Modeling sand behavior using a critical state model implemented in FLAC

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ABSTRACT
A critical state constitutive model previously developed at the University of Alberta for the prediction of the behavior of sands was implemented in the computer program FLAC using its specific FISH programming code. Accuracy of the predictions obtained by the program was verified in the single-element scale by comparing predicted and observed results of triaxial tests on Toyoura and Syncrude sands, and in the full-scale level by modeling the field event of the CANLEX project, and comparing predictions with readings of field instrumentations. The current study showed that in order to obtain correct results from the analysis, it is important to use an appropriate soil constitutive model, to account for the simultaneous pore pressure generation and dissipation during loading, and to consider soil anisotropy.

RÉSUMÉ
Un état critique le modèle que constitutif s'est développé à l'université d'Alberta pour la prévision du comportement des sables a été ajouté au programme informatique FLAC. L'étude courante a montré l'importance d'employer un modèle approprié de sol, et vu la génération et la dissipation simultanées de pression de pore pendant le chargement, et considérer l'anisotropie de sol.

1 INTRODUCTION

Over the past decades, significant advances have been made in the development of computer technology and numerical methods, and these advances have facilitated the solution of numerous problems related to the behavior of soil structures subjected to various loading conditions. One of the problems for which, due to the complexity of the material behavior and loading condition, conventional limit equilibrium procedures can not provide realistic solutions; and therefore, more advanced solution techniques based on stress-strain behavior of the material are required, is the flow liquefaction of loose sands. Solutions to problems involving flow liquefaction can be obtained using finite element or finite difference computer programs in which appropriate constitutive models for the prediction of the behavior of sands subjected to drained and undrained loading are used.

The finite difference computer program FLAC (Fast Lagrangian Analysis of Continua) has been used widely for the advanced analysis of soil structures subjected to various loading conditions. However, the material models available in this program can only take into account limited aspects of the elasto-plastic behavior of sands, and are therefore not suitable for analyses such as those required to predict flow liquefaction. This program, however, allows implementation of user-defined constitutive models; and therefore, it is possible to add a suitable sand model to the program in order to allow flow liquefaction analysis.

In this paper, a critical state constitutive model for sands presented by Imam et al. (1995) is first briefly introduced, and its implementation into FLAC is then described. Ability of the program in predicting the behavior of sands is verified by comparing predicted and observed behaviors of Toyoura and Syncrude sands at the single-element level. The program is then used for the analysis of a structure made of sand. For this purpose, the full-scale field event of the CANLEX (Canadian Liquefaction Experiment) project is selected, and the predicted behavior using the current program is compared with the available field data collected during the field event.

2 BRIEF DESCRIPTION OF THE CONSTITUTIVE MODEL FOR SAND

A critical state constitutive model for sands was developed with emphasis on taking into account important aspects of the behavior of loose liquefiable sand. Details of the model and its formulation are described by Imam et al. (2005), and the constitutive relationships for triaxial conditions are summarized in the Appendix of this paper. The model uses a capped yield surface with the stress ratio M_p at its point of peak deviatoric stress (q) obtained from the undrained effective stress path in triaxial tests.

In the model, stress-dilatancy is based on Rowe's (1962) dilatancy relationship combined with a modified form of the equation proposed by Manzari and Dafalias (1997). The failure criterion is given by a friction angle that depends on the current state parameter (Been and Jefferies 1985) through a slightly modified version of a relationship suggested by Wood et al. (1994).

The model uses a single set of parameters to predict sand behavior over a wide range of void ratios and confining pressures. The critical state line represents the soil state at large strain, while the behavior at small and medium strains are captured by other material parameters such as those describing yielding, dilatancy, and plastic modulus, which take into account anisotropy.

Figure 1 shows a graphical representation of the yield surface, and the stress ratios at critical state (M_{cs}) and at failure (M_f) in a p-q plane normalized to the maximum mean normal stress at yielding (p_c). Values of M_p in triaxial compression and triaxial extension are referred to as $M_{p,c}$ and $M_{p,e}$, respectively. These stress ratios control the yield surface shape (i.e. width), and account for the effects of void ratio, mean normal stress, and inherent anisotropy on the yielding stresses. A small M_p results in a slender yield surface and applies to sand that is loose, subjected to high confining pressures, or loaded in a weak direction such as in triaxial extension.

