Evaluation of first mode of vibration, base fixidity, and frequency effects in resonant-column testing

Zahid Khan, P.Eng., Ph.D.
Civil Engineering Department, American University of Sharjah, Sharjah, UAE
Giovanni Cascante, P.Eng., Ph.D.
Department of Civil Engineering – University of Waterloo, Waterloo, Ontario, Canada
Hesham El-Naggar, P.Eng., Ph.D.
Department of Civil Engineering, University of Western Ontario, Canada
Carlo Lai, Ph.D.
European Centre for Training and Research in Earthquake Engineering, University of Pavia, Italy

ABSTRACT
In resonant column testing (ASTM standard), the shear strain variation is assumed to be linear and the shear wave velocity and damping ratio are assumed frequency independent. To investigate these assumptions, various specimens are tested at different confinements, shear strain levels, and frequencies. To correct the apparent reduction in shear wave velocities in stiff specimens because of a lack in base fixidity, a new model and a new calibration procedure based on a two-degree of-freedom system are proposed.

RÉSUMÉ
Dans une expérience de colonne de résonance (norme ASTM), la variation de la déformation en cisaillement est supposée linéaire et la vitesse des ondes de cisaillement et le rapport d'amortissement sont supposés indépendants de la fréquence. Pour étudier ces hypothèses, différents échantillons sont testés à différents confinements, niveaux de déformation en cisaillement et fréquences. Pour corriger la réduction apparente de la vitesse des ondes de cisaillement dans les échantillons rigides en raison d'un manque de fixité de la base, un nouveau modèle et une nouvelle procédure de calibration basée sur un système a deux degrés de liberté sont proposés.

1 INTRODUCTION

The resonant-column (RC) test is the ASTM standard to evaluate dynamic properties of soils under different confining and shear strain conditions (ASTM 2000). In this technique, a fixed-free cylindrical soil specimen is excited in torsion. The resonant frequency and the damping ratio are obtained from the analysis of the input excitation and the response of the specimen in both time and frequency domains. At resonance, a fixed-base specimen is assumed to vibrate in the first torsional mode without significant participation of other modes. In addition, the deformation along the specimen is assumed to be linear if the ratio of the mass polar moment of inertia of the specimen to that of the driving plate is smaller than 0.1 (Woods 1978). The assumption of a linear mode is critical for the analysis of resonant column test results (Drnevich 1967). This assumption, however, has not been properly verified, especially for stiff specimens and large-strain testing ($10^{-4} < \gamma < 10^{-3}$). For large strains, it is important to account for the non-linear behaviour of the soil; and for stiff specimens, the coupling conditions between the specimen and the end-platens of the RC device should be evaluated.

Conventional resonant-column testing is based on the determination of the resonant frequency of a soil specimen by measuring the specimen response at different excitation frequencies (frequency sweep). The solution of the equation of motion for a column-mass system is used to determine shear wave velocity from the resonant frequency. Furthermore, material damping ratio can be measured by curve fitting the frequency (transfer function) or time domain (free-vibration decay) data with the corresponding theoretical equations, or by the half-power bandwidth method (Cascante et al. 2003). During a frequency sweep (constant excitation voltage), the imposed shear strain levels are not constant for all frequency components. Thus, the measurement of frequency dependant dynamic properties of soils (shear wave velocity and damping ratio) is difficult to perform in conventional RC tests. The non-resonance (NR) method has been recently used to measure the dynamic properties of soils as a function of frequency (Lai et al. 2001). The NR method is based on the solution of the equation of motion governing the forced vibration of a continuous, homogeneous, and linear viscoelastic cylinder representing a soil specimen. The method allows determining simultaneously the shear wave velocity and material damping ratio at the same frequency of excitation. Since these parameters can be determined at different frequencies of excitation, the NR method is well suited to investigate the frequency dependence laws of these important soil parameters.

Direct comparison and validation of results from the NR method and the corresponding results from the standard RC method and the transfer function method (ASTM D4015-92, 2000) are not available in the literature. To show this comparison, a new equation for the transfer function (NTF) between the torque and the rotation of a soil specimen is developed. The NTF allows independent assessment of shear modulus and damping ratio of soils at different frequencies assuming a single-degree-of-freedom model.

The main objectives of this paper are to: to evaluate the assumed linearity of the first mode of vibration and the assumed base fixidity in resonant column testing, and finally to evaluate variation of dynamic properties of soils using the non-resonant method. Clean dry sands, mine tailings, and cemented sand specimens are tested using different specimen-platen coupling conditions at different shear strain levels. Aluminum and PVC probes are tested to evaluate the assumption of base fixidity. Resonant column tests are performed on two sands and on a sand-bentonite-mud (SBM) mixture at different confinements, frequencies, and shear strain levels using the RC, NR, and newly introduced NTF methods.

