Site response analysis of high contrast shear wave velocity soil profiles using the generalized reflection/transmission method

Kasgin Khaheshi Banab
Department of Civil and Environmental Engineering, Carleton University, Ottawa, ON, Canada
Siva Sivathayalan
Department of Civil and Environmental Engineering, Carleton University, Ottawa, ON, Canada
Dariush Motazedian
Department of Earth Sciences, Carleton University, Ottawa, ON, Canada

ABSTRACT
The surficial geology of the majority of Ottawa, Canada mainly consists of very loose soil with a low shear wave velocity of about 150 m/sec, underlain by very hard bedrock with a shear wave velocity of around 2500 m/sec. This remarkably large shear wave velocity contrast (15 to 20) between the soft soil and very hard bedrock can trap seismic waves, which results in multiple reverberations and large seismic soil amplification. To assess the seismic amplification in the Ottawa region, we modified the generalized method of reflection/transmission (R/T) coefficients by implementing a proper soil damping scheme in the R/T algorithm, based on the desired level of shaking. Some representative sites in the eastern part of the city were used to calibrate the R/T method with equivalent linear method (ELM). Being applicable for long range of seismic waves (SV and P), different angles of incidences and having the ability to model the wave trapping issue with a scheme to account for the internal reverberations of the reflected waves, R/T method can be used for the generating of the future seismic amplification maps in the Ottawa region which lacks available strong ground motion data.

RÉSUMÉ
En majorité, la géologie de surface d'Ottawa, Canada, est caractérisée par un sol à faible vitesse d'onde de cisaillement qui environne les 150 m/sec, et qui repose sur un fond rocheux très induré avec une vitesse d'onde cisaillante de 2500 m/sec. Ce fort contraste de vitesses (15 à 20 fois) entre le sol meuble et le fond rocheux peut trapper des ondes sismiques avec pour résultat de multiples réverbérations et une forte amplification du sol. Pour évaluer l'amplification sismique dans la région d'Ottawa, nous avons modifié la méthode généralisée du coefficient de réflexion/transmission (R/T) en implémentant un facteur d'atténuation appropriée du sol dans l'algorithme R/T basé sur le niveau désiré de secousse. Des sites représentatifs dans la partie Est de la ville ont été utilisés pour calibrer la méthode de R/T avec une méthode linéaire équivalente (ELM). Étant applicable pour une grande variété d'ondes sismiques (SV et P), différents angles d'incidences et ayant la possibilité de modéliser l'onde trappée avec la capacité de tenir compte des réverbérations internes des ondes réfléchies, la méthode R/T peut être utilisée pour générer les futures cartes d'amplification dans la région d'Ottawa qui manque de données concernant les forts mouvements de terrain.

1 INTRODUCTION

During many large earthquakes (e.g. Mexico City in 1985 and Kobe in 1995), the damage distribution in sedimentary basins has been attributed to the local site effects; and, seismic amplification has been one of the main rationales in the explanation of the observed structural failures and intense ground accelerations. This problem of seismic amplification has gained attention amongst geotechnical engineers and seismologists; and, many studies have been conducted to investigate the site response effects in sedimentary basins, using different analytical/numerical methods. Analytical/numerical methods constitute a significant part of the site response studies, especially when strong ground motion data is not available to implement empirical/semi-empirical analysis techniques. A good review of the methods that use earthquake records/strong motion data to delineate site amplification can be found in Safak (2001). Most of one, two and three dimensional analytical/numerical methods focus mainly on site effects that occur due to the vertical or oblique incidence of seismic waves from the underlying bedrock. Some of these studies are mentioned here:

Papageorgiou and Kim (1993) have shown one dimensional and two dimensional seismic site effects for deep layers of sediments on a half-space (bedrock) for different incidence angles of seismic waves, using the discrete wave number boundary element method (Kawase, 1988). Their study encompassed a relatively low velocity contrast (ratio of bedrock shear wave velocity to soil velocity) of about 2 between the soil layer and the underlying bedrock. Zeng and Benites (1998) developed a boundary integral discrete wave number approach to study the seismic response of two dimensional basins for the incidence of plane shear waves. They considered low velocity contrasts (<3) between the bedrock and its upper soil layer for their multilayer basins. Mossessian and Dravinski (1990) used an indirect boundary integral method to investigate the amplification of 2-D and 3-D valleys upon the incidence of the seismic waves. They studied valleys with a low velocity contrast (<2) between the half-space and the overlying valley sediment.

