# Seismic site response modelling of representative soil columns in the Quebec City region

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#### **ABSTRACT**

This paper describes site effects investigation carried out as part of a comprehensive study of dynamic soil behaviour in the Quebec City region. Response of five typical surficial sediment columns, with total thickness ranging between 10 and 80 m, to artificial and recorded accelerograms was modelled using a nonlinear soil deformation model SIREN. A number of analyses were conducted applying various intensity levels by scaling the peak ground accelerations from 0.01 g to 1.0 g. Higher spectral accelerations on ground surface are obtained for the artificial accelerograms (for periods≥0.1s), for stiffer soil columns, and for longer strong motion durations.

# RÉSUMÉ

Cet article présente les résultats d'une investigation des effets de site menée dans le cadre d'une étude du comportement dynamique des sols de la région de Québec. La réponse de cinq colonnes de formations superficielles typiques de la région (avec des épaisseurs totales variant de 10 à 80 m) à des accélérogrammes enregistrés et synthétiques a été modélisée à l'aide de SIREN, un modèle numérique de déformation non linéaire de colonnes de sols. Une série d'analyses a été effectuée en appliquant des séismes de magnitude variable, caractérisés par des accélérations de pointe variant de 0,01 à 1,0 g. Des accélérations spectrales au sol plus élevées ont été obtenues dans le cas d'accélérogrammes synthétiques (pour des périodes ≥ 0.1s), de colonnes de sol plus rigides et de mouvements sismiques forts de plus longue durée.

#### 1 INTRODUCTION

It is well known that seismic motions at the surface of a soil column can have significantly different characteristics from those of the motions at the underlying bedrock. Depending on the thickness, density and shear modulus of the soil deposit as well as the intensity, frequency content and duration of the bedrock motion, the seismic motion can be amplified or deamplified at the ground surface. These site effects have been considered in the National Building Code of Canada in correlation with the shear wave velocity of the upper 30 m of the soil deposits (NBCC 2005). In the code, the seismic input for a given location is defined through a design response spectrum and a procedure is proposed to scale the site class C design spectrum for different soil classes by using frequency and intensity dependent scale coefficients (Fa and Fv).

The Quebec City region is known for its intense seismic activity (Lamontagne, 2008). Historically, strong earthquakes ( $m_b \geq 6$ , epicentral distances ~100 km) caused damages on several occasions: Charlevoix - Kamouraska (1663, 1791, 1860, 1870, and 1925) and Saguenay (1988). A number of local earthquakes with lower magnitudes also occurred in the past. Major earthquakes may occur again and this emphasizes the need for preparing a program to minimize future damages.

The bedrock consists mainly of sedimentary rocks, generally slate and sandstone in the Appalachians and shale and limestone in the St. Lawrence Lowlands (Globensky, 1987). Sedimentary rocks are underlain at depth by older rocks of the Canadian Shield which crop out in the northern part of the city. Based on preliminary field results, the sedimentary bedrock formations

underlying the soil deposits in Quebec City are classified as site class B (Lin et al., 2007). Glacial till is ubiquitous at the base of the Quaternary sequence. It is generally overlain by younger fine grained to medium sands, marine silty clays and deltaic sands. In the northern part of the study area, glacial till is directly overlain by marine silty clays and beach sands.

Several authors studied the geotechnical properties (Cockburn, 1984) and site response characteristics in the Quebec City region (Doré, 1984; Michaud, 2006). Nastev et al. (2008a) described a procedure for the generation of two sets of acceleration time histories at bedrock level, i.e. scaled accelerograms of recorded seismic motions and synthetic accelerograms consistent with the uniform hazard spectrum for site class B. For the ongoing dynamic soil behaviour study, however, acceleration time histories at the ground surface are needed to evaluate the structural response and the vulnerability of city infrastructure (Leboeuf et al., 2006).

This paper discusses results from numerical simulations of the seismic site response conducted using five representative soil deposits and applying both input data sets as seismic excitation. One-dimensional simulations were carried out using the nonlinear computer (www.oasys-software.com). program SIREN objectives of this study are: i) to estimate the characteristics of the site responses expected in the region, ii) to verify which of the data sets is more conservative (generates higher spectral accelerations) and more appropriate to be used, and iii) to compare the computed results with the design spectra defined in NBCC 2005. All the comparisons were made in terms of 5% damped spectral acceleration for the considered soil columns and levels of seismic shaking.

