# Experimental Evaluation of the Subgrade Reaction and Soil Modulus Profiles for Granular Backfills



# M. GadElRab Hussein, Graduate Student

Department of Civil Engineering, Al-Azhar University, Nasr City, Cairo, Egypt Current Visiting Research at McGill University, Montreal, Quebec, Canada

## ABSTRACT

In this study, five different backfill materials of known relative densities have been prepared in a large scale testing facility. Plate-load tests have been conducted to asses the effect of soil density on the subgrade reaction and soil modulii. The relationship between the applied pressure and the corresponding soil movement has been established at the surface as well as at different depths. The modulus of subgrade reaction is determined based on the measured stress-settlement relationship at the surface whereas the Young's modulus profile has been established using the measured soil displacements at different depths. A comparison between the measured soil modulus and subgrade reaction and those reported in the literature is presented.

# RÉSUMÉ

Cinq matériaux de remblayages différents, donc la densité relative est connue, sont préparés dans un circuit d'essai à grande échelle. Des essais de chargement à la plaque sont effectués pour déterminer l'effet de la densité du sol sur la réaction de la sous-fondation ainsi que le module du sol. Le rapport entre la contrainte appliquée et les mouvements du sol sont établis en surface et en profondeur. Le module de la sous-fondation est déterminé en surface en considérant la relation contrainte-tassement établi. Le profile du module de Young est établi en évaluant les tassements à différentes profondeurs. Une comparaison des modules du sol et de la sous-fondation obtenu par cette étude et ceux rapportés dans la littérature est aussi présentée.

# 1 INTRODUCTION

The ground response to loading imposed by different foundation systems is known to be a complex soilstructure interaction problem. Reliable estimates of the soil modulus or subgrade reaction modulus are generally needed for structural design purposes. Bending moments, shear forces, and deflections can only be computed if these soil reaction values are available to the designer (Ulrich et al., 1988).

Structural loading causes stresses to increase in the subsurface soil layers and settlement of the supported structure. The magnitude of elastic settlement depends directly on the values of the elastic parameters (Young's modulus,  $E_s$ , and Poisson's ratio,  $v_s$ ), (Holtz, 1991).

There are several methods to determine the modulus of subgrade reaction,  $k_s$ , and Young's modulus,  $E_s$ . One way of determining  $E_s$  is to conduct a laboratory triaxial or unconfined compression tests on representative undisturbed samples extracted from the depths required (Murthy 2002). Triaxial experimental evidence (Hanna and Adams 1968, Soderman 1968, Leonards and Bozozuk, 1972) suggests a common approximation for the ratio of  $E_s/(\sigma_1-\sigma_3)$  ranging between 250 to 500.

Since it is practically impossible to obtain undisturbed sample of cohesionless soils, the laboratory methods of obtaining  $E_s$  can be ruled out.

Several investigators suggested analytical approximations to determine the modulus of subgrade reaction using the elastic modulus and Poisson's ration (Biot 1937, Hogg 1938, Vesic 1961, Barden 1963, Vlasov and Leontiev 1966). Young (1960) used the consolidation test data to obtain the modulus of subgrade reaction. Other researchers (Nascimento and Simoes 1957, Recordon 1957, Black 1961, Brata 1967, Singh 1967) related the modulus of subgrade reaction to the California Bearing Ratio "C.B.R.".

Field methods are increasingly used to determine the soil strength parameters. They have been found to be more reliable than the ones obtained from laboratory tests (Bowls 1988, Das 1998, and Murthy 2006).

Most methods use the data from the standard penetration test "S.P.T." or the cone penetration test "C.P.T." and/or the plate-load test "P.L.T.". Also, there are methods that use the results of the pressure-meter test "P.M.T.". Scott (1981) proposed that the modulus of subgrade reaction for sandy soils, k<sub>0.30</sub>, can be estimated from the standard penetration test data. D'Appolonia (1970) suggested equations for estimating Young's modulus for pre-loaded and normally loaded sands using S.P.T. results. Other correlations between the modulus of elasticity and the standard penetration test results were investigated along with some other factors "Overburden pressure, depth of footing and over consolidation ratio"; (Schultze and Melzer 1965, Schultze and Sherif 1973, Bowles 1988, U.S. Corps Engineers 1990, Kulhawy and Mayne 1990).

To overcome the problem of sample disturbance in laboratory tests, the field plate-load tests are sometimes conducted for specific projects. The Transport and Road Research Laboratory (1952) suggested an equation to determine the modulus of subgrade reaction using the plate-load test data corresponding to a settlement of 0.05 inch.