Also Bard and Bouchon (1980) examined the response of sediment-filled valleys for the incidence of seismic waves. They utilized the method of Aki and Larner (1970), which is derived from Rayleigh's representation of the scattered wave field for the plane wave incidence. Their velocity models consist of a low contrast shear wave velocity model with the largest contrast ratio of 5 between the bedrock and the overlying soil sediment. Sanchez-Sesma et al. (1989) carried out research on the ground motion effects on horizontally stratified alluvial basins. They used the mentioned discrete wave number (Aki and Larner, 1970) of plane wave (SH-waves) expansion in terms of the Haskell propagator matrices (Haskell, 1953). Some of other significant studies about the analytical/numerical solutions of seismic wave incidence on sedimentary basins can be found in Khair et al. (1991), Ashford and Sitar (1997) and Sanchez-Sesma and Luzon, 1995.

The very high contrast shear wave velocity, which is one of the main concerns in this study, was not the focus of the above-mentioned studies, although they illustrate most of the features of lateral interference of seismic waves, surface topography or basin shape effect, diffraction of seismic waves and resonance modes for seismic site amplification. In addition to lower shear wave velocity contrast, the damping values were assumed in the aforementioned studies; i.e. most of the researchers have solved their site response problem based on some selected damping values and they have not emphasized on the probable effect of damping variability on the final analysis outcomes. Consequently, the effect of the level of shaking on variations of soil damping was not a point of concern. They mostly involved boundary integral solutions and finite element/boundary element discretization of two dimensional or three dimensional basins and examined the low velocity gradient cases.

It should be pointed out that the different levels of shaking will generate different shear strain levels which in turn mobilize different damping values for the soil materials. In other words, with the increasing excitation or the input motion values, more strains are induced and the damping ratio decreases. This concept is one of the most important features of soil non-linearity which is very well-known in the community of geotechnical earthquake engineering.

The geological characteristics of the near-surface sediments in the Ottawa region are quite different from those of the above-mentioned studies. The surficial geology of the city consists of very loose and very low shear wave velocity post-glacial deposits (Holocene age sediments) underlain by very high shear wave velocity bedrock (see Hunter et al., 2006 and Motazedian et al., 2008). As an example, according to different seismic tests carried out at a site in Barrington Park of the Orleans area located in the eastern part of the city (See Figure 1), the average shear wave velocity of soil is about 146 m/sec; and, the average shear wave velocity of the underlying bedrock is about 2500 m/sec, which causes a very high shear wave contrast of 17 for this site. This unusual contrast can cause multiple internal reflections of seismic waves, leading to large seismic amplification values. Thus, suitable methods that are capable of addressing this issue should be sought for soil modeling in the study area.

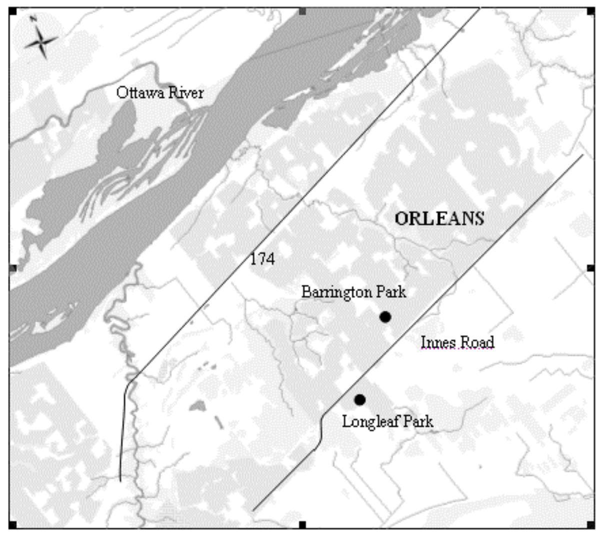

Figure 1. Two examined sites in the Orleans area located in the eastern part of the city of Ottawa.

