



# A numerical approach for electro osmotic consolidation in soft soils

V. Jeyakanthan,  
*School of Aerospace, Civil and Mechanical Engineering  
 University of New South Wales @ ADFA, Canberra, Australia*  
 C.T. Gnanendran and S.R. Lo  
*School of Aerospace, Civil and Mechanical Engineering  
 University of New South Wales @ ADFA, Canberra, Australia*

## ABSTRACT

Two-dimensional electro osmotic consolidation of a clayey soil is investigated using a finite element numerical model and the results are compared with the corresponding experimental results. Modified Cam Clay elasto-plastic model has been embedded in to the formulation and implemented into the finite element code. Two electro osmotic triaxial tests at different initial conditions were conducted and the pore water pressure response and axial deformation were measured and compared with predicted numerical results. An excellent agreement is noticed between experimental and numerical results. The numerical solution agrees well with the Esrig's (1968) one-dimensional electro osmotic consolidation solution.

## RÉSUMÉ

Two-dimensional electro osmotique la consolidation d'un sol argileux est étudié en utilisant un modèle numérique par éléments finis et les résultats sont comparés avec les résultats expérimentaux. Mis à jour le Cam Clay élasto-plastique modèle a été incorporé à la formulation et la mise en oeuvre dans le code éléments finis. Deux électro osmotique triaxial tests à différentes conditions initiales ont été menées et les pores de la pression de l'eau de réponse et la déformation axiale ont été mesurés et comparés avec les résultats numériques prédit. Un excellent accord est constaté entre les résultats expérimentaux et numériques. La solution numérique d'accord avec la Esrig's (1968) one-dimensional electro osmotique solution de consolidation.

## 1 INTRODUCTION

Application of electro-osmosis for the improvement of soil was first introduced by Casagrande in 1940s. Since then numerous experimental and field studies have been carried out in different soils and it was found the electro osmosis is more effective in fine grained soils like clayey soils. When naturally present net negative charges in the surface of the clay particle come into contact with the pore water, a layer of cations is formed in the pore water adjacent to the soil particle surface. These cations start to move towards the cathode terminal and drag the surrounding water, when an electric field is applied to the soil. If drainage is provided at both cathode and anode ends of the electric field application, then pore water flows continuously towards the cathode end. This flow is called electro osmotic flow in the soil. When the flow is prevented at the anode, a non uniform negative pore water pressure starts to develop in the soil which leads to electro osmotic consolidation of the soil.

According to Terzaghi's theory of effective stress, change in total stress ( $\Delta\sigma_{ij}$ ) can be written as,

$$\Delta\sigma_{ij} = \Delta\sigma'_{ij} + \delta_{ij} \Delta u \quad [1]$$

Where  $\Delta\sigma'_{ij}$  is change in effective stress,  $\Delta u$  is change in pore water pressure and  $\delta_{ij}$  is Kronecker delta. If there is no change in total stress, the change of the pore water

pressure would be equal and opposite to the change in the effective stress. Therefore development of negative pore water pressure ( $u_{eo}$ ) by electro osmotic consolidation at anode would lead to an increase in effective stress. i.e,

$$\Delta\sigma'_{ij} = -\delta_{ij} \Delta u_{eo} \quad [2]$$

As such, the soil gets preconsolidated. Increase in the preconsolidation pressure can be related to increase in undrained shear strength (Bjerrum et al. 1967; Rittirong and Shang 2008). This is the manner how the soil is strengthened by electro-osmotic consolidation.

The theory of one dimensional electro osmotic consolidation was first provided by Esrig (1968). This theory was developed based on the assumption that electro-osmotic flow, which is developed due to the voltage gradient, and hydraulic flow, which is developed due to hydraulic gradient according to Darcy's law, can be superimposed. Validity of this assumption can be found elsewhere in the literature (Gray and Mitchell 1967). Consolidation was coupled with the flow by means of effective stress theory.