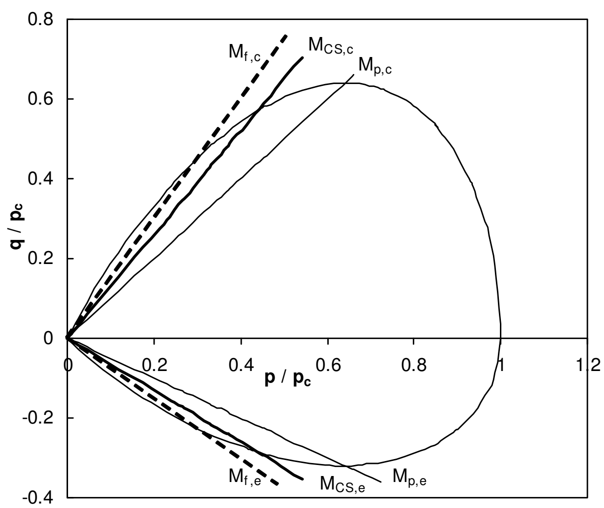

Figure 1. Yield surface of isotropically consolidated sand.

Stress-induced anisotropy is represented by the stress ratio α, at which the tangent to the yield surface is parallel to the q-axis. This stress ratio is non-zero only in anisotropically consolidated sand. Size of the yield surface is determined by p_c.

The model described above was implemented in the computer program FLAC using its specific programming language FISH. Ability of the program to predict the behavior of sands at the single element level is shown in the next section by comparing results of triaxial compression and extension tests on Toyoura and Syncrude sands.

3 PREDICTING SAND BEHAVIOR AT THE SINGLE ELEMENT LEVEL

In triaxial compression tests, loading is applied to the sample by moving the upper platen at a constant speed. The axisymmetric element was used to model the behavior of cylindrical triaxial samples. During isotropic consolidation of samples, the cell pressure is gradually increased in order to apply an equal pressure in all directions; however, consolidation of anisotropically consolidated samples is carried out by incrementally increasing cell pressure and axial load such that a constant ratio of axial to radial stress is maintained. Therefore, in the FLAC modeling of anisotropically consolidated samples, a higher axial stress was applied to the top of the sample compared to the radial stress, such that the ratio between these stresses remained constant and equal to the ratio needed to induce the required stress-induced anisotropy.

Since in the critical state model used, parameters obtained at previous step are used for calculations in the next step, applying large strain increments leads to errors that gradually accumulate and increase. Therefore, the size of the steps, or the speed of movement of the nodes at the top of the sample should be selected small enough to avoid larger than acceptable errors.

In modeling triaxial compression tests, the upper nodes were given downward speeds and in modeling triaxial extension tests, they were given speeds in the upward direction. As with other critical state models, all predictions for the various consolidation pressures, void ratios, consolidation stress ratios, and directions of loading (compression v.s. extension) are made using a single set of model parameters for each of the Toyoura and Syncrude sands.

3.1 Modeling the behavior of Toyoura sand

Toyoura sand is a subangular, predominantly quartzic sand with maximum and minimum void ratios of 0.977 and 0.597, respectively (Ishihara, 1993). Imam et al. (2005) modeled its behavior with calculations carried out using spreadsheets. A variable stress ratio (Mp), as suggested in the model, was used. However, in the current FLAC calculations, a constant value for this stress ratio throughout shearing was used in order to simplify the calculations and reduce the analysis time. As a result, some of the model parameters obtained for Toyoura sand in the current study are different from those obtained by Imam et al (2005) as shown in Table 1. All model predictions shown in this paper are compared with the results of triaxial tests on Toyoura sand presented by Ishihara (1993).

Figure 2 shows the observed response of Toyoura sand consolidated to a void ratio of 0.833 and confining pressure of 2000 kPa, subjected to triaxial compression, along with FLAC predictions. The figure shows a good match between the observed and predicted behaviors.

Table 1. Model parameters for Toyoura sand

Parameter type	Parameter name	Toyoura sand (Imam et al., 2005)	Toyoura sand (current study)
Peak state	K_p	1.2	1.2
	φ_μ	21	21
	a_P	0.18	0.21
Stress-dilatancy	φ_{cs}	31	31
	K_{PT}	0.75	1.75
	a_{PT}	0.15	0.05
Plastic Stiffness	H	1	1
Elasticity	G_a	5000	5000
	K_a	8500	8500