2 THEORETICAL BACKGROUND

In the fixed-free configuration of the resonant column, the variation of angular rotation along the specimen is given by a quarter-sine function if the mass polar moment of inertia of the driving plate tends to zero (Io = 0). Conversely, the variation of angular rotations approaches a straight line as the ratio of the mass polar moment of inertia of the specimen and the drive plate I/Io tends to zero. The solution for the fixed-free torsional resonant column is (e.g., Richart, et al., 1970):

$$\frac{I}{I_o}=\frac{\omega_n H}{V_s}\tan\left(\frac{\omega_n H}{V_s}\right) \tag{1}$$

where ω_n is the natural frequency, H is the height of the specimen and V_s is the shear wave velocity. The approximate solution for the torsional resonant frequencies using the Raleigh method is

$$\omega_n^{\ 2}=G\ J\left[\left(\frac{I}{3}+I_o\right)H\right]^{-1} \tag{2}$$

where J is polar moment of inertia and G is the shear modulus. If the RC device is modelled as two-degree-of-freedom system (TDOF) and the stiffness k_2 represents the stiffness at the base of the RC and k_1 the torsional stiffness of the specimen ($k_1 = G\ J/H$), then the natural frequencies of the system are given by:

$$\omega_{1,2}=\sqrt{\frac{1}{2}\left[\frac{k_1}{I_1}+\frac{(k_1+k_2)}{I_2}\right]\mp\sqrt{\frac{1}{4}\left[\frac{k_1}{I_1}-\frac{(k_1+k_2)}{I_2}\right]^2+\frac{k_1^{\ 2}}{I_1\,I_2}}} \tag{3}$$

The subscript 1 refers to properties of the specimen, whereas, the subscript 2 refers to the characteristics of the base and the combined mass polar moment of inertia of driving plate and the specimen is $I_1 = (I/3 + Io)$. The standard D4015-92 (ASTM 2000) indicates that the RC can be assumed fixed at base when the ratio $I_2/I_1 > 100$. Ashmawy and Drnevich (1994) considered the base of a RC fixed if the mass of the base is 100 times larger than the mass of the specimen.

The analytical solution for a resonant-column test is obtained using a viscoelastic Kelvin-Voigt model for a material characterized by weak energy dissipation (i.e. low material damping ratio). Thus, the basic equation of motion for a rod undergoing torsional oscillations is given by (Hardin 1965):

$$\frac{d^2\theta}{dt^2}=\frac{G}{\rho}\frac{d^2\theta}{dz^2}+\frac{c}{\rho}\frac{d^3\theta}{dz^2dt} \tag{4}$$

where ρ is the mass density, θ is the angular rotation at the distance z from the bottom of the specimen, and c is the viscous damping coefficient. The solution of Eq. 4 is obtained using the separation of variables method and appropriate boundary conditions which are i) zero rotation at the fixed end of the soil column and ii) at the top of the specimen $z = H$, the torque T_o must be equal to the applied torsional excitation (Lai et al., 2001):

$$\frac{T_o}{\varphi(H)}=J_P\ \frac{\rho\,\omega^2 H}{\beta\tan(\beta)}-I_o\,\omega^2 \tag{5}$$

where ω is the angular frequency, I_o is the mass polar moment of inertia of the driving plate, J_P is the area polar moment of inertia of the, H is the height of the specimen, $\varphi(H)$ is the rotation angle at $z = H$, and β is a variable that depends on the is the shear wave velocity V_s and is given by (Hardin 1965):

$$\beta=\frac{\omega\,H}{V_S} \tag{6}$$

By replacing the elastic shear modulus with the complex-valued shear modulus $G_s{}^*(\omega)$, which is expressed in terms of the real, $G_1(\omega)$, and the imaginary, $G_2(\omega)$, components as

$$G_s^{\ *}(\omega)=G_1(\omega)+i\,G_2(\omega) \tag{7}$$

where i is the complex unit ($i^2 = -1$); thus, the complex value of the parameter β^* is given by:

$$\beta^*=\sqrt{\frac{\rho\,\omega^2H^2}{G_s^{\ *}(\omega)}} \tag{8}$$

Substitution of Eq. 8 in Eq. 5 yields an equation that can be solved iteratively for the complex shear modulus as a function of frequency, if the torque and rotation angle at the top of the specimen are known for a given excitation frequency. The torque and the angle of twist are in general out of phase because of energy dissipation, thus Eq. 5 can be rewritten as (Lai and Rix 1998):

$$\frac{T_o(\omega)\, e^{-i\theta(\omega)}}{\varphi(\omega)} = \frac{I\,\omega^2}{\left[\sqrt{\frac{\rho\,\omega^2 H^2}{G_S^{*}(\omega)}}\,\tan\left(\sqrt{\frac{\rho\,\omega^2 H^2}{G_S^{*}(\omega)}}\right)\right]} - I_o\,\omega^2 \qquad (9)$$