According to Esrig's one dimensional electro-osmotic consolidation theory, the developed negative pore water pressure at a particular time is given by [3],

$$u_{eo}(x,t) = -\frac{k_e}{k_h} \gamma_w E(x) + \frac{2k_e \gamma_w E_m}{k_h \pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+\frac{1}{2})^2} \sin \frac{(n+\frac{1}{2})\pi x}{L} [\exp - (n+\frac{1}{2})^2 \pi^2 T_v] \quad [3]$$

Where  $u_{eo}(x,t)$  is the pore water pressure developed at distance  $x$  from cathode at a particular time  $t$ ,  $k_e$  is electro-osmotic permeability,  $k_h$  is hydraulic permeability,  $\gamma_w$  is specific unit weight of pore water,  $E(x)$  is voltage at distance  $x$  from cathode,  $E_m$  is voltage at anode,  $T_v (= C_v t / L^2)$  is the time constant and  $L$  is the distance between anode and cathode.

Following Esrig's work, an analytical model was developed by Wan and Mitchell (1976) to account for the combined effects of load application and electro-osmotic consolidation. Though these theories are well established, the usage of these theories is limited to one dimensional case and for simple geometries or electrode configurations.

A two dimensional finite element model for the electro osmotic flow was formulated by Lewis and Garner (1972) but this method was limited to the analysis of confined flow problems only. However, an extended work was presented later to solve problems involving deformation (Lewis and Humpheson 1973). Su and Wang (2003) presented an analytical solution for two dimensional electro-osmotic consolidation and Shang (1998) presented a two dimensional analytical model for electro-osmosis enhanced preloading consolidation using vertical drains. However, these analytical solutions are applicable to certain electrode configurations, and boundary and geometric conditions. More recently, a numerical model was presented by Rittirong and Shang (2008) by using finite difference method to analyze electro osmotic consolidation problems in two dimensional electric field.

A numerical model has been developed by the authors using the finite element method for predicting the electro osmotic consolidation behaviour of clay in which the stress strain behaviour of the clay is modeled as an elasto-plastic Modified Cam Clay (MCC) material. Pertinent details of this numerical model and the predicted results of typical electro-osmotic consolidation problems are presented in this paper. In particular, this paper presents results obtained from electro-osmotic triaxial tests at different initial conditions and the predicted results. An example problem is analyzed using constant hydraulic permeability and compared with Esrig's one-dimensional theory.

## 2 NUMERICAL MODELING

### 2.1 Governing equations

Finite element formulation for the coupled electro-kinetic and hydrodynamic flow through a porous media has

been derived by Lewis and Garner (1972) for a two-dimensional case. In this formulation, coupled fluid flow and current flow equations were written in terms of hydraulic and voltage gradients and finite element formulation were formed. This model is capable of predicting the electro osmotic head developed between the electrodes. There is no provision given to accommodate the stress strain behavior of the soil mass during the electro osmotic consolidation. Based on Biot's consolidation theory, a finite element form of normal two-dimensional consolidation has been presented by Britto and Gun (1987). A combination of fluid flow equation under transient condition and equilibrium equation has been used in their formulation. Indeed, equilibrium equation has been used in the form of virtual work and the constitutive relations are included to account the stress strain behavior. An extended form of the virtual work principle has been adopted in this research along with the fluid flow equation and current flow equation.

### 2.2 Association of Modified Cam Clay model

Modified Cam Clay (MCC) is a widely accepted elasto-plastic model to describe stress strain behavior of saturated clay. Yield surface of MCC is elliptical in  $q$ - $p'$  space and given by the equation,

$$q^2 + M^2 p'^2 = M^2 p' p_c' \quad [4]$$

Where  $q$ ,  $p'$ ,  $p_c'$  and  $M$  are deviator stress, mean normal effective stress, preconsolidation pressure and frictional constant respectively. It is well understood that during the electro osmotic consolidation, a negative pore water pressure ( $u_{eo}$ ) develops in the soil. This pore water pressure is not uniform and it varies spatially as well as with time according to the electrode configuration, electro osmotic properties of the soil and applied voltage.  $u_{eo}$  at any point in the soil mass is determined at each time step with the numerical model developed by the authors and the  $u_{eo}$  is input to the MCC model through the coupling equations.