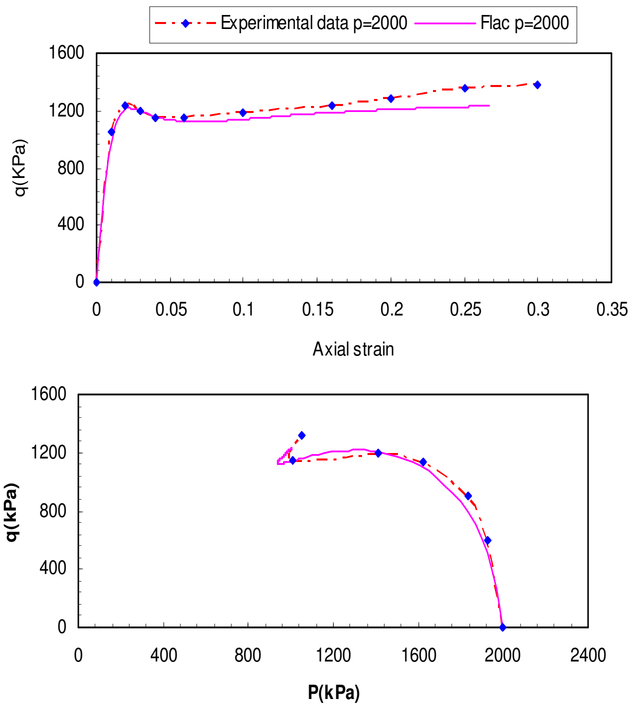

Figure 2. Comparison of measured and FLAC results of Toyoura sand with Pc=2000 and e = 0.833.

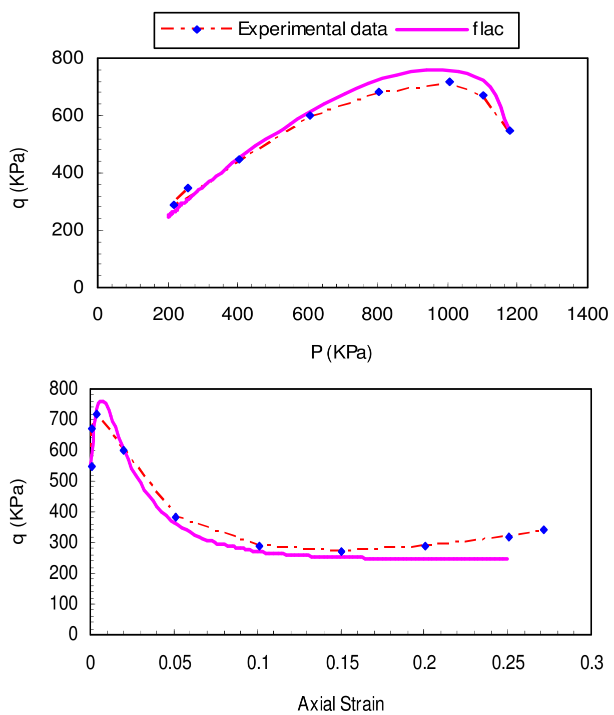

Figure 3. Anisotropically consolidated Toyoura sand with e=0.900.

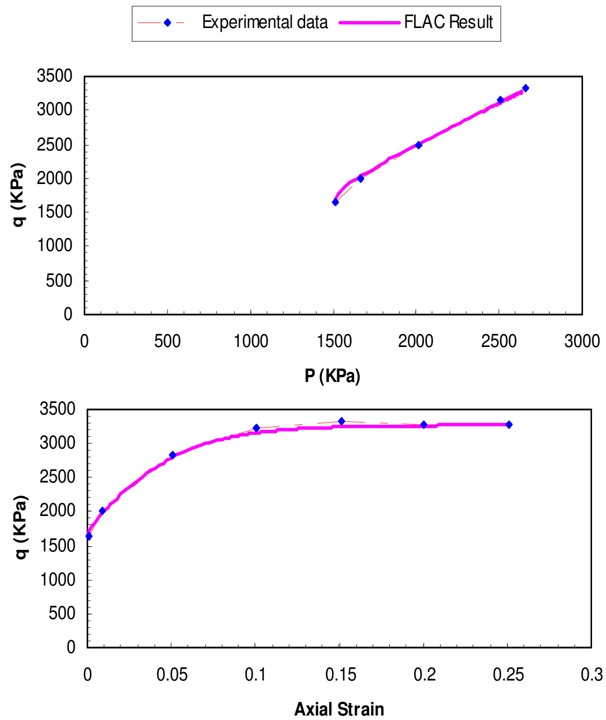

Figure 4. Anisotropically consolidated Toyoura sand with e=0.748 and Pc=1500.