Using the equation of motion for a cylindrical specimen subjected to a torsional excitation, the Lorentz force equation (proportionality between the force applied to the system by the magnetic field in coils and the current in the coils), and representing the soil specimen in a resonant column test by a single-degree-of-freedom (SDOF) system, the transfer function between the torque T_o and the angle of twist φ can be computed as function of frequency ω and the damping ratio ξ as (Khan et al. 2007)

$$\frac{T_o(\omega)\, e^{-i\theta(\omega)}}{\varphi(\omega)} = \left(I_o + \frac{I}{3}\right)\omega_o^{2}\left[1 - \left(\frac{\omega}{\omega_o}\right)^{2} + i\,2\,\xi\left(\frac{\omega}{\omega_o}\right)\right] \qquad (10)$$

3 EXPERIMENTAL PROGRAM

An experimental program is designed to examine the linearity of the first mode of vibration for low ($\gamma < 10^{-6}$) and mid-shear strains ($10^{-4} < \gamma < 10^{-3}$) in dry sands, cemented sands and mine tailings; the coupling effects for stiff specimens; and the base fixidity of the RC device. The maximum displacement at resonance along the height of different specimens is measured at four locations with a miniature accelerometer (2 grams, PCB 353B65 Figure 1). The accelerometer at the top of the specimen on the driving plate (PCB 352A78) is used as a reference for all measurements, whereas the miniature accelerometer is located sequentially at the selected elevations during testing. A cemented-sand specimen is tested by using three different coupling agents between the specimen and the end platen coupling. The testing procedures are similar to the procedures used for the dry sands and mine tailings. Eleven PVC and six aluminium calibration probes of different stiffness (f_o = 8 Hz to f_o = 171 Hz) are tested to study the fixidity of the base of the RC.

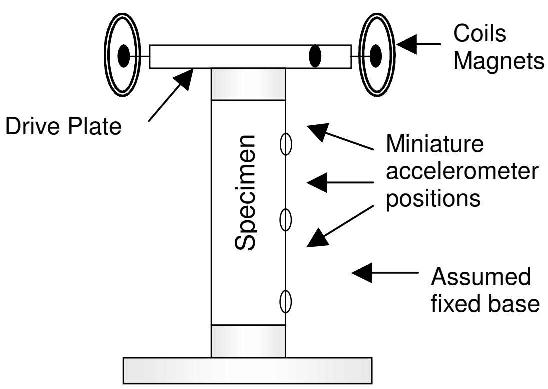

Figure 1. Measurement of first mode shape. Miniature accelerometer at locations 1, 2 and 3. Reference accelerometer at location 4.

The results of RC tests for two silica sand specimens and one sand-bentonite-mud (SBM) specimen are presented in this study. Sand 1 is a rounded to sub-rounded, clean dry sandy specimen (D_{50} = 0.18 mm, C_u = 2); whereas, sand 2 denotes an angular, clean dry sandy specimen (D_{50} = 0.5 mm, C_u = 1.3). The bentonite mud (164 % water by weight of bentonite) is mixed with sand 1 (SBM) to enhance the viscous properties of the soil (estimated viscosity = 2,700 cP). In the SBM specimen, 96% of the void space of the sand matrix (e = 0.646) is filled with the bentonite mud to simulate a clayey sand behaviour (higher viscosity specimen). The specimens are tested at different effective confining pressures (σ_o = 32 kPa to 260 kPa) and shear strain levels ($10^{-6} < \gamma < 10^{-2}$). At each confinement and shear strain level, the dynamic properties are measured using the RC, the NR, and the NTF methods. To reduce the effects of large-strain cyclic loading, the specimens (both sand and SBM specimens) are compacted before testing at the maximum achievable relative density $d_R \approx 96$ % for sand 1 and $d_R \approx 97$ % for sand 2, and pre-strained at the maximum strain levels ($\gamma \approx$ 0.03 %) at different confining pressures..

To measure the degradation of dynamic properties with shear strain level using the NR and NTF methods a single excitation frequency (close to resonant frequency at low-strain levels, $\gamma < 10^{-5}$) is selected after measuring the dynamic properties of the soil specimens at different shear strain levels using the RC method. In the RC method, the transfer function between the applied torque (current) and the angular acceleration is obtained using a sinusoidal-chirp excitation (with bandwidth = 50 Hz). This sinusoidal-excitation has constant amplitude. However, the induced shear strain is not constant for all frequencies in the frequency bandwidth used during the tests. The measured transfer function curve is then fitted with the theoretical transfer function to obtain the resonant frequency and damping ratio of the specimen (Cascante et al. 2005). The shear wave velocity is then computed using Eq. 2.

