Contaminant Isolation By Cutoff Walls: Reconsideration Of Mass Fluxes

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ABSTRACT
Cutoff walls are used frequently to isolate contaminants at both controlled and uncontrolled hazardous waste sites. Neville and Andrews (2006) presented a containment criterion for contaminant isolation by a cutoff wall. Their analysis yields the Darcy flux required to achieve containment, based on the condition that long-term advective and diffusive mass fluxes across the wall are balanced. In this paper we show that the condition of zero net mass flux represents only one particular case. Straightforward expressions for the long-term mass fluxes across a cutoff wall can also be developed from the same theory for any Darcy flux. The expressions for the long-term mass fluxes may be used to estimate the mass flux to the environment in cases where it is difficult to satisfy the criterion of zero net mass flux.

RÉSUMÉ
Des barrières souterraines imperméables sont souvent utilisées pour confiner les sources de contamination présentes sur les terrains contaminés. Neville et Andrews (2006) ont présenté, pour de telles barrières, un critère de confinement des sources de contamination. Le résultat de leur analyse indique quel est le flux de Darcy permettant d'atteindre le confinement, en se basant sur l'équilibre, à long-terme, entre les flux de masse advectifs et dispersifs à travers la barrière. Cet article démontre que la condition d'équilibre, où le flux de masse net de contamination traversant la barrière est nul, représente un cas particulier. Des simples expressions mathématiques décrivant les fluxes de masse à travers une barrière peuvent êtres obtenues pour toutes les valeurs de flux de Darcy. Des expressions quantifiant les débits massiques à long-terme peuvent être utilisée pour estimer la charge de contamination relâchée dans l'environnement souterrain, dans les cas où il est difficile d'obtenir un parfait confinement.

1 INTRODUCTION

Cutoff walls are used frequently to isolate contaminants at both controlled and uncontrolled hazardous waste sites. Neville and Andrews (2006) presented a containment criterion for contaminant isolation by a cutoff wall. Their containment criterion builds on the analysis of Devlin and Parker (1996) and is developed from the assumption that containment is achieved when the long-term advective and diffusive mass fluxes across the wall are balanced so that the net mass flux is zero.

The conceptual model of Neville and Andrews (2006) is shown schematically in Figure 1. Their analysis may be used to estimate the Darcy flux required to achieve containment according to this criterion. In this paper, we show that the assumption of zero net mass flux represents only one particular case. The same theory may be used to develop a straightforward expression for the long-term net mass flux across a cutoff wall for any Darcy flux. This expression may be used to estimate the net mass flux to the environment in cases where it may not be possible to satisfy the criterion of zero net mass flux.