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# 2 DESCRIPTION OF THE METHOD

#### 2.1 Soil columns

A suite of five soil columns (Figure 1, #1 through #5) with corresponding depths to bedrock of 10, 20, 30, 50 and 80 m is considered in this study. Soil columns were carefully selected to represent the typical soil conditions in the Quebec City region. Column # 1 is fairly representative of the northern rim of the city where the foothills of the Laurentians are characterized by relatively shallow depths to bedrock (0~30 m). In that region which lies below marine limit, relatively thin offlap beach sands commonly overlie marine silty clays which in turn overlie glacial sediments and bedrock.

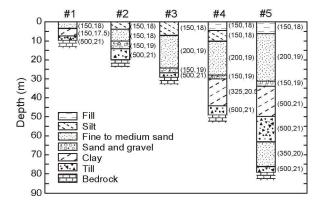


Figure 1. Schematic soil columns. Numbers in parentheses indicate assumed typical shear wave velocity (m/s) and unit weight (kN/m³) for the soil materials.

In the rest of the study area, glacial sediments are ubiquitous at the base of soil columns consist of compact. generally matrix-dominated, diamictons with variable thickness and clast lithology, thought to have been deposited during the last glaciation. Occasionally, older sediments (till and medium to fine grained sand) can be found, e.g. column #5 representative of the buried valley underlying the actual lower town), but their extent and age are still being investigated. Preliminary data suggest that it may represent an interstadial fluvial or marine unit and may be overlain at least locally by till. The next overlying unit consists of marine silty clays whose thickness may reach over 15 m in the deepest parts of the buried valley (column # 5) but is commonly less than that (column # 4). In the buried valley of the lower town, these clays are overlain by deltaic sands (columns # 4 and 5) and are thus seldom exposed at the surface. Elsewhere in the Québec City region, these marine clays are frequently exposed at or near the surface. Columns # 2, 3, 4 and 5 depict common situations in the buried valley where medium to coarse-grained deltaic sands constitute the main subsurface unit but are almost never exposed at the surface. In most situations, these sands are overlain either by a few meters of intertidal silts (columns # 2 and 3), by fill (column # 5), or both (column # 4).

#### 2.2 Numerical model

The one-dimensional simulations were carried out using the program SIREN (www.oasys-software.com), based on the solution of the wave equation for vertically travelling shear waves in the horizontally layered soil rock system. The soil column is specified as a series of layers represented with lumped masses each with its own material type characterised by a stress-strain relationship and a bulk density, and elastic-plastic springs. The program operates in the time domain and simulates directly nonlinear soil properties including hysteretic damping. At each time step, the shear stress in each soil layer is calculated from the shear strain existing in that layer at that time. The acceleration of each lumped mass is calculated as the net force acting on that mass (difference of the shear stress above and below the mass) divided by the mass. The displacement and velocity are obtained by numerical integration of the acceleration. Having calculated the displacement of the masses, the shear strain and consequently the shear stress at the next time step for each layer can be determined and the process is repeated until the total specified time is achieved.

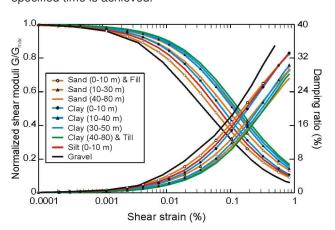


Figure 2. Normalized modulus decay and hysteretic damping curves as functions of the strain. Values in parentheses indicate approximate depth for given soils and corresponding unit weights.

# 2.3 Soil dynamic properties

The degree of nonlinearity within each soil column is determined by use of strain dependent shear modulus and damping curves. The applied shear modulus reduction curves for sands, silts and clays were based on Zhang et al. (2005). The influence of the mean effective stress acting at a given depth was taken into account in their study. For the purpose of this study the value of the coefficient of effective earth stress at rest taken as 0.5. For gravels, the mean shear modulus reduction curves proposed by Rollins et al. (1998) were used. Finally, as tills in the study area have fine grained matrix, it was assumed that their dynamic behaviour can be approximated by that of clays. For comparison, a similar study of the site response with fully nonlinear analysis is currently underway. For both studies hysteretic damping

curves were calculated. The applied shear-modulus degradation and damping ratio curves for the considered soils materials are shown in Figure 2.