4 RESULTS AND DISCUSSIONS

Figure 2 summarizes the mode shapes for the sand and mine paste specimens for large (10-4 < γ< 10-3) shear strain levels at σo = 100 kPa. Similar results are obtained for low strain measurements. These results confirm a linear variation of the shear strains as predicted by the theory.

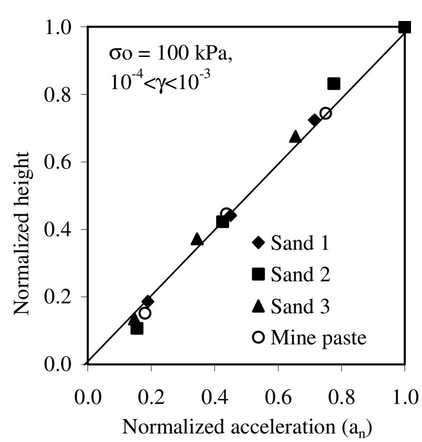

Figure 2. Measured mode shapes for sand and mine tailing specimens at large strains

Figure 3 summarizes the measured mode shapes for the cemented-sand specimen when the coupling is achieved with different coupling agents. The coupling between the specimen and the end-platens is enhanced by using three cementing agents: gypsum based cement (SR), Portland cement (PC), and epoxy resin; the measured mode shapes are plotted in Fig. 14 ($\gamma < 10^{-6}$). The linearity of mode shapes improves by using cementing agents, especially with the epoxy resin. Significant slippage is observed when friction is used for coupling.

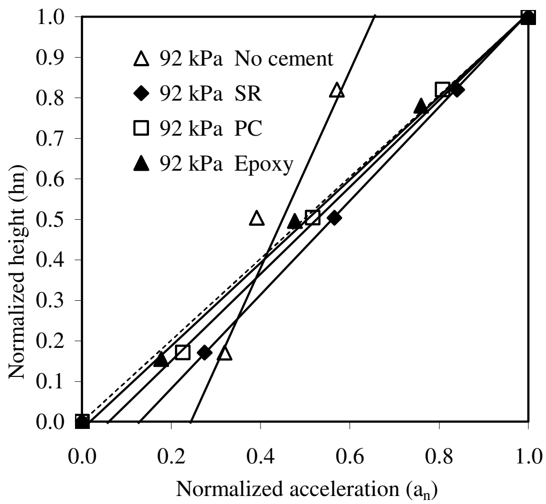

Figure 3. Measured mode shapes for cemented-sand specimen with different coupling cementing agents at 92 kPa (SR: Gypsum cement, PC: Portland cement).

The dynamic properties measured with the top accelerometer are presented in Fig. 4; SR and Portland cements produce a smaller change than the epoxy resin, as these agents do not adhere well to the stainless steel.

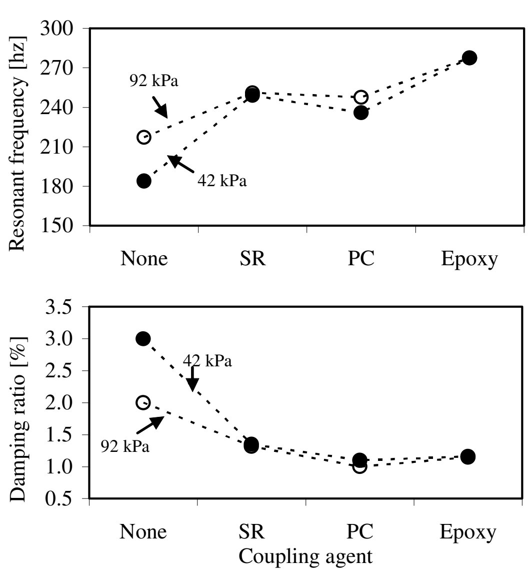

Figure 4. Dynamic properties for cemented sand specimen with different coupling agents (σ_o = 42 and 92 kPa, H = 145 mm).

Similar observations were reported by Lovelady and Picornell (1990). The resonant frequency increases by at least 50 % when the epoxy resin is used and the damping ratio decreases up to 200 %. These dynamic properties are not corrected for the effect of RC base fixidity and eddy current damping; because, the results of Fig. 4 are used only to confirm that the epoxy resin provides adequate coupling as concluded from Fig. 3.

Figure 5 presents the variation of normalized shear wave velocities and damping ratios for eleven PVC and six aluminium calibration probes. Equation 3 is used to curve fit the measurements to obtain the values of base stiffness (k_2) and MPMI (I_2).

