional and 3 dimensional analyses of convex slopes in jointed rock mass. It is shown that 2D modeling of convex slopes may lead to unsafe design. The influence of the coefficient of lateral pressure as well as joint spacing and joint stiffness on the stability of convex slopes is also investigated.

RÉSUMÉ

Dans ce papier, utilisant la méthode d'élément distincte, une comparaison est faite entre 2 à dimensions et 3 à dimensions analyse de pentes convexes dans la masse de rocher démontable. Il est montré que 2D modelage de pentes convexes peut mener à la conception dangereuse. L'influence du coefficient de pression latérale de même que l'espace de joint et la raideur commune sur la stabilité de pentes convexes est aussi examinée.

1 INTRODUCTION

The curvature of rock slopes can affect its stability and its displacements. In this article, the behaviour of concave jointed rock slopes which mainly exist in the bends of roads and highways is investigated.

Some studies on the effect of the curvature of slopes on their stability have been made. Jenike and Yen (1961) presented the results of a limit theory analysis performed on the slopes of rigid-perfectly plastic material. They determined S-shaped critical profiles that describe the theoretical failure shape for different radii of curvature. They showed that, as the radius of the slope increases, the profile of the stable slope in axial symmetry approaches the profile of the slope in plane strain. Hoek and Brown (1997) concluded that the analysis assumptions were not applicable to rock-slope design. Piteau and Jennings (1970) studied the influence of plan curvature on the stability of slopes in four diamond mines in South Africa. Hoek and Bray (1981) summarized their experience with the stabilizing effects of concave slope curvature. Yoon et al. (2002) presented a kinematic analysis method for the slip failure of multi-faced rock slopes.

The review of the previous studies indicate that most of the studies are related to the concave slopes especially in the open pit mines and the issue of convex rock slopes have not been dealt with sufficiently.

In this article, the use of discrete element modeling technique is presented for the stability analyses of convex rock slopes. Universal Distinct Element Code (UDEC) is used for the analyses in two dimensions and 3DEC (Itasca, 2004) is used for the three dimensional analyses.

2 FUNDAMENTALS OF 3D DISTINCT ELEMENT METHOD

The 3DEC software is the three dimensional version of Distinct Element Method (Cundall, 1971, 1988). The 3DEC model consists of several discrete blocks which

represent the discontinuum medium and discontinues act as the boundaries between the blocks.

Large displacement such as a slip or separation can be simulated on discontinues.

Relative displacement along discontinues is determined by linear displacement- force relationship for both shear and normal stresses. The program uses an explicit solution scheme which results in a stable solution for the unstable physical processes (Itasca, 1998).

The aim of distinct element method is calculation of stress and strain in discontinuum media such as jointed rock masses. Basic equations which are used in distinct element method are a force-displacement law in the form of constitutive law and motion equations according to the Newton's Second Law. For deformable blocks, constitutive laws are used for the block material to calculate stresses in blocks together with the nodal forces in the nodal points. These constitutive laws use the finite difference technique to calculate stresses in blocks. In addition to the relative displacements between blocks, the blocks have interaction with each other on their contact surfaces.

Block displacements are computed using unbalanced moments and forces which act at the center of gravity of the blocks. The resultant forces include boundary forces and are applied to the edge of the blocks. Blocks acceleration, w(t) are related to the resultant forces by Newton's Second Law:

$$\frac{\rho}{m} = \frac{F(t)}{m} + \beta$$
[1]

Where m is the block mass, t is time, and g is gravitational acceleration.

The new arrangement of the blocks generates new conditions and thus new contact forces at the block boundaries. Applied constitutive laws for the contacts are:

$$\Delta \sigma_n = K_n \Delta u_n,$$
 [2]

$$\Delta T = K_s \Delta u_s$$
[3]

Where K_n and K_s are normal and shear stiffness per unit area, respectively, $\Delta\sigma_n$, $\Delta\tau$, Δu_n and Δu_s are normal and shear stress variations and normal and shear displacement variations, respectively. Stresses are calculated using the selected yield criterion (e.g., Mohr-Coulomb criterion) in the nodes along the contacts.

$$\tau = \sigma_n \tan \phi + C$$
 [4]

Where $\boldsymbol{\phi}$ is the angle of internal friction and C is cohesion.