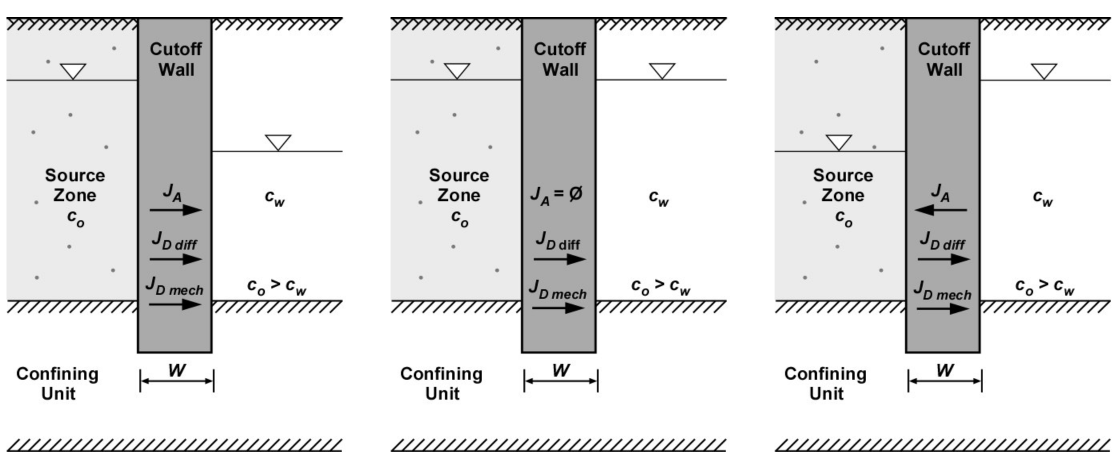

Figure 1. Conceptual model of source isolation

2 NEVILLE AND ANDREWS (2006) CONTAINMENT CRITERION

The analysis of Neville and Andrews (2006) assumes a constant concentration within the source zone and steady one-dimensional flow and dispersive diffusive transport across the cutoff wall. In Figure 1, c_0 and c_w denote the steady concentrations along the inside and outside faces of the wall, respectively, and w is the thickness of the wall. The terms J_A, $J_{D\,diff}$, and $J_{D\,mech}$ represent the advective, diffusive, and mechanical-dispersive mass fluxes, respectively.

The advective mass flux, J_A, is thus defined:

$$J_A = qc \quad [1]$$

where c is the concentration and q is the Darcy flux (positive outwards from the source zone). The steady-state dispersive diffusive mass flux, J_D, is given by:

$$J_D = -\theta D \frac{dc}{dx} \quad [2]$$

where θ is the effective porosity and D is the dispersion coefficient.

The diffusive flux in Equation 2 is a lumped representation of the mechanical-dispersive and diffusive mass fluxes. Both process are assumed to be Fickian processes and additive; therefore, the dispersion coefficient D can be interpreted as the sum of the mechanical-dispersive and diffusive fluxes (Bear, 1972):

$$D = \alpha_L |v| + D^* \quad [3]$$

where α_L is the longitudinal dispersivity, v is the groundwater velocity, and D^* is the effective diffusion coefficient. The mechanical-dispersive flux accounts for variations in the groundwater velocity across the wall that are beneath the scale of resolution of the analysis. For a properly constructed wall, this flux should be negligible.

Neville and Andrews (2006) derived a containment criterion for any given wall design (wall thickness, diffusion coefficient, and Darcy flux across the wall). Following the approach of Devlin and Parker (1996), they assumed that containment is achieved when the net mass flux on the outside face of the wall is zero:

$$J_A + J_D = 0 \quad [4]$$

Given a known concentration of the source and a target concentration of outside of the cutoff wall, the Neville-Andrews containment criterion yields the Darcy flux that is required to achieve a long-term net mass flux of zero:

$$q = \frac{\theta D}{w} \ln\left\{\frac{c_w}{c_0}\right\} \quad [5]$$

3 GENERAL INTERPRETATION OF THE NEVILLE AND ANDREWS (2006) ANALYSIS

The analysis of Neville and Andrews (2006) accommodates groundwater seepage in either direction, and any combination of specified concentrations over both faces of the cutoff wall. However, Equation 5 represents only one specific case, in which the dispersive diffusive and advective mass fluxes are balanced. It is important to note that steady-state conditions may be attained for <u>any</u> Darcy flux. To demonstrate this point, we consider the more general case of transient conditions.

The governing equation for transient transport is:

$$\theta \frac{\partial c}{\partial t} = -q \frac{\partial c}{\partial x} + \theta D \frac{\partial^2 c}{\partial x^2}, \quad 0 \le x \le w \quad [6]$$

The initial and boundary conditions are:

$$c(x,0) = 0 \quad [7a]$$

$$c(0,t) = c_0 \quad [7b]$$

$$c(w,t) = c_w \quad [7c]$$

The general solution for transient conditions is derived by generalizing the derivation of Al-Niami and Rushton (1977):

$$c(x,t) = c_0 \frac{\sinh\left\{\frac{qx}{2\theta D}\right\}}{\sinh\left\{\frac{qw}{2\theta D}\right\}} EXP\left\{\frac{q(x-w)}{2\theta D}\right\}$$
$$+ c_w \frac{2\pi D}{w^2} EXP\left\{\frac{q(x-w)}{2\theta D} - \frac{q^2 t}{4\theta^2 D}\right\}$$
$$\square \sum_{n=1}^{\infty} \frac{(-1)^n n}{\left(\frac{Dn^2\pi^2}{w^2} + \frac{q^2}{4\theta^2 D}\right)} EXP\left\{\frac{-Dn^2\pi^2 t}{w^2}\right\} \sin\left\{\frac{n\pi x}{w}\right\} \quad [8]$$