It is noted that the  $u_{eo}$  gives an increment in the effective stress and it can be related to a change in mean normal stress  $p'$ , i.e.,

$$\Delta p' = \frac{1}{3} \sum_{k=1}^3 \Delta \sigma_{kk} = \Delta u_{eo} \quad [5]$$

In the MCC model the size of the yield surface is defined by the preconsolidation pressure  $p_c'$ . In normally consolidated clays, the increment in mean effective stress,  $p'$ , will tend to expand yield surface when no external force is applied, i.e., for a constant  $q$ . The effectiveness of the electro osmotic consolidation in the strength improvement for over consolidated clay depends on the over consolidation ratio and the developed negative pore water pressure,  $u_{eo}$ .

If the soil is highly over consolidated, the particular  $u_{eo}$  would give either a small or no expansion in the yield surface.

### 3 EXPERIMENTAL WORK

An experimental program was devised mainly for two purposes. One is for finding appropriate input parameters for the numerical program and the other is for calibrating the numerical results with experimental findings.

The soil used for the tests was taken from Raymond Terrace which is located 15 km north of Sydney, Australia. This sample was recovered at a depth of 2.5 to 3 m from a construction site. To maintain consistency throughout the experiments, samples were reconstituted from slurry made from the raw soil sample. A 60 mm diameter spilt mould was used to prepare samples. Properties of the test soil are summarized in Table 1.

Table 1 Properties of soil

| Properties                          | Value     |
|-------------------------------------|-----------|
| Moisture content (%)                | 86        |
| Liquid limit (%)                    | 101       |
| Plastic limit (%)                   | 35        |
| Soil particle specific gravity      | 2.57      |
| Pore water salinity (g/L)           | 14        |
| Pore water pH                       | 2.8 - 3.5 |
| Cation exchange capacity (meq/100g) | 17.6 - 30 |
| Organic content (%)                 | 2.6       |

Since the MCC model is part of the numerical analysis, it is required to determine the MCC model parameters such as frictional constant  $M$ , slopes of the normal consolidation and swelling lines ( $\lambda$  and  $\kappa$ ), critical state void ratio  $e_{cs}$  and Poisson ratio  $\nu'$ . In addition, electro osmotic flow parameters such as hydraulic conductivity ( $k_h$ ), electro osmotic permeability ( $k_e$ ) and electrical conductivity ( $1/\rho$ ) of the given soil also need to be determined. One dimensional consolidation tests were conducted to obtain the parameters  $\lambda$ ,  $\kappa$ ,  $e_{cs}$ ,  $C_v$  and  $k_h$  (Britto and Gunn 1987).  $k_e$  and  $1/\rho$  were determined from an electro osmotic triaxial test. Effective friction angle  $\phi'$  was assumed to be  $25^\circ$ , and  $M$  was calculated from the relationship between  $M$  and  $\phi'$  given by Britto and Gun (1987).

#### 3.1 Electro osmotic triaxial test

The conventional triaxial test set up was modified to facilitate electro osmotic consolidation for this test. Two plastic cylindrical platens, which were attached with two

circular perforated copper electrodes, were used to hold the sample. A filter paper and a geotextile layer were placed between the sample and electrode to provide good drainage and to prevent the soil particle movement, i.e. the electrophoresis is controlled. Electrodes were connected with wires which were taken out of the electro osmotic cell. Two set of burette - differential pressure transducer (DPT) combinations were used to measure the volume of water going into the cell and coming out of the sample. A linear variation differential transformer (LVDT) was used to measure axial deformation. Three pressure transducers were used; two of them were used to read cell and back pressure respectively and the other one was used to monitor the pore water pressure variation at the anode.

The reconstituted samples of 60mm diameter and 120mm height were trimmed out for the electro-osmotic triaxial tests. Two tests, hereafter referred as EO1 and EO2, were conducted with various initial conditions. Samples were initially consolidated in the triaxial cell to reach target initial effective stresses of 50kPa for test EO1 and 150kPa for test EO2. A hanging load, which is slightly higher than the force exerted on the vertical axle, was applied axially to aid the measurement of axial deformation. Once the initial consolidation was finished, electric potential was applied to the electrodes by means of DC current supply. Data was recorded automatically with the aid of a data acquisition system.