Values for the shear wave velocity  $v_s$  (m/s) and unit weight γ (kN/m<sup>3</sup>) have been assumed for the different soils present in the Quebec City region (Figure 1) based on previous studies (Cockburn, 1984; Doré, 1984; Michaud, 2004). The dynamic shear modulus  $G_{max}$  $(kN/m^2)$  was computed as  $G_{max}=v_s^2\gamma/g$ , where g is the acceleration due to gravity (9.81 m/s2). The average shear wave velocity for each of the soil columns, vsa, was estimated as a weighted average over the individual shear velocities and corresponding layer thicknesses according to the following equation  $v_{sa}=H/(\Sigma h_i/v_{si})$ , where, H is the total thickness of the soil column measured from the bedrock level up to the ground surface, and  $h_i$  and  $\nu_{si}$ are thickness and shear wave velocity of ith soil layer, respectively. A soil column tends to vibrate in its own natural frequency in which maximal spectral accelerations (amplification of the bedrock shaking) are expected. The approximate first natural period of ground vibration for elastic soil layer is related to the average shear wave velocity by the well known quarter-wavelength relation: T=4H/v<sub>sa</sub>, (Dobry et al., 1976). The calculated values for the average  $v_{sa}$  and the fundamental site period T are shown in Table 1. According to the properties of each of the soil materials, the average shear wave velocity for the soil columns varies between 190 and 281 m/s. The soil column #1 has the lowest  $\nu_{\text{sa}}$ , whereas soil column #5 is the stiffest mainly due to the thick compacted clay deposits located at depths between 35 and 60 m. Based on the soil classification criteria given in NBCC 2005 and the average shear wave velocity for the first 30 meters, all five soil columns correspond to the site class D stiff soils  $(v_{sa-30} = 180 \sim 360 \text{ m/s}).$ 

Table 1. Average shear wave velocity and fundamental

site periods

site period	3			
Soil	Total	Average shear		Fundamental
column	thickness	wave velocities		site period
	H (m)	$\nu_{\text{sa}}$	$v_{\text{sa-30}}$	T(s)
		(m/s)	(m/s)	
#1	10	190	190*	0.21
#2	20	211	211*	0.38
#3	30	199	199*	0.60
#4	50	227	183	0.88
#5	80	281	189	1.14

<sup>\*</sup> v<sub>sa-30</sub>=v<sub>sa</sub> for H≤30 m

The base layer, or the so called engineering bedrock, was defined with average shear wave velocity  $\nu_s\!=\!1200$  m/s.

# 2.4 Seismic input

The input seismic motions at bedrock level consist of two sets of 10 accelerograms (Nastev et al., 2008a; Nastev et al., 2007). The first set consists of real accelerograms scaled to fit (exceeding for approximately 10%) the design spectrum for soil class B for Quebec City at a given period. The second set was derived by modifying the frequency content of the accelerograms of the first set such that their 5% damped spectra match approximately

the design response spectrum for soil class B over the entire period range of interest. The results from these data sets are indicated in the following figures with 'scaled' and 'synthetic' respectively.

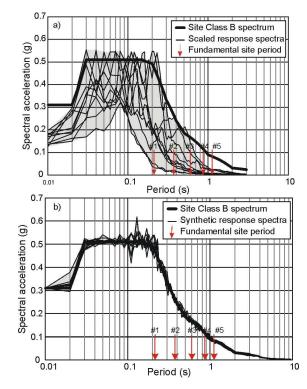


Figure 3. Acceleration response spectra (5% damping) for a) selected strong motion records; and b) generated artificial accelerograms. Arrows indicate fundamental site periods of the soil columns.

Figure 3 shows the acceleration response spectra (RS) of both data sets together with the site class B spectrum for Quebec City and the fundamental site periods of the five soil columns. It can be observed that the individual RS of the scaled data set cover the target spectrum in the vicinity of respective peak spectral acceleration. However, they are considerably below the target spectrum at low periods ≤0.03 s and at periods ≥0.3 s. On the other hand, the RS of the synthetic data set match closely the target spectrum. Near the fundamental site periods of the soil columns, the spectral accelerations of the synthetic data set are considerably higher than spectral accelerations of the scaled data set.