FLAC-predicted and measured behaviors of anisotropically consolidated loose and dense samples of Toyoura sand subjected to triaxial compression loading are shown in Figures 3 and 4, respectively. In both cases, predicted and measured responses are close. However, Figure 3 indicates that at higher strains (above 15 percent), experimental results show increasing strength with strain, while FLAC results show that the sample reaches critical state condition and experiences large shear strains at constant mean normal and shear stresses. This discrepancy can result from non-uniform strains that often occur in triaxial samples at higher shear strain due to strain concentration or development of shear bands. As a result, measured behavior based on displacements obtained at sample boundaries can no longer represent the actual strain distribution within the sample and the resulting stress-strain behavior of the sample. The FLAC results which are based on uniform, single-element behavior is therefore different from those obtained from test results.

Drained behaviors of Toyoura sand samples consolidated to a confining pressure of 100 kPa and void ratios of 0.966, 0.917 and 0.831 as predicted by FLAC and measured from test results are shown in Figure 5. Predicted initial shear stiffnesses of the denser samples are smaller than those measured. This is likely due to the use of a constant value for the yield surface parameter M_p instead of a variable parameter as suggested in the original model.

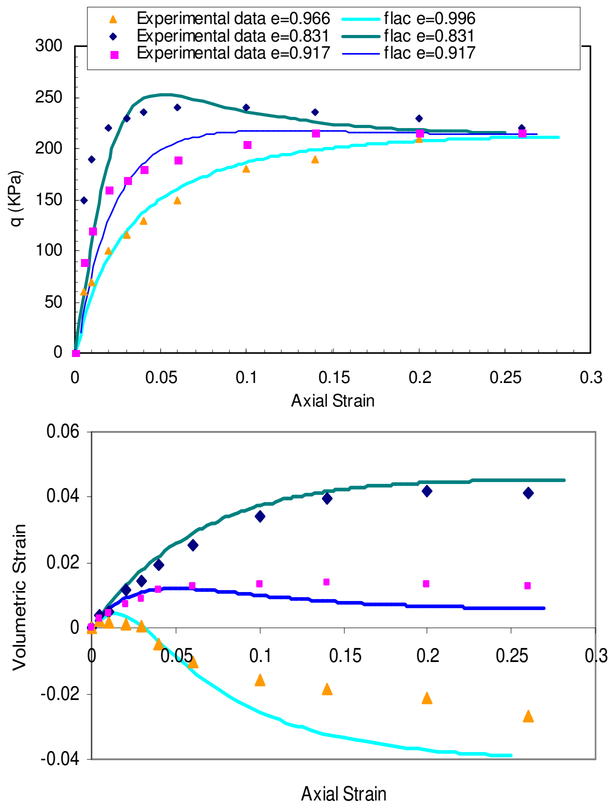

Figure 5. Drained behavior of Toyoura sand as Predicted by FLAC and measured from triaxial compression tests.

3.2 Modeling the behavior of Syncrude sand

Syncrude sand is a tailings sand produced from the extraction of oil from the Alberta (Canada) oil sand. It is a subangular sand with maximum and minimum void ratios of 0.93 and 0.55 respectively (Sladen and Handford, 1987). Since the full scale field event of the CANLEX project was carried out in an environment made predominantly of this sand, in this section we use the model to simulate the Syncrude sand behavior and determine the parameters needed for its modeling in the single-element, triaxial loading condition, and use these parameters in modeling the actual full- scale event of the CANLEX project later. All predictions shown here are compared with triaxial test results presented by the University of Laval (Wride and Robertson, 1997).

Figure 6 shows FLAC outputs of predicted behavior of a sample of Syncrude sand consolidated to a confining pressure of 294.6 kPa and a void ratio of 0.819 together with measured results. Good agreement is observed between predicted and observed behaviors for this test.

In order to examine the program performance for denser sands, FLAC results and observed behavior are compared in Figures 7 for a sample consolidated to a confining pressure of 201.6 kPa and void ratio of 0.724 and subjected to undrained shearing under triaxial compression condition. Good match is also observed between observed and predicted behaviors. Model parameters used in the predictions of the Syncrude sand behavior are shown in Table 2.