##### 3.1.1 Electro-osmotic parameters

Coefficient of electro-osmotic permeability,  $k_e$ , is one of the important parameters in the electro osmotic consolidation and accurate measurement of  $k_e$  is crucial. Electro-osmotic permeability was measured at different initial stresses and voltages. During the measurement, valves at both the anode and cathode were opened for free flow of water and the flow rate was determined. Since the applied voltage gradient is known, electro-osmotic permeability was calculated from the following equation.

$$k_e = \text{Volume flow rate (m/s)} / \text{Voltage gradient (V/m)}$$

Voltage loss at the electrodes was measured at the end of the treatment with the aid of a multi-meter by measuring voltages at 5 mm away from the electrodes. This voltage loss at the electrodes was taken in to account when  $k_e$  was calculated.

Electrical conductivity was determined from the measurements of current densities and the applied electrical potential. Another parameter required to fulfill the formulation is streaming current conductivity. According to Onsager's reciprocal relation, the streaming current conductivity can be equated to electro osmotic permeability values (Beddiar et al. 2002; Lewis and Garner 1972). Since the samples were prepared from a thoroughly mixed slurry, it was assumed that the properties in the x and y directions are the same.

### 3.1.2 Electro-osmotic consolidation

Once the initial triaxial consolidation was finished, the valve at the anode was closed and an electric potential was applied. In the mean time, data recording was also started. Prevention of water flow at anode and open flow at cathode caused the sample to consolidate. Measured values of axial deformation and pore water pressure variation at anode were compared with numerical solutions in the following section. Observation on the lateral deformation was also compared with the predicted deformed shape of the sample.

## 4 RESULTS AND DISCUSSION

The solution obtained from the numerical analysis was checked against the experimental results. A finite element mesh representing the triaxial sample was generated with 6-noded triangular elements (linear strain triangles) and the mesh is shown in Figure 1. Since the problem is axi-symmetric, half of the mesh was considered for the analysis. Appropriate boundary conditions and initial conditions were input to the analysis along with the input parameters given in Table 2. The problem was analyzed using Element 94, which was newly added to the element library, of the authors' version of AFENA (A Finite Element program for Numerical Analysis) (Carter and Balaam 1995) program to analyze axi-symmetric electro-osmotic consolidation. Material properties in x (horizontal) and y (vertical) directions were considered to be same.

Table 2 Parameters used for the analysis

| Properties                                | Value                  |
|-------------------------------------------|------------------------|
| $\lambda$                                 | 0.356                  |
| $\kappa$                                  | 0.058                  |
| $e_{cs}$                                  | 3.235                  |
| $M$                                       | 0.984                  |
| $\nu'$                                    | 0.25                   |
| $k_e$ (m <sup>2</sup> /Vs)                | $1.197 \times 10^{-9}$ |
| Electrical conductivity, $(1/\rho)$ (S/m) | 0.048                  |

Measured pore water pressure changes at anode and measured axial deformations from tests EO1 and EO2 were compared with predicted pore water pressure at node A (anode) and measured axial settlement at node B (Cathode). Applied voltage in both tests was 4.48 V.

It is well known that coefficient of hydraulic permeability,  $k_h$ , varies with the void ratio of the soil (Gulhati and Datta 2005; Nagaraj and Miura 2001; Tavenas et al. 1984). In AFENA,  $k_h$  was related to the void ratio by a power function. The  $k_h$  values were determined from the oedometer test and plotted against the void ratio ( $e$ ). The fitted relationship was:

$k_h = 3.256E^{-11} \times e^{3.22}$  with  $R^2 = 0.981$ , where unit of  $k_h$  is in m/s.

### 4.1 Pore water pressure response

Figure 2 and 3 show the variation of pore water pressure with time obtained for tests EO1 and EO2 respectively. For easy comparison, the results predicted from the numerical analysis are also superimposed in these figures. In each test, the experiment was stopped when the applied back pressure reached nearly zero at the anode. Thus, the possibility for cavitations, which may lead to false measurement of pore water pressure, was minimized. Voltage loss at electrodes has been noticed by many researchers (Lefebvre and Burnotte 2002; Lo and Ho 1991; Mohammedhassan and Shang 2001; Rittirong and Shang 2008). Therefore a voltage measurement was made at the end of treatment to account the loss at electrodes. It was noticed that 60% of the applied voltage was effective and this effective voltage was used in the analysis.

It can be seen that the numerical model predicts the pore water pressure response so closely in both tests despite a small lack in the prediction at the beginning. This discrepancy could be attributed to the higher voltage gradient at the beginning of the test rather than the effective voltage gradient used for the prediction. In the test EO2, data was not recorded for a period of time (2<sup>nd</sup> -3<sup>rd</sup> day) because of malfunction of the data acquisition system.