The solution for the steady-state concentration profile can be derived directly from Equation 8 and is given in Neville and Andrews (2006) as:

$$c(x,\infty) = c_0 - (c_0 - c_w)\frac{\left(1 - EXP\left\{\frac{qx}{\theta D}\right\}\right)}{\left(1 - EXP\left\{\frac{qw}{\theta D}\right\}\right)} \quad [9]$$

4 STEADY-STATE NET MASS FLUX

The Neville and Andrew (2006) containment analysis is generalized by deriving expressions for the advective and dispersive diffusive mass fluxes for any magnitude of the Darcy flux across the wall.

The advective mass flux across the outside face of the wall is:

$$J_A = qc\big|_{x=w} = qc_w \quad [10]$$

The steady-state dispersive diffusive mass flux across the outside face of the wall is defined as:

$$J_D = -\theta D \frac{dc}{dx}\bigg|_{x=w} \quad [11]$$

Differentiating the general steady-state solution yields:

$$\frac{dc(x,\infty)}{dx} = c_0\left(\frac{qv}{\theta D}\right)\left[\frac{EXP\left\{\frac{qx}{\theta D}\right\}}{1 - EXP\left\{\frac{qw}{\theta D}\right\}}\right]$$
$$- c_w\left(\frac{q}{\theta D}\right)\left[\frac{EXP\left\{\frac{qx}{\theta D}\right\}}{1 - EXP\left\{\frac{qw}{\theta D}\right\}}\right] \quad [12]$$

Evaluating the derivative at $x = w$ and simplifying yields:

$$\frac{dc(w,\infty)}{dx} = \left(\frac{q}{\theta D}\right)(c_0 - c_w)\left[\frac{EXP\left\{\frac{qw}{\theta D}\right\}}{1 - EXP\left\{\frac{qw}{\theta D}\right\}}\right] \quad [13]$$

The expression for the diffusive flux is obtained by substituting Equation 13 into 11:

$$J_D = -q(c_0 - c_w)\left[\frac{EXP\left\{\frac{qw}{\theta D}\right\}}{1 - EXP\left\{\frac{qw}{\theta D}\right\}}\right] \quad [14]$$

The net mass flux is defined as the sum of the advective and diffusive mass fluxes:

$$J_{net} = J_A + J_D \quad [15]$$

Substituting Equations 10 and 14 into Equation [15] yields the expression for the steady-state net mass flux across the outside face of the cutoff wall:

$$J_{net} = qc_w - q(c_0 - c_w)\left[\frac{EXP\left\{\frac{qw}{\theta D}\right\}}{1 - EXP\left\{\frac{qw}{\theta D}\right\}}\right] \quad [16]$$

5 EXAMPLE CALCULATIONS

To illustrate the characteristics of the analysis, transient and steady-state concentration profiles across the wall are calculated for two cases. The parameters for the calculations are listed in Table 1.

Table 1. Parameters for example calculations

Parameter	Value
D^*	2.592×10^{-5} m²/d
α_L	0.0 m
θ	0.3
w	1.0 m
c_0	1.0
c_w	0.01
q	Case 1: -6×10^{-5} m/d (inward flow) Case 2: $+6\times10^{-5}$ m/d (outward flow)

The results for Case 1 are shown in Figure 2. As shown in Figure 2, groundwater flow directed inwards across the wall gives rise to a steady-state concentration profile that is concave-inwards. As shown in Figure 3, groundwater flow that is directed outwards across the wall gives rise to a steady-state concentration profile that is concave-outwards. The length of time required for steady concentrations to evolve differs for the two cases, but a steady profile is eventually attained in both cases.