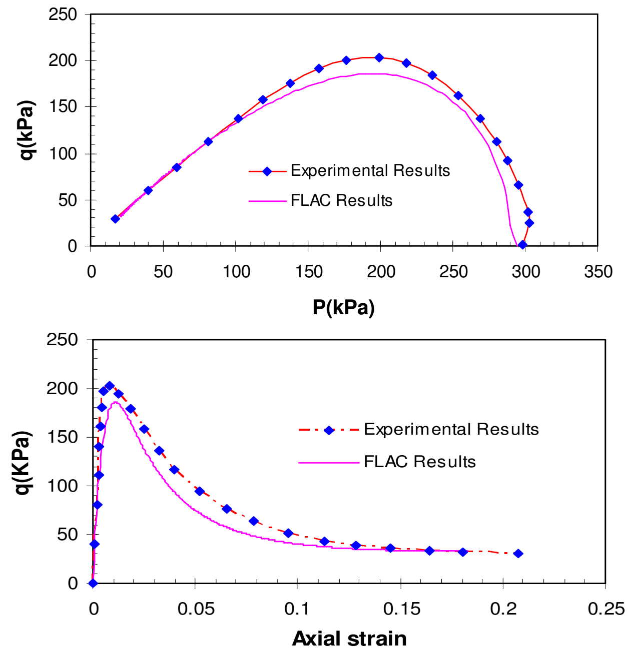

Figure 6. Comparison between observed and predicted undrained triaxial compression behaviors of Syncrude sand consolidated to 294.6 kPa and e=0.819.

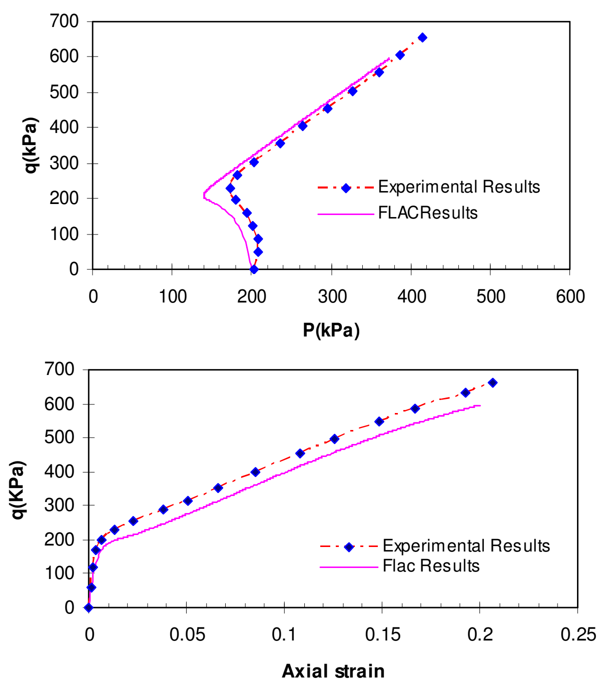

Figure 7. FLAC prediction and observed behavior of Syncrude sand consolidated to 201.6 kPa and e=0.724.

Table 2. Model parameters for Syncrude sand

Parameter type	Parameter name	Syncrude sand
Peak state	K_p	1.25
	φ_μ	23
	a_P	0.07
Stress-dilatancy	φ_{cs}	39
	K_{PT}	0.5
	a_{PT}	-0.15
Plastic Stiffness	H	1
Elasticity	G_a	4500
	K_a	4000

4 THE CANLEX FIELD EVENT

4.1 Description of the event

CANLEX (Canadian Liquefaction Experiment) was a collaborative 5-year project (1993-1998) that aimed at coordinating Canadian geotechnical expertise on the topic of soil liquefaction. The collaboration included a number of Canadian universities and geotechnical consultants. Project activities involved in-situ testing, laboratory testing, numerical modeling and a full scale field event conducted to study the liquefaction of loose sand.

An abandoned borrow pit at the Syncrude Canada Ltd. site (J-pit) was used to carry out the field event (Figure 8). The foundation sand was placed hydraulically into standing water up to elevation 318 m and was referred to as the beach below water sand (BBW sand). A level platform was then formed at elevation 321 m by placing tailings sand above the water and was referred to as beach above water sand (BAW sand). A clay dyke 8 m high with side slopes of 2.5:1 (horizontal to vertical) was constructed slowly over the tailings so as to allow drainage of the sand to occur. A 10 m high compacted sand cell containment structure was then constructed to form an enclosure.

Rapid loading was applied by pumping tailings (contained sand) behind the clay dyke. Plan and section of the site are shown in Figure 8. Water and tailings were poured into the cell behind the clay embankment. The water level was raised to a height of 7.5 m and the sand was placed to a height of 7 m. Rapid loading took about 36 hours, but did not lead to a flow failure. However, as a result of loading, the clay dike experienced a maximum displacement of 0.054m at the toe, with an average movement of the dike estimated at 0.020 m (Natarajan et al. 1996). Pore pressure rises were also measured by those instrumentations that remained functional during the event.

Changes in pore pressures during loading were measured at five instrumentation lines under the clay dike. Many of the instrumentations did not function during the event, but those in Line 1 performed best. Figure 9 shows the location of instrumentations along Line 1.