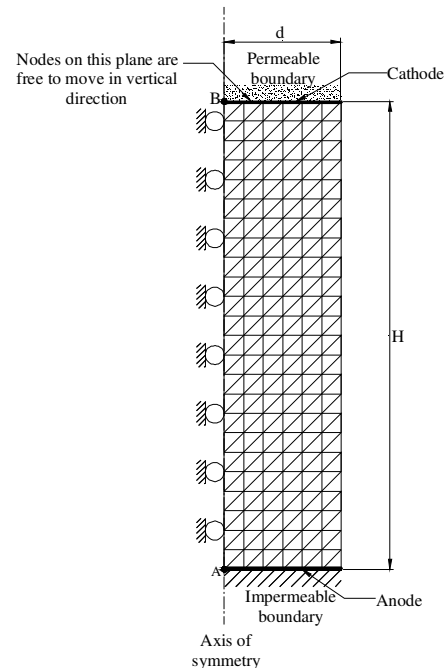


Figure 1 Finite element mesh of the triaxial sample

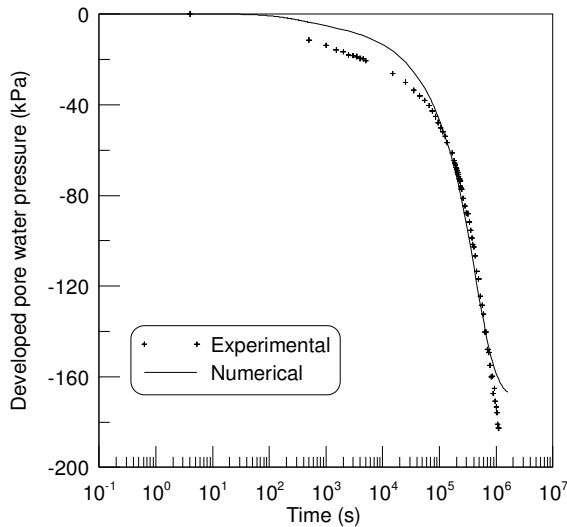


Figure 2 Variation of pore water pressure with time for test EO1

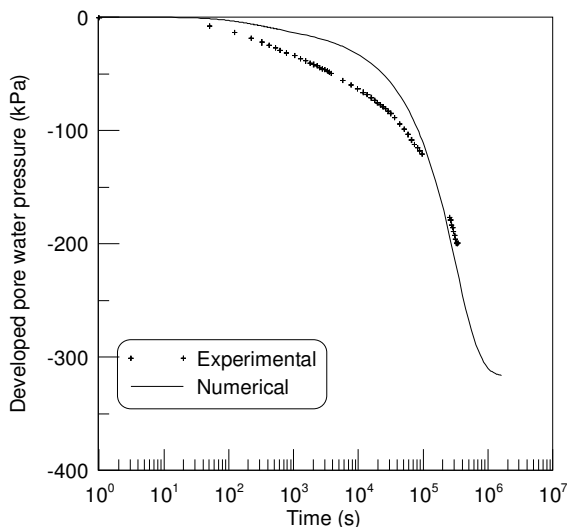


Figure 3 Variation of pore water pressure with time for test EO2

Considering the pore water pressure response obtained from tests EO1 and EO2, it is obvious that time required to develop a certain amount of negative pore water pressure was largely different in each test although the same voltage was applied. It took around 13 days to reach its maximum developed negative pore water pressure of 185kPa in test EO1 (i.e. for sample consolidated at 50 kPa). In test EO2 (i.e. for sample consolidated at 150 kPa), the same negative pore water pressure developed in 3 days and 6 hours, which is 25% time of that in the test EO1. This remarkable variation in the time response can be attributed to the varying hydraulic permeability. It is well known that void ratio reduces with the increase in mean effective stress, thus

hydraulic permeability reduces. The negative pore water pressure that is developed is inversely proportional to hydraulic permeability (Esrig 1968; Wan and Mitchell 1976). This is why the time required for attaining a certain amount of negative pore water pressure was higher in the low initial mean effective stress case than in the higher initial mean effective stress case.