For Case 1, the inwards flow opposes the concentration gradient. The steady-state net mass flux is given by:

$$J_{net} = \left(-6\times10^{-5}\ m/d\right)(0.01) - \left(-6\times10^{-5}\ m/d\right)\left((1.0)-(0.01)\right)$$
$$\bullet\left[\frac{EXP\left\{\dfrac{\left(-6\times10^{-5}\ m/d\right)(1.0\,m)}{(0.3)\left(2.592\times10^{-5}\ m^2/d\right)}\right\}}{1-EXP\left\{\dfrac{\left(-6\times10^{-5}\ m/d\right)(1.0\,m)}{(0.3)\left(2.592\times10^{-5}\ m^2/d\right)}\right\}}\right]$$

$$= -6\times10^{-7} + 2.65\times10^{-8} = -5.74\times10^{-7} \qquad [17]$$

In this case, the net mass flux is negative, indicating that the inward advective mass flux exceeds the outward diffusive mass flux.

The results for Case 2 are shown in Figure 3 In this case, the net mass flux is directed inwards across the wall. For Case 2, the outward flow is in the same direction as the concentration gradient.

The steady-state net mass flux is given by:

$$J_{net} = \left(6\times10^{-5}\ m/d\right)(0.01) - \left(6\times10^{-5}\ m/d\right)\left((1.0)-(0.01)\right)$$
$$\bullet\left[\frac{EXP\left\{\dfrac{\left(6\times10^{-5}\ m/d\right)(1.0\,m)}{(0.3)\left(2.592\times10^{-5}\ m^2/d\right)}\right\}}{1-EXP\left\{\dfrac{\left(6\times10^{-5}\ m/d\right)(1.0\,m)}{(0.3)\left(2.592\times10^{-5}\ m^2/d\right)}\right\}}\right]$$

$$= 6\times10^{-7} + 5.94\times10^{-5} = +6.00\times10^{-5} \qquad [18]$$

In the second case, diffusion and advection both direct solute towards the outside of the wall. The solution for Case 2 does not predict that the entire wall attains a concentration of the source c_0. Rather, solute is drawn off continuously at $x = w$ to maintain the concentration at the fixed level c_w. As the outwards advective flux increases, the concentration across the wall becomes nearly uniform at c_0 and declines abruptly to c_w at the boundary.

The net mass flux for a range of Darcy fluxes is plotted in Figure 4. The condition for zero net mass flux is also indicated. These results reinforce the point made previously; the condition of zero net mass flux is only one state along a continuous spectrum of possibilities.

If the aquifer outside of the wall is extensive, it is reasonable to assume significant dilution of the solute that migrates out of the wall. The worst case with respect to the predicted outward mass flux will arise when the outside concentration c_w is assumed to be zero. In this case, there is no advective flux to counteract the outward diffusion of solute and the expression for the net mass flux reduces to:

$$J_{net} = -qc_0\left[\frac{EXP\left\{\dfrac{qw}{\theta D}\right\}}{1-EXP\left\{\dfrac{qw}{\theta D}\right\}}\right] \qquad [19]$$

Results with Equation 20 for a range of values of c_w are plotted in Figure 5. For reference purposes, results are included from the previous example c_w/c_0 = 0.01. Two general observations can be made regarding the results. When the Darcy flux is directed inwards (negative), the net mass flux is only weakly sensitive to the outside concentration. When the Darcy flux is directed outwards, the net mass flux is essentially independent of the outside concentration. The assumption of an outside concentration c_w of zero provides an appropriate preliminary estimate of the net mass flux. The solution given by Equation 15 is straightforward in interpretation and implementation, and yields conservative estimates of the net mass flux.