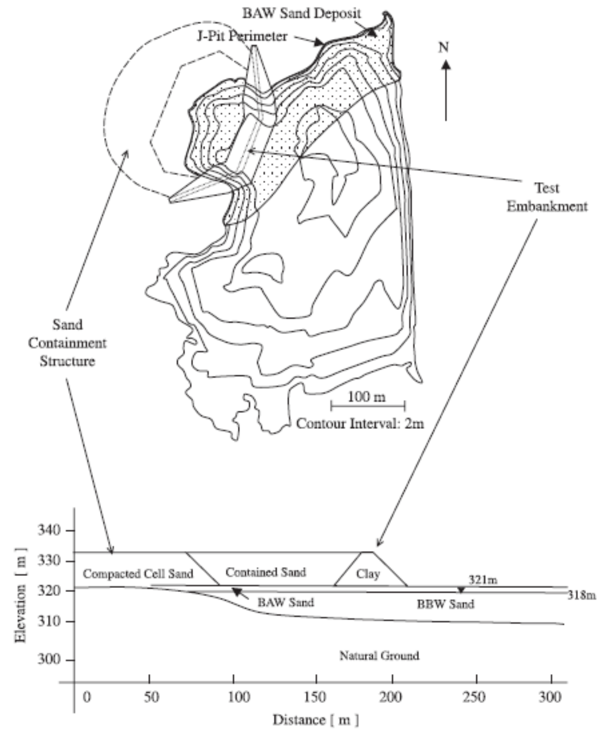

Figure 8. Plan and section of the CANLEX field event site (modified after Byrne et al. 2000).

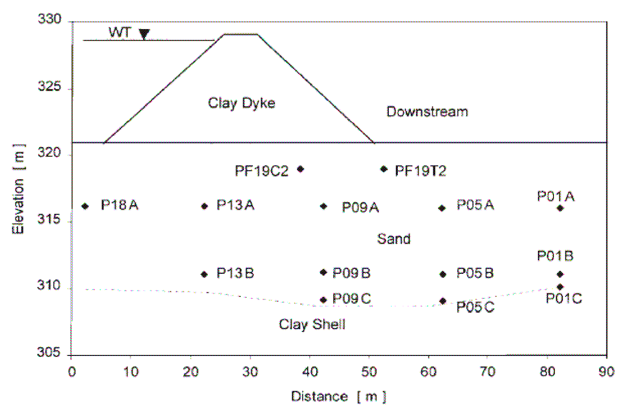

Figure 9. Location of instrumentations along Line 1 (modified after Byrne et al. 2000).

4.2 Analyses of the field event

Analyses of the field event were carried out at the University of Alberta (U of A) using the PISA software and the Cathro and Gu (1995) material model; and at the University of British Columbia (UBC) using the FLAC software and the UBCSAND model (Byrne et al. 2000). The U of A analyzed the data obtained from Line 1 and the UBC analyzed data obtained from Line 2. Since more data were obtained from Line 1, this line is selected here for analysis. Data obtained from Line 2 will be analyzed and discussed in more details in a later publication.

In the current analysis, the Imam et al (2005) model implemented in FLAC was used for the foundation sand and the Mohr Columb model was used for the material in other parts of the section. Model parameters were obtained using the test results presented by the University of Laval on Syncrude sand shown in Table 2. The foundation sand was assumed to have an average relative density of 34% (Byrne et al 2000). The surcharge sand was added in 15 layers such that model parameters can be updated after application of the previous sand layer, and coupled flow-stress analyses were carried out after application of each layer.

4.3 Deformation pattern obtained from the analyses

Figure 10 shows the deformation pattern obtained from the current analysis. As expected, deformations in the foundation sand decrease with depth. Average horizontal deformation of the dike during the field event was estimated at about 20, and maximum horizontal deformation was 54 millimeters measured at the toe of the dike. The current analysis uses an anisotropic sand behavior with softer response for the sand when loaded horizontally and stronger response when loaded vertically. As a result, zones of the foundation soil on the upstream side of the dike are loaded predominantly in the vertical direction while zones on the downstream side are loaded predominantly horizontally. This difference in loading direction results in horizontal deformations to be higher on the downstream side close to the toe of the embankment compared to the upstream side, as observed during the field event. Previous analyses of this event, described before, obtained horizontal components of deformations that decreased from the upstream to the downstream side.