#### 4.2 Axial deformation

Figure 4 and Figure 5 show the variation of axial deformation with time obtained for tests EO1 and EO2 respectively. The results predicted from the numerical analysis are also superimposed in these figures for easy comparison. It can be clearly seen that the predicted and measured axial deformation for both tests were in excellent agreement in the early stage of the consolidation though axial deformation is slightly under predicted in the later stage of test EO1. Unfortunately, the experiment had to be stopped before it reached equilibrium and hence the behavior cannot be discussed after this stage. It can be noticed that prediction of pore-water pressure response is not consistent at the early state unlike axial deformation, which is possibly because the pore-water pressure and axial deformation responses are measured at different locations.

Considering overall results discussed so far, it can be concluded that the model predicts the responses of the pore water pressure and axial deformation reasonably well.

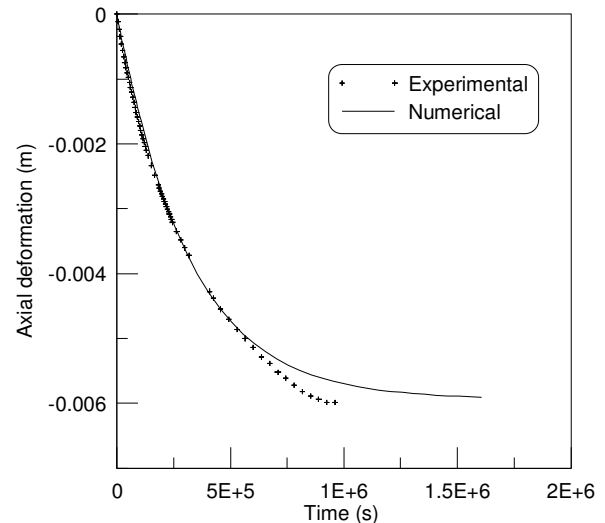


Figure 4 Variation of axial deformation with time for test EO1

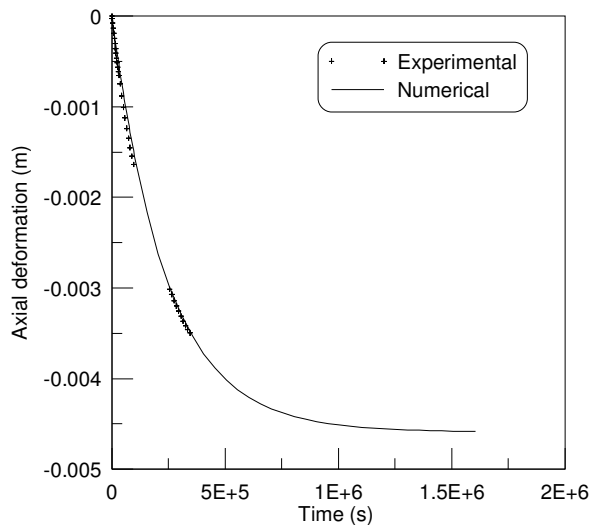


Figure 5 Variation of axial deformation with time for test EO2

#### 4.3 Lateral deformation

As the electro osmotic consolidation progresses, apparently there will be a change in the volumetric strain and the percentage of volumetric strain added from lateral deformation is significant (Win et al. 2001). Experimental measurements and observation from this current work justified this phenomenon and the numerical results also reflected this fact. The deformed sample at the end of consolidation and deformed mesh from AFENA together with the mirror image about the axis of symmetry are shown in Figure 6 and Figure 7 respectively. In the finite element analysis of this triaxial problem, displacements in the x direction at anode and cathode were restrained considering the surfaces are rough enough to restrict movement in the x direction. Lateral displacement at the edge of the sample is plotted in Figure 8. However, quantitative measurements of these values could not be compared because end of treatment measurements were taken after letting the developed negative pore water pressure to dissipate. During this period, considerable change in the dimensions of the sample can be expected.