In order to represent various levels of seismic shaking in the simulations, the target spectrum was scaled to peak ground accelerations (PGA) of 0.01g, 0.05g, 0.1g, 0.2g, 0.3g, 0.4g, 0.5g, 0.75g and 1.0g. The corresponding time histories of both data sets were also scaled by the same rate. Lower shaking levels, between 0.01g and 0.1g, are assumed to produce predominantly elastic response of the soil columns, whereas for stronger shaking levels (0.4g ~1.0g), non-linear strains are expected. The shaking level of 0.3g corresponds to the prescribed PGA at bedrock level for Quebec City (Adams and Halchuk, 2003; NBCC, 2005).

#### 3 SIMULATION RESULTS

The one-dimensional analyses of the site response were conducted for the five soil columns, the nine shaking levels (0.01~1.0g), and both sets of accelerograms. In total 900 simulations were considered in this study.

# 3.1 Synthetic vs. scaled mean RS

The mean RS, computed as an arithmetic average of the RS of the individual time histories at the ground surface, for all input shaking levels considered are given in Figure 4a. Comparing the results shown in Figures 4a, it can be noticed that 'stiffer' subsoil conditions with shorter fundamental site periods (#1, #2) generate higher spectral accelerations than 'softer' subsoil conditions with longer vibration periods (#3, #4 and #5). This is due to the coincidence of the fundamental site periods of the stiffer columns with comparably higher accelerations of the input motions (Figure 3). The frequency content of both sets of RS is quite different. Whereas the spectral energy of the scaled accelerograms is concentrated at shorter periods (≤0.1s), the mean synthetic RS are much broader. It is also noticed that the synthetic RS identify the fundamental site periods, although the maximal spectral acceleration may be associated with other vibration modes at shorter periods. A gradual increase of the predominant response frequencies with the increase of the level of shaking can also be observed (Figure 4a). The shifting of the period can be explained by the degradation of the dynamic shear modulus of soils due to the increased strains experienced during intense earthquake shaking. For periods ≤ 0.1 s, both sets of accelerograms generate comparable site responses (Figure 4b). Above this threshold, the ratio between the synthetic and scaled RS increases rapidly and the maximum is attained for periods of 3~4 s.

# 3.2 Computed RS for 0.3g shaking level

It is important to focus on soil responses for bedrock shaking level of 0.3 g. Note that this shaking level corresponds to the site class B (bedrock) and is used as input design level for Quebec City. The resulting RS and the comparison with the site class B and D (ground surface) are given in Figures 5. The mean synthetic RS is higher than the mean scaled RS for all soil columns and for almost all the periods.

Compared to the site class B spectrum, amplification of the seismic shaking at the ground surface due to the synthetic accelerograms is highest at and near the fundamental periods of the soil columns (Figure 5b). For the scaled accelerograms, amplification occurs for periods ≤0.2s, whereas deamplification of the seismic signal is observed for longer periods.

The comparison with the target site class D spectrum indicates that the mean scaled spectral accelerations are usually lower (below the 1.0 level in Figure 5c) with the exception of several isolated spectral periods for soil columns #1 and #2. For shorter periods (≤0.2s), the spectral accelerations of the D spectrum are comparable

to the synthetic spectral values for soil columns #1 and #2. For soil columns #3, #4, and #5, the D spectrum is generally more conservative for periods ≤0.2s. The mean synthetic RS is, however, much higher for spectral values centered at the fundamental site frequencies of the soil columns (Figure 5c).

# 3.3 Effects of time duration on non-linear response

In addition to the response spectrum characteristics, the non-linear response of soil columns may be influenced by the time-domain character of the time history (e.g., shape, sequence, and number of pulses). Since these time domain characteristics vary greatly for time histories with similar spectral content, it is interesting here to investigate the influence of the strong motion duration of the synthetic accelerograms on the response characteristics.

Table 2. Strong motion duration of the synthetic accelerograms

Accel.	Strong motion duration T <sub>d</sub> (s)			
Accel.				
#	VanMarcke and Lai	Trifunac and Brady		
1*	0.35	17.58		
2	5.1	17.97		
3	7.77	23.51		
4	4.96	19.45		
5	6.17	18.89		
6	3.88	22.65		
7*	1.96	18.85		
8*	0.27	37.96		
9	4.04	17.93		
10	5.77	16.1		

\* short duration ≤0.2s

For this study, the strong motion duration was computed according to the method proposed by VanMarcke and Lai (1977). It is based on the random vibration analysis and considers the earthquake motion as a stationary stochastic process. The strong-motion duration is defined proportional to the ratio of the Arias intensity and the maximum ground acceleration. The strong motion duration for the ten synthetic accelerograms is given in Table 2. For comparative purposes, the strong motion duration was also estimated according to the method proposed by Trifunac and Brady