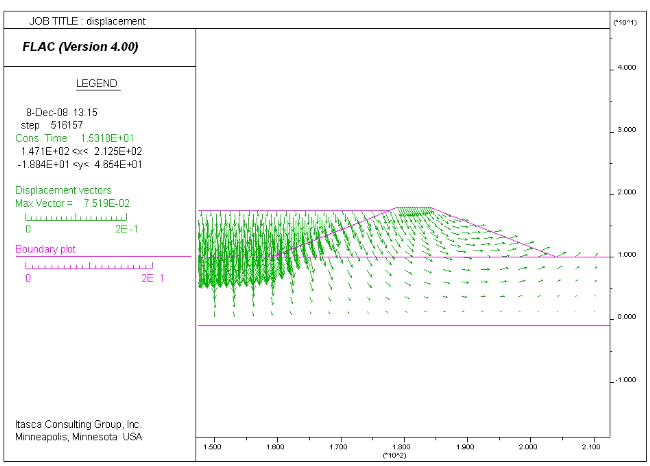

Figure 10. Deformation pattern of field-event obtained from the current analysis

4.4 Pore pressures obtained from the analyses

Values of pore pressures generated following application of the surcharge load as calculated by FLAC and measured at the locations of instrumentations in Line 1 are shown in Table 3 and plotted in Figure 11. Calculated and measured pore pressures are close, and represent improvements compared to previous analyses.

Table 3. comparison of excess pore pressures in line 1 instrumented section

	P18A	P13A	P13B	PF19C2	P09C2	PF19T2
Measured	68	34	38	18	16	7
Current Analysis	65.1	41.8	45.3	23.7	20.6	5.8

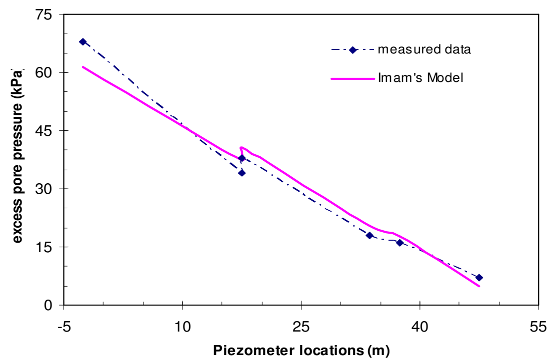

Figure 11. Pore pressures obtained from the current analysis and those measured during the field-event.

5 CONCLUSIONS

A critical state constitutive model implemented in FLAC was used to model the full-scale field event of the CANLEX project. The study showed the importance of using an appropriate soil constitutive model which takes into account soil anisotropy, and taking into account the simultaneous pore pressure generation and dissipation during loading. Compared to methods previously used, predictions obtained from the current analysis are closer to those measured.

ACKNOWLEDGEMENTS

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APPENDIX

The yield surface is defined using the following equation:

$$f = (\eta - \alpha)^2 - M_\alpha^{\ 2}\left[1-\left(\frac{p}{p_c}\right)^{\frac{1}{2}}\right] = 0 \qquad \text{[A-1]}$$

$$M_\alpha^{\ 2} = (5M_p - \alpha)(M_p - \alpha) \qquad \text{[A-2]}$$

in which, for triaxial compression (TC) and triaxial extension (TE) we have:

$$M_{p,c} = \frac{6\sin\varphi_{p,c}}{3-\sin\varphi_{p,c}} \qquad \text{in TC} \qquad \text{[A-3a]}$$

$$M_{p,e} = \frac{6\sin\varphi_{p,e}}{3+\sin\varphi_{p,e}} \qquad \text{in TE} \qquad \text{[A-3b]}$$

and $\varphi_{p,c}$ and $\varphi_{p,e}$ are the friction angles at the point of peak q in TC and TE, respectively, and are obtained from:

$$\sin\varphi_{p,c} = \sin\varphi_\mu - k_p\,\psi \qquad \text{in TC} \qquad \text{[A-4a]}$$

$$\sin\varphi_{p,e} = \sin\varphi_\mu - k_p\,\psi - a_p \qquad \text{in TE} \qquad \text{[A-4b]}$$

in which φ_μ is the friction angle corresponding to $\psi_p = 0$ in TC and is typically close to the inter-particle friction angle of the sand; k_p and a_p are material parameters, and ψ is the state parameter. A Mohr-Coulomb type failure criterion, expressed in the following form, is used:

$$\sin\varphi_f = \sin\varphi_{cs} - k_f\,\psi \qquad \text{[A-5]}$$

in which φ_{cs} is the critical state friction angle and k_f is a material parameter taken to be 0.75. Friction angles obtained from [A-5] are converted to equivalent stress ratios at failure $M_{f,c}$ and $M_{f,e}$ for TC and TE as in [A-3]. These are the maximum stress ratios attainable at the current soil state, and may not be equal to the current stress ratio η. It is only at critical state ($\psi = 0$) where the current and failure stress ratios coincide ($\eta = M_f = M_{cs}$). The flow rule is described by the following relationships:

$$d = \frac{d\varepsilon_p^{\ p}}{d\varepsilon_q^{\ p}} = A\,(M_{cs}-\eta) \qquad \text{[A-6]}$$

$$A_c = 9/(9 - 2M_{PT,c}\eta + 3M_{PT,c}) \qquad \text{in TC} \qquad \text{[A-7a]}$$

$$A_e = 9/(9 - 2M_{PT,e}\eta - 3M_{PT,e}) \qquad \text{in TE} \qquad \text{[A-7a]}$$

and $M_{PT,c}$ and $M_{PT,e}$, are as follows:

$$\sin\varphi_{PT,c} = \sin\varphi_{cs} + k_{PT}\,\psi \qquad \text{for TC} \qquad \text{[A-8a]}$$

$$\sin\varphi_{PT,e} = \sin\varphi cs + a_{PT} + k_{PT}\,\psi \qquad \text{for TE} \qquad \text{[A-8b]}$$

Hardening during shearing is determined from:

$$\frac{\partial p_c}{\partial \varepsilon_q^{\ p}} = \frac{hG}{(p_f - p_c)_{ini}}(p_f - p_c) \qquad \text{[A-9]}$$

in which h is a non-dimensional material parameter related to soil stiffness during shearing, G is the elastic shear modulus, and $(pf - pc)_{ini}$ is the initial value of $(p_f - p_c)$ at the end of consolidation and prior to shearing. The value of p_f is obtained by substituting the current M_f for η in Equation [1]. Elastic moduli are defined as follows:

$$G = G_r\,\frac{(2.973-e)^2}{1+e}(p/p_a)^{1/2} \qquad \text{[A-10a]}$$

$$K = K_r\,\frac{(2.973-e)^2}{1+e}(p/p_a)^{1/2} \qquad \text{[A-10b]}$$

in which G_r and K_r are reference values that depend on the units used and may be obtained from the elastic moduli corresponding to the atmospheric pressure p_a.

NOTATION:

a_p, a_{PT}: Difference between sinφ at peak point of the yield surface and at PT in TC and TE, respectively

d,: Soil dilatancy

e, e_{cs}: current and critical state void ratios, respectively

f : Yield function

G, G_a: Elastic shear modulus at current and atmospheric mean normal stresses, respectively

h : Material parameter related to plastic shear stiffness

K, K_r : Elastic and reference bulk moduli, respectively

$K_0 = \sigma_h / \sigma_v$ Coefficient of lateral earth pressure

k_p, k_f, k_{PT} : Slope of variation of $\sin\varphi_p$, $\sin\varphi_f$ and $\sin\varphi_{PT}$ with state parameter, respectively

M_{cs}, $M_{cs,c}$, $M_{cs,e}$: Stress ratios q/p at critical state, and its values in TC and TE, respectively

M_p, $M_{p,c}$, $M_{p,e}$: Stress ratio at the peak point of the yield surface, and its values in TC and TE, respectively

M_μ, M_f: Stress ratio q/p corresponding to inter-particle friction and failure, respectively.

p, p_a, p_c, p_p, p_f : Effective mean normal stress (= σ_1 + $2\sigma_3$)/3, and its values at atmospheric pressure and at consolidation, respectively.

q : Deviatoric stress ($=\sigma_1-\sigma_3$)

α : Stress ratio q/p at which tangent to yield surface is perpendicular to the p-axis

ε_1 , ε_3 : Major and minor principal strains respectively

ε_p , ε_q, ε_p^p, ε_q^p : Volumetric ($\varepsilon_p = \varepsilon_1 + 2\varepsilon_3$)and shear ($\varepsilon_q = 2(\varepsilon_1 - \varepsilon_3)/3$) strains and their plastic components, respectively

φ_f : Mobilized friction angle at failure

φ_{PT}, $\varphi_{PT,c}$, $\varphi_{PT,e}$: Friction angle at PT, and its values in TC and TE, respectively

φ_p, $\varphi_{p,c}$, $\varphi_{p,e}$: Friction angle at peak point of the yield surface, and its values in TC and TE, respectively

φ_μ, φ_{cv} : Inter-particle and constant volume friction angles, respectively

ψ: State parameter = e – e_{cs}