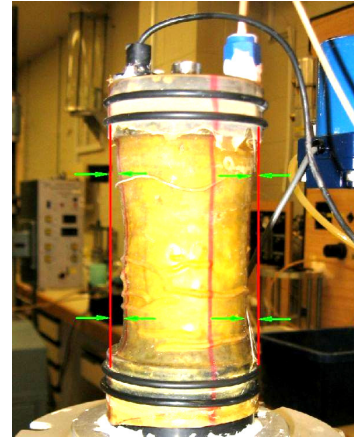


Figure 6 Photograph of the sample at the end of electro osmotic consolidation

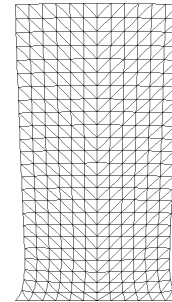


Figure 7 Deformed mesh of the triaxial sample

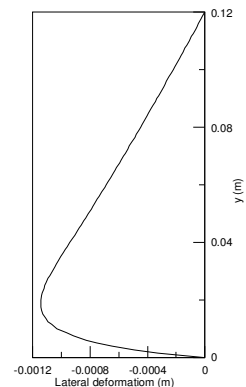


Figure 8 Displacement in x-direction at the edge of the sample

#### 4.4 Comparison with Esrig's 1-Dimensional theory

Though the formulation was derived for two-dimensional cases, simple one-dimensional problems can be analyzed by setting appropriate boundary conditions. The same finite element mesh, which was used in the previous section, was used after changing the boundary conditions. Displacements in the x direction were restricted in all the boundaries and displacement in the y direction was allowed in all boundaries except at the bottom boundary (anode). Predicted pore water pressure response was compared with the solution obtained from

the one dimensional electro osmotic consolidation theory. In this analysis, the hydraulic permeability was considered constant during the treatment because Esrig's theory considers a constant hydraulic permeability.

In Esrig's one dimensional electro osmotic consolidation theory, coefficient of consolidation,  $C_v$ , is the key parameter which is determining the rate of consolidation since the Esrig's theory was developed from the concept of Terzaghi's one dimensional consolidation theory. However, it has been argued that coefficient of electro osmotic consolidation,  $C_{ve}$ , should be used instead of conventional coefficient of consolidation,  $C_v$  in design calculations. Possible changes in chemical and mechanical properties of the soil during the electro osmotic treatment may affect the rate of consolidation (Shang and Ho 1998). Inconsistent values for  $C_{ve}$  in relation to  $C_v$  have been presented in the literature. Shang and Ho (1998) reported much lower value of  $C_{ve}$  compared to  $C_v$  from tests conducted on Gloucester Clay and Wallaceburg Clay. Win et al (2001) reported that  $C_{ve}$  values, compared to  $C_v$ , to be higher in low voltage tests and lower in high voltage tests for the Singapore Marine Clay.

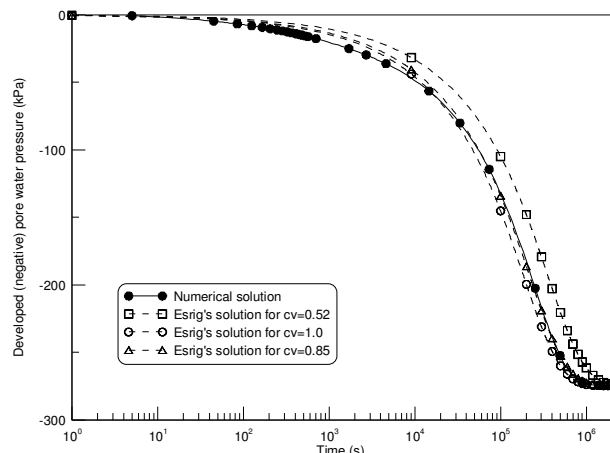


Figure 9 Pore water pressure variation with time

For the particular clay used in the current research, it was found from oedometer tests that  $C_v$  values vary in the range of 0.17 m<sup>2</sup>/yr to 0.52 m<sup>2</sup>/yr. A sensitivity analysis was done to find out the suitable value for  $C_{ve}$ . Esrig's theoretical solution was obtained with the aid of program MATLAB 7.5.0 for different  $C_{ve}$  values starting from 0.52 m<sup>2</sup>/yr. Initial mean effective stress of 150kPa was considered for this problem and corresponding  $k_h$  was determined from the relation between hydraulic permeability and void ratio and the relationship between mean effective stress and void ratio obtained from oedometer tests. Soil was considered as normally consolidated. Figure 9 presents the plots of rate of

change of pore water pressure at anode obtained from AFENA and Esrig's theory. From Figure 9, it is apparent that the predicted maximum pore water pressure (at equilibrium) at anode is the same from both numerical and theoretical solution. From this plot it has been found that  $C_{ve} = 0.85\text{m}^2/\text{yr}$  gives an excellent agreement with the numerical result.