Terzaghi (1955) proposed that  $k_s$  could be obtained for full-sized footings from plate-load tests and provided a set of equations applicable to different types of soil based on the footing shape and size. Singh (1967) suggested that the bearing plate at the plate-load test should be loaded up to 0.70 kg/cm<sup>2</sup> (10 lb/in<sup>2</sup>) within 10 seconds and the pressure is held until there is no increase of settlement (or the rate of increase in settlement becomes less than 0.05 mm/min). Then the average settlement ( $\delta$ ) of the plate is measured corresponding to (10 lb/in<sup>2</sup>) and k<sub>s</sub> is consequently calculated. The value of modulus of subgrade reaction is commonly based on the secant modulus at the maximum working stress (Henry, 1985). Lin et al., (1998) conducted series of plate-load tests to investigate the load-settlement characteristics of a gravelly cobble deposit and estimates the modulus of subgrade reaction.

In this study, a series of plate-load tests has been conducted in a large scale testing facility that has been designed and built to allow for the load-displacement relationships to be measured at different depths within the backfill material. The measured responses are then used to determine the modulus of subgrade reaction and Young's modulus for different granular backfill materials.

# 2 EXPERIMENTAL PROGRAM

The large scale setup used throughout this experimental study consisted mainly of the following components:

- Loading frame [4 m x 4 m x 3.5 m]
- Square rigid box to contain the tested material as shown in Figure (1).
- Pressure gauges.
- Dial gauges with pipes and rod extensions to measure settlement at different depths.
- Rigid circular plate.
- The applied load was located at the center of the tank at a distance of 1.73 m (in x and y directions) from the rigid boundaries. This minimizes the effect of the rigid boundary of the test tank on the recorded results.
- The backfill materials have been prepared as follow:
- Fine sand; 0.06 mm to 0.20 mm; [FS].
- Graded sand; 0.06 mm to 2.00 mm; [GS].
- Crushed stone + sand [ratio 1:1]; 0.06 mm to 40 mm; [CSS1].
- Crushed stone + sand [ratio 2:1]; 0.06 mm to 40 mm; [CSS2].
- Crushed stone; 2 mm to 40 mm; [CS].

The open box was filled with the backfill material and compacted in layers to reach the desired density. To allow for the settlement to be measured at the surface as well as at different vertical and horizontal locations [0.25B, 0.50B, 0.75B, B, 1.25B] as shown in Figure (2), a system that consists of steel plates embedded at the selected locations and connected to steel rods that extend to the ground surface has been used. The steel rods were placed inside steel pipes lined with Teflon layer to minimize friction. Four dial gauges were used to directly measure surface settlement.



Figure 1. Schematic of the testing facility



Figure 2. Setup of pipes and steel rods

In addition, a total of 20 dial gauges were placed such that the vertical movement is measured at distances 0.25B, 0.50B, 0.75B, B, and 1.25B from the edge of plate as shown in Figure (2).

The steel rods (10 mm in diameter with 22 mm head diameter) moves freely inside the steel pipe (32 mm in diameter) as shown in Figure (3).

In the present study a rigid circular plate manufactured and machined according to the ASTM standards with the following properties was used:

- · Plate diameter: 305 mm,
- Plate thickness: 32 mm, and
- Plate weight: 14.5 kg.

The load has been applied using a steel frame which was fixed to the ground. The load was measured using a pressure gauge connected to a hydraulic jack.



Figure 3. Details of steel pipes and steel rods

#### 3 TEST PROCEDURE

During the plate-loading tests, the following procedure was adopted:

- The granular soil was placed and compacted in layers of 7.5 cm each.
- The unit weight of each layer was determined using sand cone test.
- Standard Proctor tests have been done on different samples.
- Twenty steel pipes containing steel rods were placed at different depths as mentioned above.
- The steel plate (305 mm plate diameter) was placed on the prepared surface.
- The hydraulic jack was placed over the steel plate.
- Twenty dial gauges were placed on the heads of the steel rods using magnetic arms.
- Four dial gauges were placed on the plate surface.
- The initial readings were recorded for all dial gauges.
- The load was applied using steel frame which loaded by kentledge. Each load was maintained constant until the settlement rate reached 0.02 mm/min and maintained no less than one hour.
- Loads were applied in increments as presented in Table (1).