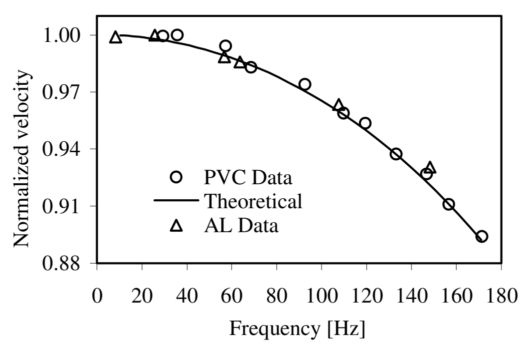

Figure 5. Variation of dynamic properties with frequency for PVC and Aluminum (AL) probes

The shear modulus of the lowest frequency specimen is used to compute stiffness (k_1) at other frequencies. The computed MPMI of the base platen from the geometrical dimensions of the RC device is I_{2b} = 4.58 x 108 g-mm^2 compared to I_2 = 6 x 106 g-mm^2 from the curve fitting analysis. This difference indicates that the inertia of the bench supporting the RC device also affects the results.

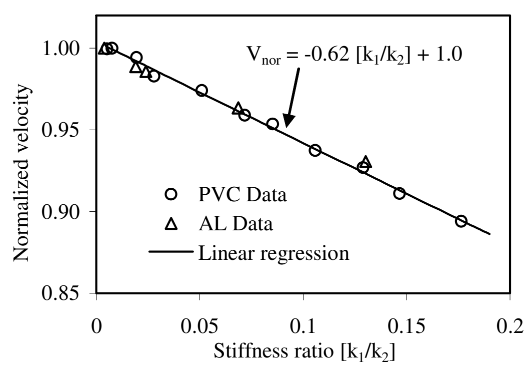

Figure 6. Variation of shear wave velocity with the stiffness ratio k_1/k_2 for PVC and aluminum probes.

The back-calculated value for the base stiffness is k_2= 4.6 x 1011 (g-cm^2/s^2). Figure 5 indicates that the curve fitted model (Eq. 3) correctly predicts the variation in shear wave velocity when a calibration mass is added to the driving plate. This agreement supports the physical validity of the TDOF model for the frequency range studied (8 Hz to 171 Hz).

The computed shear wave velocities for the probes (Eq. 2) shows a linear decrease with the increase in stiffness ratio k_1/k_2 (Fig. 6). The shear wave velocity decreases up to 11 % (23 % reduction in shear modulus); therefore, the RC results should be corrected accordingly. Given the variations in base conditions from one laboratory to another, a correction curve (Fig. 6) should be generated for each resonant column device.

Figure 7 shows typical results of the RC method using a sinusoidal chirp excitation keeping the input voltage constant (ASTM D4015-92), and a sinusoidal excitation with constant strain levels. The curve fitting of the measured data using the transfer function between force (current) and acceleration is also shown (Cascante et al. 2003).

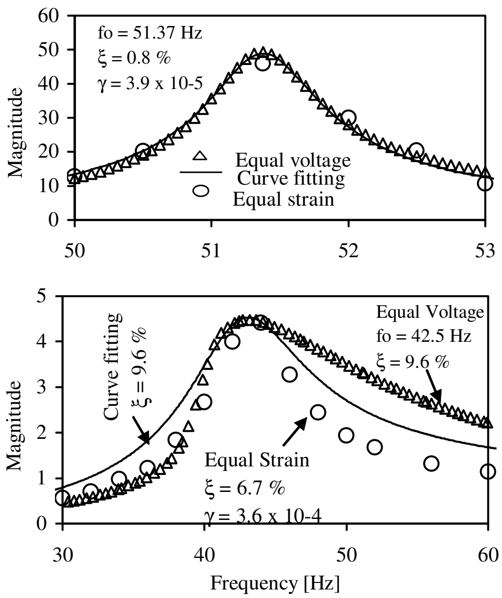

Figure 7. Comparison of the measured transfer functions using the RC method with constant excitation voltage (sinusoidal chirp) and constant strain excitation (sinusoidal sweep) for sand at 33 kPa of confinement.

At low strain levels ($\gamma \leq 3.9 \times 10^{-5}$), the results from the standard RC method (constant-excitation amplitude) are well predicted by the second order transfer function. The measured resonant frequencies when the constant-strain or the constant-excitation amplitude RC methods are used differ by less than 2 %. However, with the increase in shear strain level, the results from the standard RC method (constant-excitation amplitude) deviate from the theoretical transfer function. Damping ratios differ up to 30 % at large shear strain levels. Therefore, the shear strain levels should be constant in the RC method to avoid the overestimation of material damping ratio.