The internal deformations of the blocks are calculated using a mesh of tetrahedral elements. Newton's second Law is applied for the calculation of velocities and displacements at any node. Strains are determined using the displacement and the stresses are computed using the selected constitutive law by the user.

3 NUMERICAL MODELLING

3.1 2D versus 3D Analysis

The first step in the creation of a model is to decide between performing the analyses in two dimensions or three dimensions. In many problems associated with tunnelling and mining, the specific conditions don't allow the use of plane strain condition and therefore three dimensional analyses are required. These conditions include (Lorig and Varona, 2000):

- Direction of principle geologic structures does not strike within 20° to 30° of the strike of the slope.
- Axis of anisotropy does not strike within 20° to 30° of the slope.
- Directions of principal stresses are thought to be not parallel or not perpendicular to the slope.
- Distribution of geomechanical units varies along the strike of the slope.
- Slope geometry in plan cannot be represented by two-dimensional (i.e., axisymmetric or plane strain) analysis.

3.2 Creation of Model Geometry

To simulate the convex slope in 3DEC, a FISH subprogram has been used. A sample of the geometry of the models used in the X-Y Plane is presented in Figure 1(a). In Figure 1(b), the various curvatures of slopes involved in the analyses are shown. The curvatures have been obtained from the hyperbolic relationship of $z=k.x^2$ in which k is the coefficient of the hyperbola and x and z are the coordinates as presented in Figure 1(b). As shown in Figure 1(a), for modeling, 3 zones of A, B and C have been used. In Zone B, the joint spacing is 4 times the joint spacing in Zone A and in Zone C; the joint spacing is twice the joint spacing in Zone B.

To evaluate the effects of the coefficient of lateral pressure, normal stiffness and shear stiffness of the joints, the slope height is taken as H=50m and the slope curvature is taken as a circle with a radius (r_c) of 50m (Figures 4 to 6). In order to investigate the effect of slope curvature (k), the slope height is taken as 50m with joint spacing of 6m (Zone A) (Figure 9).

To investigate the influence of joint spacing, joint sets with spacing of 4, 8, 15 and 30m (Zone A) are used for two slopes with heights of 30m and 50m. To evaluate the effects of the above-mentioned parameters, one joint set (J_1) is involved. The properties of this joint set are presented in Table 1.

The effects of coefficient of lateral pressure, normal stiffness, and shear stiffness have also been evaluated for slopes with two joint sets. For this purpose, another joint set (J_2) , perpendicular to the joint set J_1 has been considered in Zone A. The properties of this joint set are also presented in Table 1.

Table 1. Characteristics of joint sets

Joint set	Dip Direction	Dip(β)	Spacing
	[°]	[°]	[m]
J_1	90	40	Various
J_2	270	50	30

The slope face angle, (α) is 60°.

A zero velocity has been applied in the horizontal direction to the vertical boundaries. The bottom boundary is fixed in the vertical direction and gravitational acceleration of 10m/s² is applied. Computations have been performed with sufficient time steps until the model reaches equilibrium.

To avoid slip or separation during initial loading to reach equilibrium, high shear and tensile strength were allocated to the joints. After reaching equilibrium, for the analyses, real values of strength were assigned to the joints. The stability of the blocks was evaluated with the gradual decrease of coefficient of internal friction and cohesion until displacements increase and model become unstable.



Figure 1. The model used for the convex slope; (a) two dimensional model in the x-y plane (section A-A), (b) various curvatures of the slope

3.3 Material Properties

The height the slope is low compared to the strength of the intact rock; therefore, for the intact rock, elastic, isotropic deformation is considered. For the joints, the failure criterion of Mohr-Coulomb is used.

The mechanical properties of intact rock and joints are shown in Tables 2 and 3 respectively.