### 4 TEST RESULTS

The plate-load tests were carried out on the above mentioned soils. The sieve analysis and the standard proctor tests were also carried out for these types of soils, and the results are shown in Figures (4) and (5), respectively. The settlement was measured at the surface and at different depths [0.25B, 0.5B, 0.75B, B, 1.5B] along the center line of the plate.

Table 1. Load increments used in plate-load tests

Pressure (bar)	Load (kN)	Soil pressure (kN/m <sup>2</sup> )
2.50	4.160	58.90
5.00	8.330	117.8
7.50	12.50	176.8
10.0	16.66	235.6
12.5	20.83	294.5
15.0	24.99	353.4
20.0	33.33	471.2
22.5	37.49	530.1



Figure 4.Grain size distribution for the five tested samples



Figure 5. Proctor test results for the five tested samples

4.1 Determination of the Modulus of Subgrade Reaction

The modulus of subgrade reaction,  $k_s$ , was determined using two different methods based on the stress-settlement relationship. The first method was proposed by

Lin et al. (1998) based on the ultimate bearing capacity of the soil such that:

$$k_s = \frac{q_a}{\delta_a}$$

Where,

 $k_s$  = Modulus of subgrade reaction, kN/m<sup>3</sup>  $q_a$  = Allowable bearing capacity, kN/m<sup>2</sup>

 $\delta_a$  = Allowable settlement corresponding to (q = q<sub>a</sub>), m.

The allowable bearing capacity can be determine by dividing the ultimate bearing capacity by a factor of safety of three.

The ultimate bearing capacities of the tested soils were determined from the relationships between the applied stress and the measured settlement as shown in Figure (6). This method was implemented for the five tested soils.



Figure 6. Relationship between stress and settlement at the surface for fine sand backfill with relative density of 80.20%

The relationships between the relative density and the modulus of subgrade reaction for the five tested granular soils are shown in Figure (7). From this figure it can be seen that the modulus of subgrade reaction increases with increasing the relative density of the soil. Also, the modulus of subgrade reaction for the crushed stone is greater than that of the sand material.

The second method used to determine the modulus of subgrade reaction was based on the range of applied load for each stress level and finding the corresponding displacement. The obtained relationship between the relative density and the modulus of subgrade reaction for fine sand is shown in Figure (8).



Figure 7. Relationship between subgrade reaction modulus and relative density using Lin et al. (1998)



Figure 8. Relationship between subgrade reaction modulus and relative density

#### 4.2 Determination of Young's Modulus (E<sub>s</sub>)

In order to determine the Young's modulus at the surface as well as at different depths (0.25B, 0.50B, 0.75B, B, 1.50B), the British Standard (B.S. 5930, 1999) method was adopted assuming a uniformly loaded rigid plate on a semi-infinite elastic isotropic solid. The relationship is expressed by:

$$E_{s} = \frac{\pi q B (1 - v_{s}^{2})}{4\delta}$$

Where,

 $E_s$  = Young's modulus, kN/m<sup>2</sup> B = Plate width, m  $\delta$  = Settlement under applied pressure, m q = Applied pressure between plate and soil, kN/m<sup>2</sup> v<sub>s</sub> = Poisson's ratio

The average Young's modulus was determined at the surface as well as at different depths under different applied stresses that range between  $58.9 \text{ kN/m}^2$  and  $530.1 \text{ kN/m}^2$ . The relationships between the stress and Young's modulus for fine sand at the surface as well as at depth (B) below the surface are shown in Figures (9) and (10). This relationships figured out that the soil modulus increases with depth for granular material.



Figure 9. The changes in Young's modulus for different stress levels



Figure 10. The changes in Young's modulus for different stress levels

Young's Modulus generally increased with increasing the soil relative density as shown in Figure (11). However the Young's modulus of the crushed stone was found to be greeter than that of the sand material.



Figure 11. Relationship between Young's modulus and relative density

The variation of Young's modulus with depth is presented in Figure (12). The figure shows that Young's modulus increases with depth for granular soils and the percentage of the modulus at the surface is around 25% of that at a depth (B) below the surface.



Figure 12. Relationship between Young's modulus and depth for crushed stone + sand 1:1 (CSS1) under different relative densities

The available values for the modulus of subgrade reaction are generally for sand material. Little data are available for gravely soils. Thus, in the present study a comparison between the values of the modulus of subgrade reaction for sand which obtained by the two methods and the values reported in the literature (Terzaghi 1955, Indian Standards, 1979, Bowles, 1988 and Das, 1998) are compared in Figures (13) and (14).