The variation of the dynamic properties of sand (round sand) with frequency and shear strain level using the NR and the NTF methods is summarized in Figure 8. Shear wave velocities from the NR and the NTF methods are practically identical. For the NR method however, the damping ratios tend to increase slightly with frequency as the shear strain level increases. These results are in agreement with previous results that show a practically frequency-independent damping ratio and phase velocity for sands at low strain levels (Kim and Stokoe 1995).

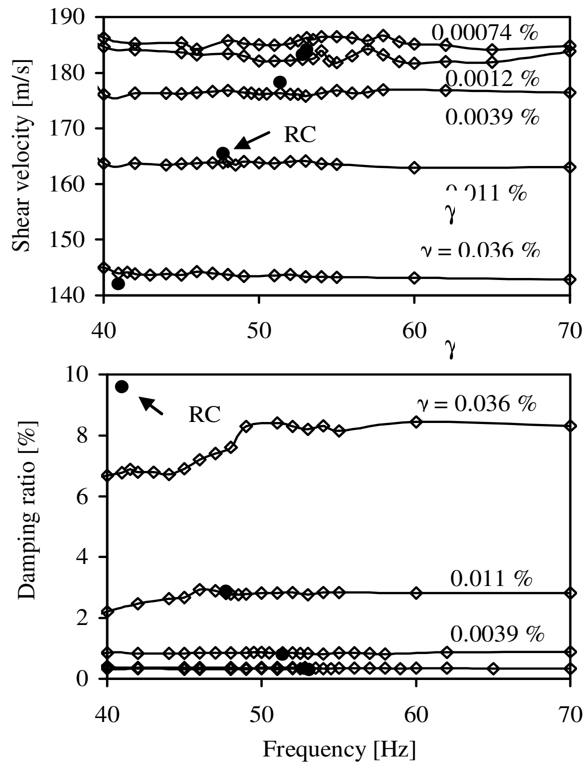

Figure 8. Dynamic properties as a function of frequency at different shear strain levels using NR method (sand 2, σ_o = 33 kPa).

At larger strain levels, damping ratios and shear wave velocities tend to deviate from a linear trend most likely because of the effect of other modes of vibration of the resonant-column apparatus. The damping ratio from the NR method, at large strain levels, is smaller than the measured values using the RC method because of the non-constant strain level conditions inherent in the RC method. The results from the NTF method are practically identical to the ones presented in Figure 8 for the NR method. Although, the NR method is based on linear viscoelasticity theory, it can still be used at large strains in view of the equivalent linear response of the specimen when the strain level is constant for all frequencies tested.

Typical degradation curves from the NR and RC methods for sand are shown in Figure 9. The data are fitted with a modified hyperbolic model (Hardin and Drnevich 1972). The dynamic properties measured using the NR method are obtained for different frequencies; the variation of these properties with frequency for a given strain level is indicated in the figure by the size of the symbol. This variation increases with the increase in shear strain level most likely because of the increasing participation in the response of flexural and other vibration modes of the RC apparatus. Similar results are obtained at other confinements.

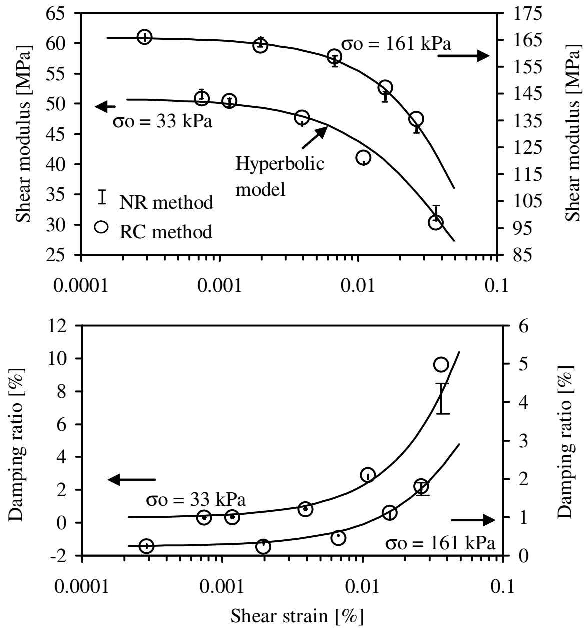

Figure 9. Dynamic properties as a function of shear strain level for the NR and the RC method (sand 2, frequency range for the NR 40 to 70 Hz).