In this study, to account for the internal reverberations of the seismic waves inside the soil layers, we applied generalized method of reflection/transmission (R/T) "(Kennett, 1974; Kennett et al., 1979) for two sites (See Figure 1) in the Ottawa region to obtain the seismic soil amplification for incident SV-waves. These sites, which have loose sediment on hard bedrock, are good representatives of the surficial geology of the eastern part of the city. The shear wave velocity variations of these sites are shown in Figures 2 and 3. In addition, to include the influence of the level of shaking, we improved the R/T algorithm to include soil non-linearity and introduced a damping function which is compatible with the peak ground acceleration of the simulated (artificial) ground motions for the study area.

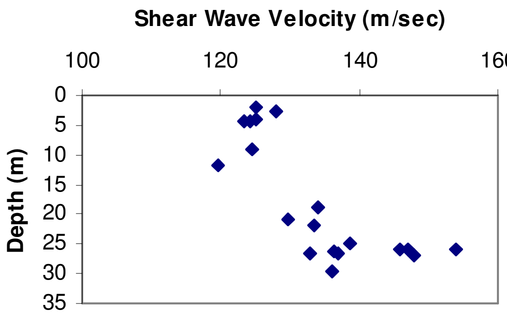

Figure 2. Shear wave velocity variation for Longleaf Park Site from reflection/refraction tests.

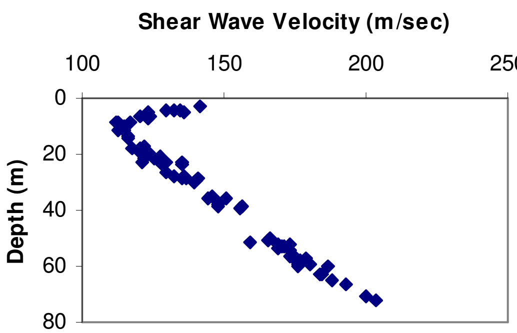

Figure 3. Shear wave velocity variation for Barrington Park Site from reflection/refraction tests.

2 MATRIX METHOD FOR SITE RESPONSE ANALYSIS

Generalized R/T method is the reformulation of the older Thompson/Haskell matrix method. Thompson (1950) introduced the matrix method to deal with the problem of incident seismic waves for a stratified medium. His matrix equation relates the stress-displacement components of different layers of soil using the total layer matrix derived from the multiplication of the layer matrices of all the soil layers.

Both of Thompson and Haskell's methods need some modifications in order to be applicable for the purpose of this paper. First, in Thompson's method, the incident and reflected wave angles should be calculated inside each layer using Snell's law. These calculations can be time-consuming when more complicated schemes of reflection and transmissions occur inside the layers. Second, neither of these two matrix methods delivers a comprehensive scheme for the problem of the internal reflections and reverberations inside a layer.

3 GENERALIZED METHOD OF REFLECTION/TRANSMISSION (R/T)

The R/T method was developed based on the two concepts of scattering matrices at the interfaces of soil layers and the propagator matrix (Gilbert and Backus, 1966). Using these concepts Kennett, 1974 introduced the famous scheme for the internal reflections of the seismic waves:

$$R^{13}_{U} = R^{23}_{U} + T^{23}_{D} R^{12}_{U} [I - R^{23}_{D} R^{12}_{U}]^{-1} T^{23}_{U} \quad [1]$$

$$T^{13}_{U} = T^{12}_{U} [I - R^{23}_{D} R^{12}_{U}]^{-1} T^{23}_{U}$$

These equations show all the internal reflections for a superposed media. R and T indicate the reflection and transmission matrices, respectively. U and D subscripts denote the up-going and down-going waves, respectively. "I" is the unit matrix. These equations can be better interpreted by expanding the inverse of matrices. The inverse matrices in equation 1 can be expanded using the matrix math. This expansion allows including all the reflection/transmission matrices without complications arising from unit matrix (I) and the inverse matrices.