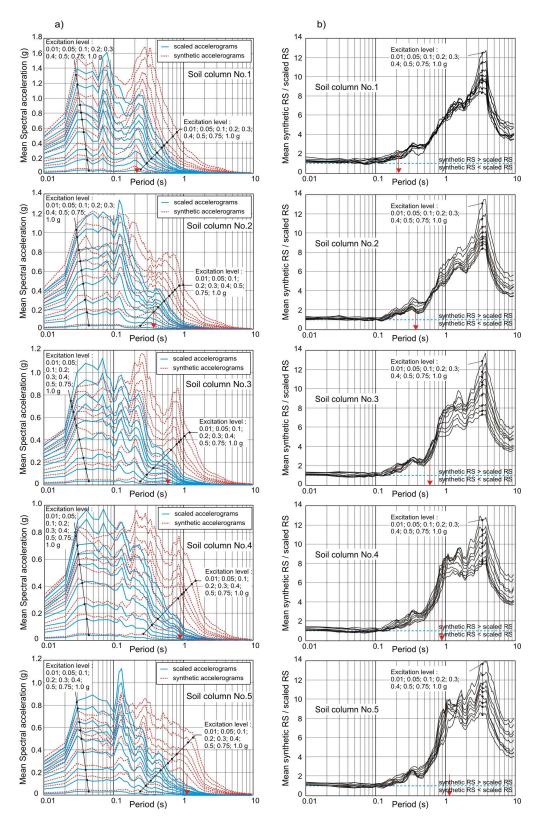


Figure 4. a) Mean site response spectra computed for both sets of accelerograms for shaking levels: 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.75 and 1.0 g; and b) ratios between the mean synthetic and scaled RS. Arrows indicate fundamental site periods.

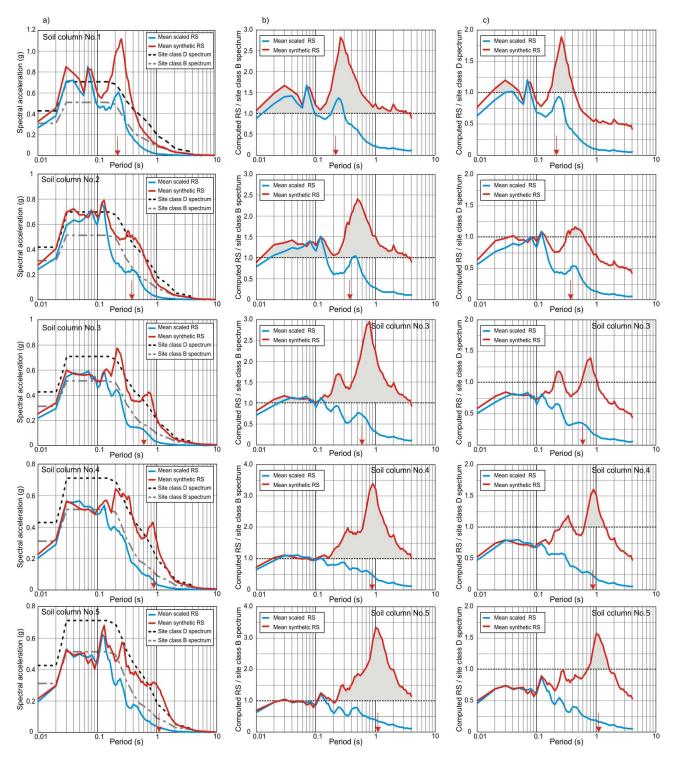


Figure 5. Site response spectra computed for shaking level 0.3 g: a) mean RS for scaled and synthetic sets of accelerograms, b) ratio between the synthetic and scaled mean RS vs. site class B spectrum, and c) ratio between the synthetic and scaled mean RS vs. site class D spectrum. Shaded zones indicate zones of amplification of the input signal (b), and zones where the computed RS are more conservative than the site class D spectrum (c). Arrows indicate fundamental site periods of soil columns.