Figure 9 shows good agreement between the results from the NR method and those of the standard RC method. The discrepancy between these two methods increase with the increase in shear strain as discussed before. The NR method allows the reduction of the number of loading cycles imposed on a specimen because of its higher signal-to-noise (S/N) ratio in comparison with the S/N ratio used in the RC method. The NR method is faster than the standard RC test because the latter requires ten or more frequency spectrum averages to obtain a smooth frequency response curve. A smaller number of spectrum averages implies a smaller number of excitation cycles. The reduction of loading cycles is an important issue in the evaluation of dynamic properties at large-strain levels because of the induced fabric changes during loading.

The measured degradation curves from the NR method (single excitation frequency), the NTF method, and the RC method for sand are summarized in Figure 10. Results from the NR method are curve-fitted with hyperbolic models. Results from the NR and the NTF methods agree well for the shear modulus.

Figure 11 shows the low-strain (γ < 10-3 %) dynamic properties for the SBM specimen using the NR and NTF methods (σ_o = 33 kPa). The results for frequencies f = 1.5 Hz to 50 Hz are obtained using two proximity probes on the driving plate. Results for frequencies f > 25 Hz are obtained using the accelerometer. Measurements for frequencies between f = 25 Hz and f = 50 Hz are performed with both types of transducers. The NR and the NTF methods show practically a linear increase in the shear modulus with frequency. The rate of increase of the shear modulus is ΔG = 0.24 MPa / Hz. The damping ratio also increases with frequency. However, this increase is underestimated by 50% in the NTF measurements (NR method: $\Delta\xi$ = 0.035 % / Hz, NTF method: $\Delta\xi$ = 0.018 % / Hz). Rix and Meng (2005) also reported an increase in the shear modulus with frequency of ΔG = 0.16 MPa / Hz and in the damping ratio of $\Delta\xi$ =0.048 % / Hz using the NR method on a remoulded Kaolin specimen. Damping ratio starts to decrease after 60Hz and becomes negligible at f=150 Hz.

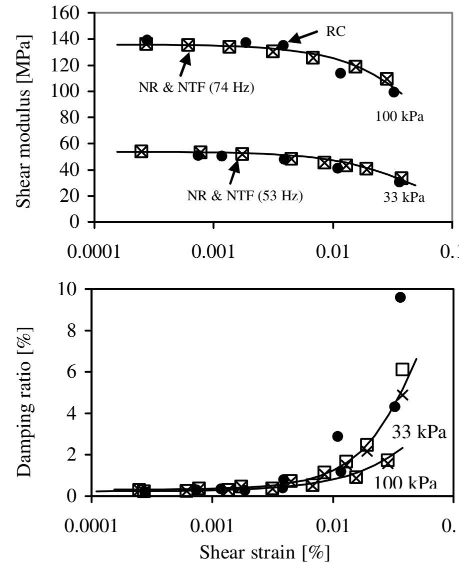

Figure 10. Degradation curves from the NR and NTF methods using one excitation frequency and the conventional RC method using sinusoidal chirp (sand 2).

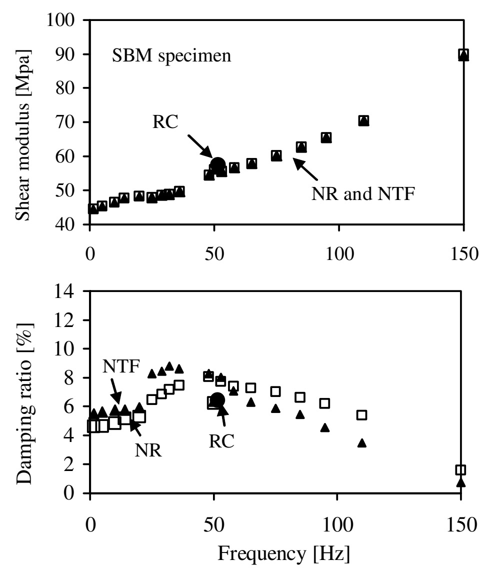

Figure 11. Dynamic properties for the SBM specimen using the RC, the NR, and the NTF methods at larger frequency bandwidth ($\gamma < 10^{-6}$, σ_o = 33 kPa).

Figure 11 shows that the dynamic properties of soils exhibiting strong viscoelastic behaviour cannot be considered frequency independent in the studied frequency bandwidth, even for low-strain level excitations (Lai et al., 2001) as it is the common practice in geotechnical engineering (Hardin and Drnevich 1972).

5 CONCLUSIONS

The resonant column test results are commonly analyzed assuming a linear variation of shear strains along the specimen height (first torsional mode). For sands and mine tailings specimens, the measured accelerations at all confining pressures and shear strain levels follow the assumed linear variation. However for cemented sand specimens (stiff specimens, fo > 200 Hz), the coupling between the specimen and end-platen has to be improved to obtain a linear mode shape.

The accuracy of the measured dynamic properties for stiff specimens improves up to 50 % for the resonant frequency and 200 % for the damping when the coupling is improved with epoxy resin.