For a given square matrix A, the expansion is:

$$[I - A]^{-1} = I + A + A^2 + \ldots \quad [2]$$

Using Equations 1 and 2, one can obtain the reflection and transmission coefficients. Thus, the reflection coefficient for the upward wave propagation, as an example, is:

$$R^{13}_{U} = R^{23}_{U} + T^{23}_{D} R^{12}_{U} T^{23}_{U} + T^{23}_{D} R^{12}_{U} R^{23}_{D} R^{12}_{U} T^{23}_{U} + \ldots \quad [3]$$

All the terms in Equation 3 can be interpreted as in the following two examples. The terms should be read from right to left. The first term (R^{23}_{U}) is the reflection from the lower level of the layer. The second term ($T^{23}_{D} R^{23}_{D} T^{23}_{U}$) shows the transmission up through the lower level, reflection by the lower level and transmission down through the lower level. This internal reverberation scheme is shown in Figure 4.

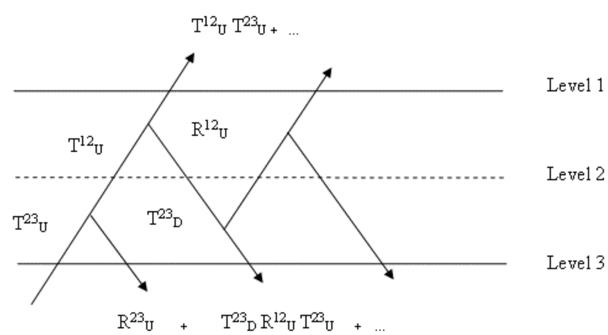

Figure 4. R/T method scheme for superposed media showing the reverberations inside the soil layers for upward wave propagation. R and T denote the reflection and transmission matrices for the corresponding layer. Levels are the boundaries of soil layers.

4 SEISMIC RESPONSE FOR THE UPWARD WAVE PROPAGATION USING R/T METHOD

The generalized method of reflection/transmission can be used to determine the seismic amplification for the upward propagation of P-SV waves. We used a three-step procedure which is summarized here:

1- All the scattering matrices are obtained for the layer interfaces, regarding the type of the incident wave and the ray parameter. Since the ray parameter remains constant during the wave traveling to the upper layers, an algorithm is made to compute all the scattering matrices at the layer interfaces.

2- Equation 3 is a key equation that is used to transfer the incident wave to the upper layers. This equation should be combined with the layer matrices and the scattering matrices to obtain and interpret the total reflection and transmission matrices at the top and bottom of the layers.

3- According to the definition of the R/T matrices, the displacement vector (U_x , U_z) at the free surface is:

$$T^{top}{}_D \; (U_x , U_z) - R^{free}{}_U = -R^{second}{}_U \qquad [4]$$

where U_x, U_z , $T^{top}{}_D$, $R^{free}{}_U$ and $R^{second}{}_U$ are the relative horizontal and vertical displacements, the transmission matrix for the downward wave at the top layer, the reflection matrix at the free surface and the reflection matrix on the second interface (the interface below the free surface), respectively. This relative displacement vector is the seismic amplification that is calculated for the corresponding excitation frequency.

5 DISCUSSION AND RESULTS

5.1 Damping ratio of soil materials

Original R/T method lacks any definition of damping. A proper definition of damping ratio (D) is needed to include the effects of energy loss in the soil materials. In the reflectivity methods, the damping of soil materials can be incorporated in the shear wave velocity model of the layered media.

A solid friction model incorporating the wave energy loss (damping ratio) can be defined mathematically, if the velocity is considered as a complex quantity (Aki and Richards, 1980). In this approach, damping ratio is included in the complex part of the velocity function, which can easily allow the making of a rigorous algorithm for wave propagation in the R/T method.

In this research, to account for the effect of different levels of shaking on damping, we used the damping-shear strain data for clay (from Sun et al., 1988) and obtained the best fit to this data shown in Figure 5. The best fit equation is:

$$D = 17.308\ \varepsilon^{0.2537} \qquad [5]$$

where ε is the shear strain.