Table 2. Mechanical properties of intact rock

Density [kg/m ³]	Bulk Modulus [GPa]	Shear Modulus [GPa]	Cohesion [kPa]	Friction angle [°]
2800	19	17	-	-

Table 3. Mechanical properties of joints

Normal Stiffness [GPa/m]	Shear Stiffness [GPa/m]	Cohesion [kPa]	Friction angle [°]	Tensile Strength [kPa]
0.12	0.01	30	30	-

4 CALCULATION OF SAFETY FACTOR USING SHEAR STRENGTH REDUCTION

For the slopes, safety factor is often described as the ratio of shear strength to the existing shear stresses. A common method for the estimation of safety factor in a discrete element program is reduction of joint shear strength until failure takes place. In this method, safety factor is the ratio of shear strength of the joint to the reduced shear strength at the point of failure. This technique of stress reduction was first used by Zienkiewicz and Humphson and Lewis (1975) for estimation of safety factor for a slope consisting of various materials. For the stability analysis using shear strength reduction technique, simulations are made for trial safety factors (F.S.) which are increased step by step. The shear strength parameters, cohesion (C) and internal friction angle (ϕ) are decreased for each step according to the relation:

$$r = \sigma_n \frac{\tan \varphi}{F.S.} + \frac{C}{F.S.}$$
[5]

5 RESULTS

The displacements of the slope in 3D analysis are shown in Figure 2. As expected, displacements take place mainly due to the slip of the joints.



Figure 2. Displacements of the slope after reaching equilibrium



Figure 3. Variation of safety factor with joint spacing for two slopes with different heights (2D and 3D Modelling)

The safety factors corresponding to the slopes with heights of 30m and 50m for K0=1 condition are presented in Figure 3. It can be noted that the value of safety factor in 3D analyses are lower than the value of safety factor in 2D analyses. This is due to the fact that in the convex slopes confinement by the adjacent materials is lower.

In order to investigate the effects of normal stiffness and shear stiffness on the analyses, sensitivity analyses were performed for the various values of these parameters. The values of normal stiffness were varied between 0.05GPa/m to 40GPa/m and the values of shear stiffness were taken as one tenth of the values of normal stiffness. As expected, lower values of stiffness resulted in higher displacements. However the whole behavior and type of failure remained unchanged. Corresponding results are presented in Figures 4 and 5.

Figure 6 shows the results pertained to the various values of coefficient of lateral pressure. It can be noted that the increase in the value of this parameter results in a decrease in the value of safety factor.



Figure 4. Variation of safety factor with joint stiffness (3D Modelling & r_c =50m)



Figure 5. Variation of displacements in X Direction with joint stiffness (3D Modelling & r_c =50m)



Figure 6. Variation of safety factor with the coefficient of lateral pressure (3D Modelling & r_c =50m)

Variations of safety factor and displacements with various values of joint stiffness are compared for one joint set and two joint sets in Figures 7 and 8.



Figure 7. Comparison of analyses results using one and two joint sets. (a) Variation of safety factor, (b) Variation of displacements (3D Modelling & rc=50m)

By comparison of these two cases, it can be noted that the safety factor varies with a similar trend; while the displacements reach a constant and almost equal value.



Figure 8. Factor of safety vs. coefficient of lateral pressure and number of joint sets (3D Modelling & $r_{\rm c}{=}50m)$

To investigate the effect of curvature of the stability of the slope, analyses are made using different values of curvatures. In these analyses one joint set (J_1) is involved. The results are presented in Figure 9.

According to these results, with the increase of the slope curvature, due to the reduction in the lateral confinement, the safety factor decreases; especially in the lower end of curvature values.

6 CONCLUSIONS

In this paper, using distinct element method, a comparison was made between 2 dimensional and 3 dimensional analyses of convex slopes in jointed rock mass. It was shown that 2D modeling of convex slopes may lead to unsafe design.



Figure 9. Safety factor vs. curvature of the slope (3D Modelling)

The influence of the coefficient of lateral pressure as well as joint spacing and joint stiffness on the stability of convex slopes was also evaluated.

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