The resonant column should not be considered fixed at the bottom when it is resting on the standard laboratory bench. Each RC device should be calibrated by testing specimens of different stiffness. For a stiff specimen (fo = 200 Hz) the under-estimation of the shear wave velocity can be as high as 16 %, which represents an under-estimation of the shear modulus of 35 %.

The applicability of the non-resonant (NR) method for the evaluation of dynamic properties of soils as a function of frequency using the resonant column device is demonstrated for a frequency bandwidth up to 150 Hz and shear strain levels between $\gamma = 10^{-6}$ and $\gamma = 10^{-4}$.

A new transfer function (NTF) method is introduced to compare the results of this conventional transfer function approach with those of the NR method. Shear modulus measurements from both the NR and the NTF methods agree well with the RC method results for different confinement pressures and shear strain levels. However, damping ratios are overestimated (up to 54 %) in the RC method at large strains because the shear strain is not constant at all frequencies.

The stiffness and damping degradation curves of sands can be measured using the NR method with a single frequency of excitation. This approach is especially attractive to evaluate the change in degradation curves with frequency, to reduce testing time (minimizing the effect of the number of loading cycles), and to study the effect of number of cycles on the dynamic properties of soils.

The dynamic properties of sands within a 25 Hz bandwidth appear to be invariant from a practical point of view. However, the dynamic properties exhibit a more pronounced change with frequency for a sand-bentonite-glycerin (SBM) specimen.

Acknowledgements

The authors would like to express their gratitude to Natural Sciences and Engineering Research Council of Canada (NSERC), OCE-Centre for Earth and Environmental Technologies, and Waterloo-Geophysics Inc. for the funding and in-kind contributions provided for this research.

REFERENCES

ASTM D4015-92, (2000), “Standard Test Methods for Modulus and Damping of Soils by the Resonant-Column Method,” American Society for Testing and Materials, Annual Book of Standards.

Ashmavy, A.K., and Drnevich, V.P., (1994), “A General Dynamic Model for Resonant Column/Quasi-static Torsional Shear Apparatus”, Geotechnical Testing Journal, ASTM, Vol. 17, No. 3, pp. 337-348.

Cascante, G., Vanderkooy, J., and Chung, W., (2005), “A new mathematical model for resonant-column measurements including eddy-current effects,” Canadian Geotechnical Journal, Vol. 42, pp. 121-135.

Cascante, G., Vanderkooy, J., and Chung, W., (2003), “Difference between current and voltage measurement in resonant column testing,” Canadian Geotechnical Journal, Vol. 40, No. 4, pp. 806-820.

Drnevich, V. P., (1967), “Effects of strain history on the dynamic properties of sand,” Ph.D. Thesis, University of Michigan, 151 pp.

Hardin, B.O., (1965), "The 6. Nature of Damping in Sands," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 91, No. SM1, pp. 63-97.

Hardin, B. O. and Drnevich, V. P., (1972). “Shear Modulus and damping in soils: Design equations and curves.” Journal of soil mechanics and foundations division, ASCE, Vol. 98, No. SM7, pp. 667.

Kim, D.-S., and Stokoe, K. H. (1995). "Deformational Characteristics of Soils at Small to Medium Strains." Earthquake Geotechnical Engineering, Tokyo, Japan, 89-94.

Lai, C. G., and Rix, G. J. (1998). “Simultaneous inversion of Rayleigh phase velocity and attenuation for near surface site characterization.” Report, National Science Foundation and U.S. Geological Survey, Georgia Institute of Technology.

Lai, C. G., Pallara, O., Lo Presti, D. C. and Turco, E. (2001). "Low-strain stiffness and Material Damping Ratio Coupling in Soils." Advanced Laboratory Stress-strain Testing of Geomaterials, Tatsuoka, T., Shibuya, S., and Kuwano, R. Eds, Balkema, Lisse, pp: 265-274.

Lovelady, P. L. and Picornell, M. (1990), “Sample coupling in resonant column testing of cemented soils.” Dynamic Elastic Modulus Measureemnt in Materials, ASTM STP 1045, Philadelphia.

Richart F. E., Hall J. R. and Woods R. D. (1970), “Vibrations of soils and foundations, Prentice Hall, Englewood Cliffs, pp: 414

Rix, G. J., and Meng, J. (2005). “A non resonance method for measuring dynamic soil properties.”, Geotechnical Testing Journal, 28(1), pp.1-8.

Woods, R.D., (1978), “Measurement of Dynamic Soil Properties”, Proceedings, ASCE Specialty Conference on Earthquake Engineering and Soil Dynamics, Pasadena, California. Vol.1, pp. 91-178.