Equation 5 shows the dependency of damping on the strain level; and, in turn, the shear strain can be related to the expected peak ground acceleration (PGA) in the region. To obtain this correlation, we used the simulated time histories for eastern Canadian earthquakes (Atkinson and Beresnev, 1998) as input motions and applied the equivalent linear method (ELM) method (Schnabel et al., 1972) for the layered-soil models in the region. We chose different sites in the eastern part of the city of Ottawa that represent the dominant geotechnical characteristics of the sedimentary sites in the study area. Artificial records with the PGAs from 0.02g to 0.5g were used; and, the maximum effective strains were obtained, as shown in Figure 6. The best fit linear-equation to the PGA- ε data is:

$$\varepsilon = 0.4752\ PGA \qquad [6]$$

Substituting Equation 6 in Equation 5, we obtain:

$$D = 14.33\ PGA^{\,0.2537} \qquad (28) \qquad [7]$$

It should be noted that this equation was developed using the simulated time histories for eastern Canada and gives an approximate correlation that helps to choose the suitable range of the damping ratio of the deposits for the study area. Now in order to modify the velocity models of R/T algorithm, damping ratio values from equation 7 can be used in the complex part of the velocity functions.

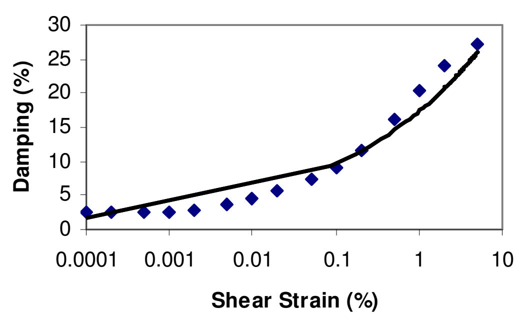

Figure 5. Damping curve for clayey soil from Sun et al. (1988). The best fit equation is D = 17.308 $\varepsilon^{0.2537}$ (R^2 = 0.9586), which is used to deduce the final damping equation. D and ε are the damping and the shear strain, respectively.

(30)

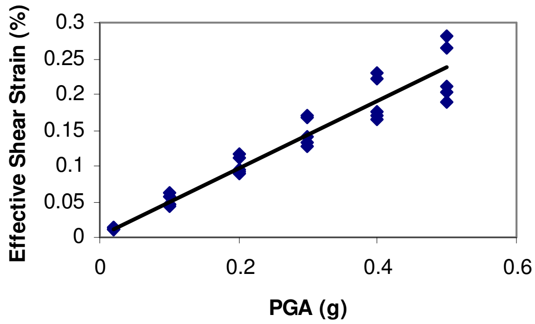

Figure 6. Peak ground acceleration from simulated time histories (generated for eastern Canada) versus effective shear strain using ELM. The best fit equation is ε = 0.4752 PGA (R^2 = 0.9289). ε and PGA are the effective shear strain and the peak ground acceleration, respectively.

5.2 Calibration of R/T method using the ELM

The R/T method was applied for two sites (Barrington Park and Longleaf Park) that represent the general surficial geology of the city of Ottawa in Orleans area (See Figure 1). These sites, which have loose sediments underlain by very high velocity bedrock, are located in the eastern part of the city. The shear wave velocity profiles and other physical parameters are available from different seismic reflection/refraction and nearby borehole measurements. Shear wave velocity profiles are shown in Figures 2 and 3. For the case of vertical incidence of plane SV-waves, R/T method was applied using the appropriate damping ratio (Equation 7) for multi-layer models with soil density (ρ) of 1600 KN/m^3 and, the horizontal amplification values were obtained.

Then, the R/T results were compared against the results of ProShake software (EduPro Civil Systems, Inc., 2001) which is based on the equivalent linear method (ELM) (Schnabel et.al. ,1972, Seed and Idriss ,1969) for the vertical propagation of S-waves. To perform a realistic equivalent linear analysis, the target sites were analyzed; and, the horizontal amplification values were obtained for a multi-layer soil model using the simulated time history records for eastern Canada (Atkinson and Beresnev, 1998). In these soil models, V_s values were extracted from Figures 2 and 3 and average soil density (ρ) of 1600 KN/m^3 was assumed. Figures 7 to 10 illustrate the results of calibration of R/T method with the ELM.