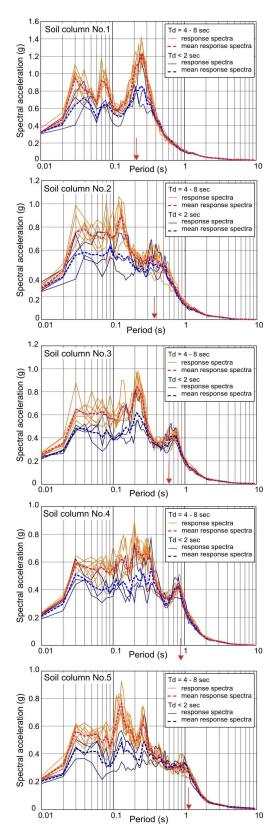


Figure 6. Effects of strong motion duration on the non-linear site response of the synthetic accelerograms.

(1975). This method is based on the computation of the time duration corresponding to the time span between 5% and 95% of the total energy arrival.

It can be seen from results given in Table 2 that the strong motion duration computed according to the VanMarcke and Lai method is more sensitive to the short intervals of high amplitudes than that computed by the Trifunac and Brady method. The later tends to overestimate significantly the strong motion duration of the records. The individual RS of the ten synthetic accelerograms for the 0.3g bedrock shaking level are given in Figure 6. As expected, the accelerograms with shorter strong motion duration yield lower RS for all soil columns. To emphasize the effects of the strong motion duration, the RS of accelerograms with strong motion duration ≤ 2.0s on one side and the RS of the remaining accelerograms on the other are grouped together and the respective mean RS computed. At the periods of interest for civil engineering structures (≥0.1s), the mean RS of the synthetic accelerograms with longer strong motion duration is approximately 40% to 60% higher than that of the accelerograms with smaller strong motion duration.

# 4 CONCLUSIONS

The non-linear site response of five representative soil columns was analysed using the one-dimensional simulations by the computer program SIREN. The seismic input at the rock outcrop was defined by two acceleration time histories: scaled accelerograms of recorded seismic motions, and synthetic accelerograms, both consistent with the uniform hazard spectrum for site class B. In order to represent the various levels of seismic shaking, the target spectrum and the time histories were scaled to peak ground accelerations (PGA) of 0.01g, 0.05g, 0.1g, 0.2g, 0.3g, 0.4g, 0.5g, 0.75g and 1.0g. The comparison of the results from this study is essential for decisionmaking on the methodology and the direction of the future work on site effects in the Quebec City region.

The results obtained from this study indicate that 'stiffer' subsoil conditions with shorter fundamental site periods (soil columns #1, #2) generate higher spectral accelerations than 'softer' subsoil conditions with longer vibration periods (soil columns #3, #4 and #5). Thus, in areas with shallower soil deposits (10 to 20 m), the earthquake shaking at the ground surface can be to stronger than in areas with thicker soil deposits (≥30 m).

Depending on the input time histories data se, the synthetic accelerograms generated more conservative (higher) responses at soil surface particularly for longer periods,  $\geq 0.1$ s. This is primarily due to the incompleteness of the scaled set, which was not effective in matching the target spectra at short ( $\leq 0.03$  s) and long periods ( $\geq 0.3$  s). The synthetic accelerograms generated also higher amplifications of the seismic shaking at the rock level.

The site class D spectrum (representative for the ground surface) was more conservative than the computed RS for the scaled set of accelerograms for

all considered periods. The same observation is valid for the synthetic set of accelerograms except for periods centered to the fundamental period of vibration of the respective soil columns, where the computed mean site RS exceed the D spectrum up to 60%. This observation will have important implications for the ongoing study. It means that spectral accelerations computed according to the NBCC 2005 will be overestimated for periods other than the fundamental site periods; they will, however, be underestimated for periods close to the predominant site period.

As all of the synthetic accelerograms have very similar RS, the duration of the strong ground motion has been shown to have significant effect on the level of site response. Accelerograms with longer strong motion duration ( $4 \sim 8$  s) yield up to 60% higher RS than those with shorter time duration ( $\leq 2$ s).

Finally, in the absence of recorded strong bedrock motions particularly during moderate to severe earthquakes which are needed for a proper earthquake resistant design, synthetic earthquake motions have proven to be effective in predicting site response parameters. Thus, having in mind that for the ongoing study acceleration time histories at the ground surface are needed to evaluate the structural response and the vulnerability of the city infrastructure, the synthetic set of accelerograms appears suitable for representing the bedrock seismic motion. It is, however, recommended that the spectral accelerations defined in NBCC (2005) be also used to estimate site response for selected periods and for different soil classes present in the study area.

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