Once a suitable value for  $C_{ve}$  has been determined, pore water pressure distribution between the electrodes at various times was plotted and the results were compared with the numerical solution. Figure 10 shows the pore water pressure distribution at 1 hr, 1 day and 7 days. Numerical solution and theoretical solutions match quite well though there are some discrepancies at the initial stage.

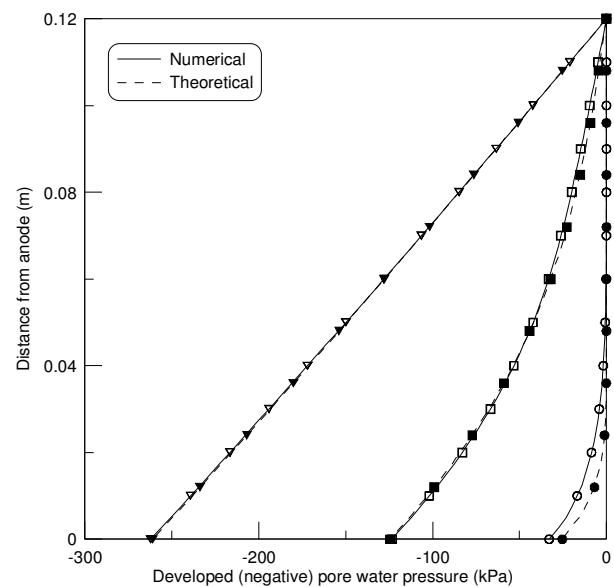


Figure 10 Pore water pressure distribution between electrodes with time

While coefficient of electro osmotic consolidation,  $C_{ve}$ , determines the rate of consolidation in the theoretical solution, hydraulic permeability is the one which decides the rate of consolidation in numerical solution. As discussed earlier, hydraulic conductivity changes with mean effective stress. Therefore, the solution has to be checked at various initial mean effective stresses. Solution at initial stresses of 100kPa and 200kPa are given in Figure 11. A very good agreement between theoretical and numerical solution was noticed.

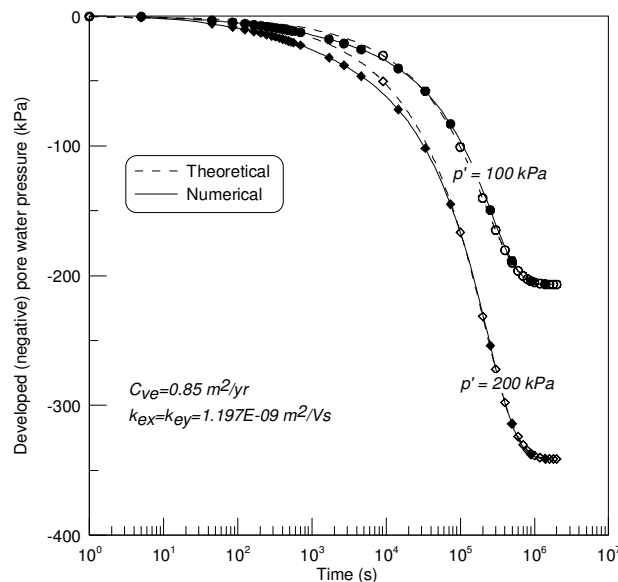


Figure 11 Pore water pressure variation with time  $p' = 100 \text{ kPa}$  and  $p' = 200 \text{ kPa}$ .

## 5 CONCLUSIONS

Two-dimensional electro osmotic consolidation of a clayey soil has been investigated using a finite element numerical model and the results are compared with the corresponding experimental results. Modified Cam Clay model has been embedded in to the formulation and implemented into the finite element code. Two electro osmotic triaxial tests at different initial conditions were conducted and the pore water pressure response and axial deformation measurements were compared with predicted numerical results. Good agreement is noticed between experimental and numerical results. The numerical solution agrees well with the Esrig's (1968) one-dimensional electro osmotic consolidation solution.

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