As profiles are shown in Figures 2 and 3, the average shear wave velocity values for the Barrington Park and Longleaf Park sites were measured 146 and 131 m/sec, respectively, using reflection/refraction method. The fundamental frequencies (approximately equal to $V_s/(4H)$) were 0.52, and 1.09 Hz for the Barrington Park and Longleaf Park, respectively, where V_s is the shear wave velocity and H is the depth to bedrock. These frequencies are in good agreement with those of the generalized R/T method which were 0.6 and 1.2 Hz for the Barrington Park, and Longleaf Park sites, respectively (Figures 7 to 10).

Figures 7 to 10 show the results of the R/T method calibration with the ELM which indicate good agreement on the fundamental frequencies of both methods. There is a slight frequency shift in the ELM results, which can be attributed to the nonlinearity effect of shear modulus softening. In addition, there is fairly a reasonable agreement on the seismic amplification values, based on both methods; and, the differences in the seismic amplification values were probably due to the variable shear modulus in ELM.

It should be noted that the R/T method can be developed for a long range of seismic waves (SV and P) and different angles of incidences of the seismic waves; whereas, ELM is limited to the vertical propagation of shear waves.

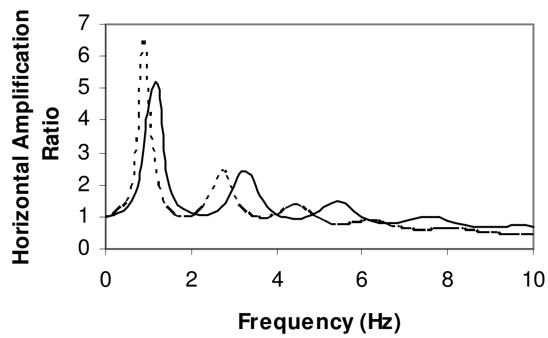

Figure 7. Amplification curves for the Longleaf Park site: Comparison of the generalized R/T method (solid line) with the equivalent linear method (dashed line). In the R/T method, the vertical incidence of the SV wave and the damping ratio compatible with a PGA of 0.2g are considered. In the equivalent linear method, the multilayer model is subjected to a simulated strong motion for eastern Canada with a PGA of 0.2g.

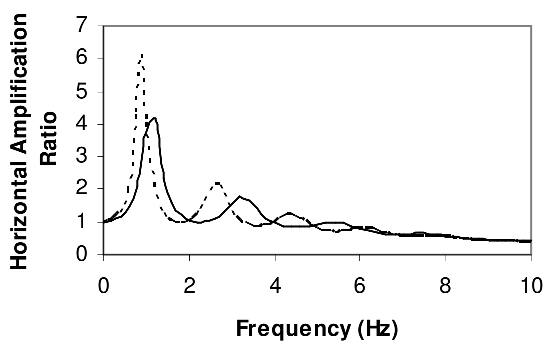

Figure 8. Amplification curves for the Longleaf Park site: Comparison of the generalized R/T method (solid line) with the equivalent linear method (dashed line). In the R/T method, the vertical incidence of the SV wave and the damping ratio compatible with a PGA of 0.4g are considered. In the equivalent linear method, the multilayer model is subjected to a simulated strong motion for eastern Canada with a PGA of 0.4g.

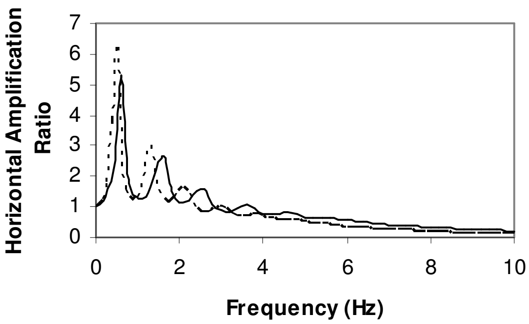

Figure 9. Amplification curves for the Barrington Park site: Comparison of the generalized R/T method (solid line) with the equivalent linear method (dashed line). In the R/T method, the vertical incidence of the SV wave and the damping ratio compatible with a PGA of 0.2g are considered. In the equivalent linear method, the multilayer model is subjected to a simulated strong motion for eastern Canada with a PGA of 0.2g.