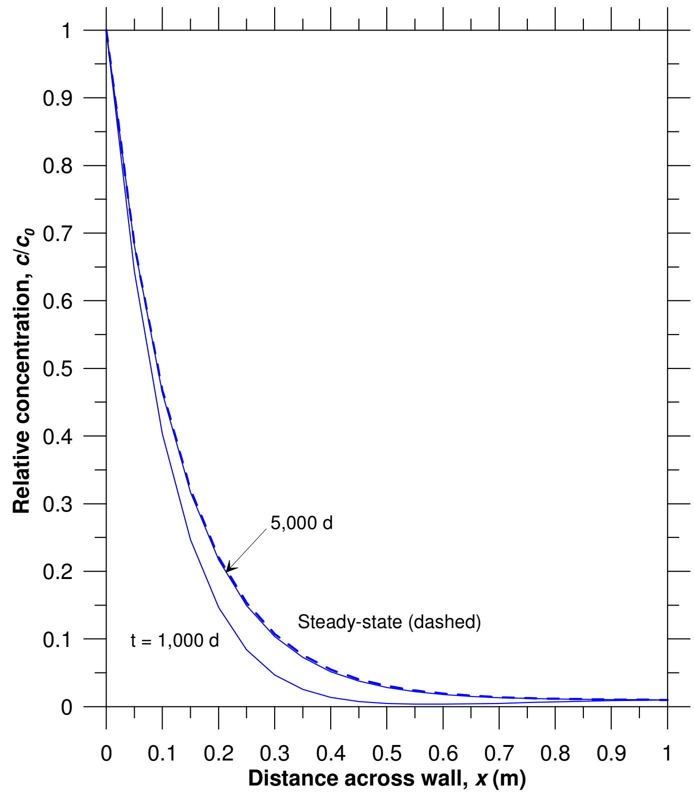

Figure 2. Concentrations for Case 1, inwards flow

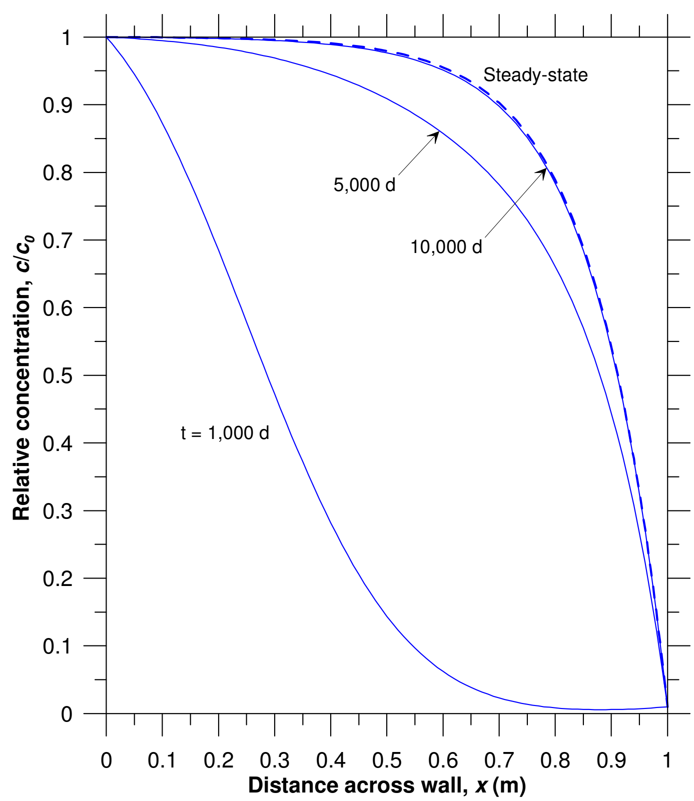

Figure 3. Concentration for Case 2, outwards flow

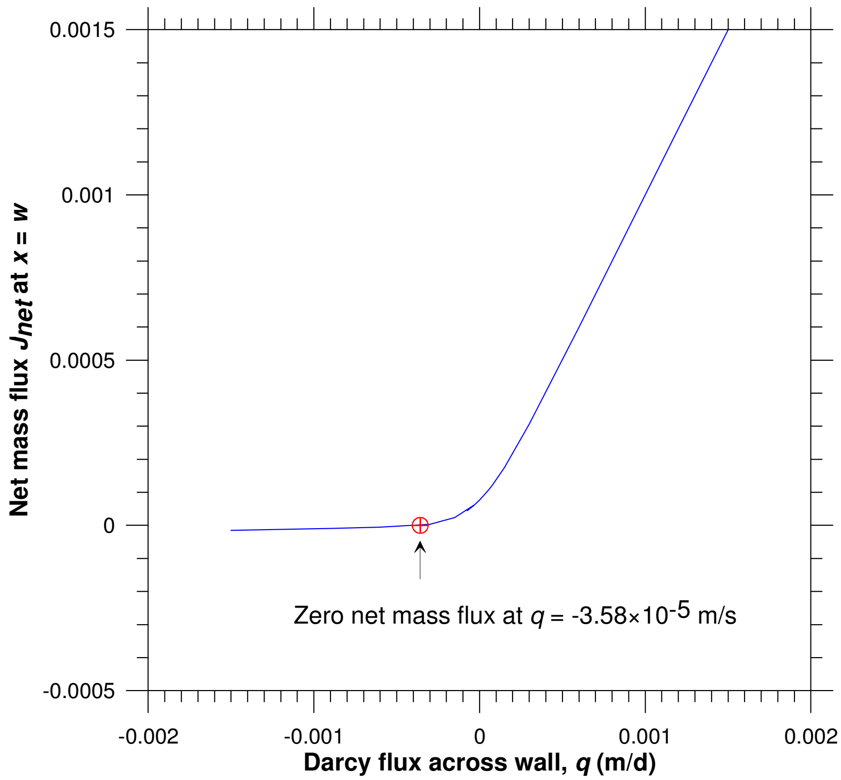

Figure 4. Net mass flux outside of the wall

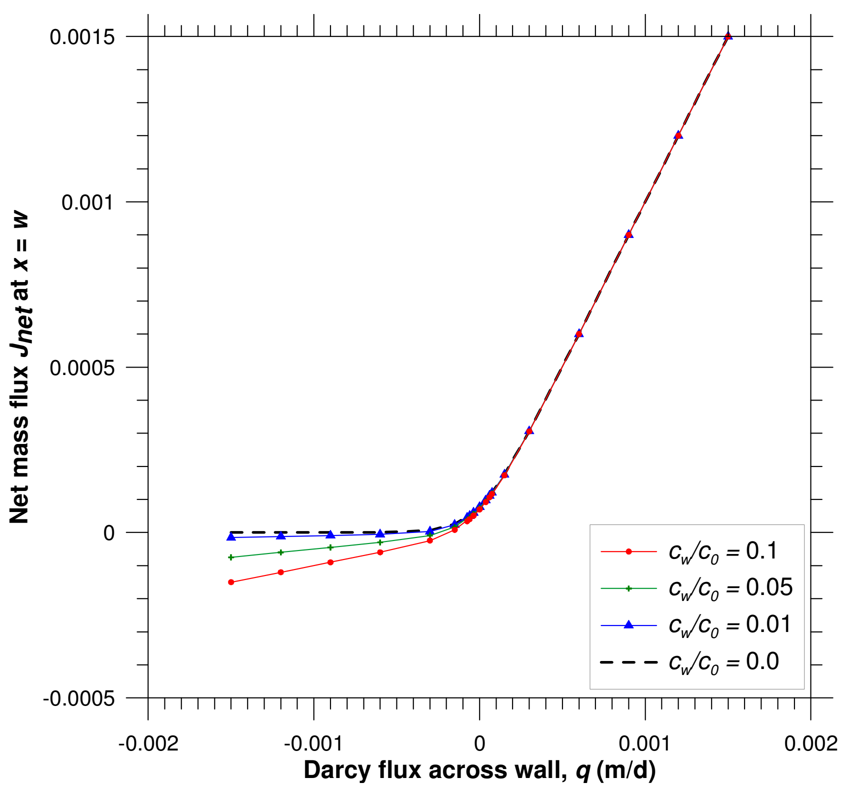

Figure 5. Net mass flux outside of the wall: Effect of c_w

6 CONCLUSIONS

The containment criterion of Neville and Andrews (2006) is based on the condition that the net mass flux across the cutoff wall is zero. A steady-state concentration profile will develop across the wall for any magnitude and direction of the Darcy flux, and the condition of zero net mass flux may be unnecessarily restrictive to achieve the containment objectives at a particular site.

This supplement to the analysis of Neville and Andrews (2006) presents an expression for the steady-state net mass flux across a cutoff wall for any value of the Darcy flux. The results of example calculations reveal that when the Darcy flux is directed inwards towards the source zone, the net mass flux is weakly dependent upon the concentration on the outside face of the wall. When the Darcy flux is directed outwards from the source zone, the net mass flux is essentially independent of the outside concentration. The assumption of an outside concentration equal to zero yields a simple expression for the net mass flux that is straightforward to interpret and evaluate, yielding a conservative estimate of the net mass flux across the wall.

7 REFERENCES

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