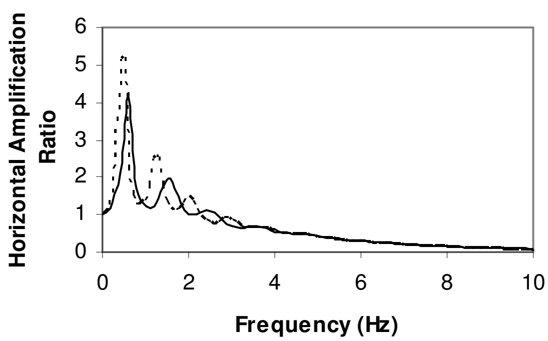

Figure 10. Amplification curves for the Barrington Park site: Comparison of the generalized R/T method (solid line) with the equivalent linear method (dashed line). In the R/T method, the vertical incidence of the SV wave and the damping ratio compatible with a PGA of 0.4g are considered. In the equivalent linear method, the multilayer model is subjected to a simulated strong motion for eastern Canada with a PGA of 0.4g

6 SUMMARY AND CONCLUSIONS

In this research, the algorithm of the generalized reflection/transmission method for upward seismic wave propagation was presented and modified. Also the R/T method was applied for two representative sites in the city of Ottawa.

Most of the city of Ottawa is covered by loose sediments underlain by very hard, high velocity bedrock; and, there is a very high shear wave velocity contrast at the boundary between the loose sediments and the bedrock, which can trap seismic waves in the soil layer. The main advantage of the R/T method is a scheme that considers all internal reverberations of the seismic waves, leading to a proper simulation of the wave trapping inside the soil layers.

The results of the amplification values and the fundamental frequencies from the R/T method were calibrated and validated using ELM for the case of vertical incidence of shear waves. The agreement between the results of two methods is relatively good, although the strain-dependent shear modulus of ELM can cause some differences between the amplification values.

We recommend the damping modified R/T method especially in the areas without any available strong ground motion data. The R/T method can be developed for a long range of seismic waves (SV and P) and for different angles of incidence. This method can be applied to develop the seismic amplification curves for different levels of shaking (e.g. PGA-based curves).

7 ACKNOWLEDGEMENTS

The authors would like to thank Professor Peter M. Shearer from the University of California, San Diego, for providing us with the useful subroutines of his wave propagation program. We thank NSERC (Natural Sciences and Engineering Research Council of Canada), ORDCF (The Ontario Research and Development Challenge Fund) and POLARIS (Portable Observatories for Lithospheric Analysis and Research Investigating Seismicity) for their financial and technical support. We also thank S.E. Pullan, J.A. Hunter, A.G. Brooks, A. Pugin, T. Cartwright, R.A. Burns, R.L. Good and M. Douma from the Geological Survey of Canada and A. Duxbury and V. Ter-Emmanuilyan, Carleton University students, for their assistance.

8 REFERENCES

Aki K. and Larner K. L., 1970, Surface motion of a layered medium having an irregular interface due to incident plane SH waves, *Journal of Geophysical Research,* 75, 933-954.

Aki K. and Richards P.G., 1980, *Quantitative Seismology: Theory and Methods* , W. H. Freeman and Co., San Francisco, California.

Ashford S.A. and Sitar N., 1997, Analysis of topographic amplification of inclined shear waves in a steep coastal bluff, *Bulletin of the Seismological Society of America*, Vol. 87, No. 3, 692-700.

Atkinson G. and Beresnev I., 1998, Compatible ground-motion time histories for new national seismic hazard maps, C*anadian Journal of Civil Engineering.*, 25, 305-318.

Bard P. and Bouchon M., 1980, The seismic response of sediment-filled valleys - Part 1, The case of incident SH waves, *Bulletin of the Seismological Society of America*, Vol. 70, No. 4, 1263-1286.

Bard P. and Bouchon M., 1980, The seismic response of sediment-filled valleys - Part 2, The case of incident P and SV waves, *Bulletin of the Seismological Society of America*, Vol. 70, No. 5, 1921-1941.

Gilbert F. and Backus, G.E, 1966, Propagator matrices in elastic wave and vibration problems, *Geophysics,*31, 326-332.

Haskell N., 1953, The dispersion of surface waves in multilayered media, *Bulletin of the Seismological Society of America*, 43, 17-34.

Hunter J.A., Burns R.A., Good R.L., Aylsworth J.M., Pullan S.E., Perret D. and Douma M., 2006, Borehole shear wave velocity measurements of Champlain Sea sediments in the Ottawa-Montreal region, *Geological Survey of Canada Open File,* #5345

Kawase H., 1988, Time-domain response of a semi-circular canyon for incident SV, P and Rayleigh waves calculated by the discrete wave number boundary element method, *Bulletin of the Seismological Society of America*, Vol. 78, No. 4, 1415-1437.

Kennett B.L.N. and Kerry N J., 1979, Seismic waves in a stratified half space, *Geophysical Journal of Royal Astronomical Society*, 57, 557-583.

Kennett B.L.N., Reflections, 1974, rays and reverberations, *Bulletin of the Seismological Society of America*, Vol. 64, No. 6, 1685-1696.

Khair K.R., Datta S. K. and Shah A. H., 1991, Amplification of obliquely incident seismic waves by cylindrical alluvial valley of arbitrary cross-sectional shape. Part 2: Incident SH and Rayleigh waves, *Bulletin of the Seismological Society of America*, Vol. 81, No. 2, 346-357.

Masaki K. and Iida K., 1980, Specific attenuation factor Q of shear waves in soil, *Bulletin of Aichi Institute of Technology.*

Mossessian T. K. and Dravinski M., 1990, Amplification of elastic waves by a three dimensional valley, Part 1, Steady state response, *Earthquake Engineering and Structural Dynamics*, 19, 667-680.

Motazedian D. and Hunter J.A, 2008, Development of a NEHRP map for the Orleans suburb of Ottawa, Ontario, *Canadian Geotechnical Journal*, 45:1180-1188.

Papageorgiou A.S. and Kim J., 1993, Propagation and amplification of seismic waves in 2-D valleys excited by obliquely incident P and SV waves, *Earthquake Engineering and Structural Dynamics*, 22, 167-182.

ProShake Software, Version 1.11, September 2001, EduPro Civil Systems, Inc., Sammamish, WA, USA.

Safak E., 2001, Local site effects and dynamic soil behavior, *Soil Dynamics and Earthquake Engineering*, 21, 453-458.

Sanchez-Sesma F.J., Heitz J.F. and Bravo M.A., 1989, Ground motion on stratified alluvial basins, *proceeding of Fourth International Conference of Soil Dynamics and Earthquake Engineering*, Mexico City, Mexico.

Sanchez-Sesma F.J. and Luzon F., 1995, Seismic response of three-dimensional alluvial valleys for incident P, S, and Rayleigh waves, *Bulletin of the Seismological Society of America*, Vol. 85, No. 1, 269-284.

Schnabel P.B., Lysmer J. and Seed H.B., 1972, SHAKE: a computer program for earthquake response analysis of horizontally layered sites, Report No. EERC72-12, University of California, Berkeley.

Sun J.I., Golesorkhi R. and Seed H.B., 1988, Dynamic moduli and damping ratios for cohesive soils, Report No. UCB/EERC-88/15 Earthquake Engineering Research Center, University of California, Berkeley.

Thompson W.T., 1950, Transmission of elastic waves through a stratified solid, *Journal of Applied Physics*, 21, 89-93.

Zeng Y. and Benites R., 1998, Seismic response of multi-layered basins with velocity gradients upon incidence of plane shear waves, *Earthquake Engineering and Structural Dynamics*, 27, 15-28.