Figure 13. Comparison between the obtained modulus of subgrade reaction for fine sand and the literature values



Figure 14. Comparison between the obtained modulus of subgrade reaction for graded sand and the literature values

These figures show an agreement between the obtained modulus of subgrade reaction for fine and graded sand and the available values in the literature for loose and medium dense sand (up to 65 % relative density). However, in the case of dense sand the obtained values

of the modulus of subgrade reaction are lower than the values reported in the literature.

Figures (15) through (17) show the variation of Young's modulus with depth up to a depth of 1.5B (B = diameter or width of foundation) for the five tested granular soils under different relative densities (Dr = 25%, 50%, 70%) respectively. From these relationships it can be concluded that soil modulus generally increase with depth at a rate that depends on the compaction degree. Also the modulus for course soils is greater than that for fine material.



Figure 15. Comparison of the variation of Young's modulus with depth for the five tested soils under relative density of 25%



Figure 16. Comparison of the variation of Young's modulus with depth for the five tested soils under relative density of 50%



Figure 17. Comparison of the variation of Young's modulus with depth for the five tested soils under relative density of 75%

Figure (18) shows that the obtained Young's modulus values for fine and graded sand are lower than the values reported in the literature for all examined densities.



Figure 18. Comparison between the obtained Young's modulus for sand and the literature values

## 5 CONCLUSIONS

From the present study the following conclusions are drawn:

- The modulus of subgrade reaction increases with increasing the relative density of the soil.
- For crushed stone the modulus of subgrade reaction is greater than that for the 2:1 mix of crushed stone and sand.

- The Young's modulus increases with increasing the relative density of the granular soil.
- The Young's modulus of crushed stone is greater than the Young's modulus for crushed stone-sand mix.
- The Young's modulus generally increases with depth for the all examined granular material.

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#### REFERENCES

- ASTM, 1997. Standard test method for nonrepetitive static plate load tests of soils and flexible pavement components, for use in evaluation and design of airport and highway pavements. ASTM D1196-93, Conshohocken, PA.
- Barden, L. 1963. The Winkler model and its application to soil. Structural Engineer, 41: 279 280.
- Biot, M. A. 1937. Bending of infinite beams on an elastic foundation. J. Appl. Mech. Trans. Am. Soc. Mech. Eng., 59:A1–A7.
- Black WPM 1961. The calculation of laboratory and insitu values of California bearing ratio from bearing capacity data. Geotechnique, 11(1):14–21.
- Bowles, J. E. 1988. Foundation analysis and design, 4<sup>th</sup> edition, Mc Graw-Hill Inc., New York.
- Bozozuk, M., and Leonards, G.A. 1972. The Gloucester test fill. In Proceedings of the ASCE Specialty Conference on Performance of Earth and Earth-Supported Structures, Purdue University, ASCE, 1(1): 299-317.
- British Standards Institute (1999). BS 5930:1999 Code of practice for site investigations. British Standards Institute (BSI), UK.
- Converse, F. J. 1962. Foundations subjected to dynamic forces, in Foundation engineering, edited by G. A. Leonards, McGraw-Hill, 769-825.
- Das, B. M. 1998. Principles of foundation engineering, 4<sup>th</sup> edition, PWS publishing company, Boston.
- D'Appolonia, D. J., and D'Appolonia, E. 1970. Use of SPT to estimate settlement of footings on sand. Proc., Symposium of Foundation Interbedded Sands, Division of Applied Geomechanics, Commonwealth Scientific and Industrial Research Organization, Australia and Western Australia of the Australian Geomechanics Society, Perth, 16–22.
- Department of the Environment Transport and Road Research Laboratory 1952. Soil mechanics for road engineers, HMSO, London. Road
- Florida standard method of test, 2000. Florida method of test for nonrepetitive static plate-load test of soils and flexible pavement components. AASHTO, T-222-78.

- Hanna, T. H., and Adams, J. I. 1968. Comparison of field and laboratory measurements of modulus of deformation of a clay. Highway Research Record, 243: 12-22.
- Henry, F. D. C. 1968. The design and construction of engineering foundations. Chapman and Hall, London.
- Hogg, A. H. A. 1938. Equilibrium of a thin plate, symmetrically loaded, resting on an elastic foundation of infinite depth. London, Edinburgh and Dublin, Philosophical Magazine and journal of Science, 25:168.
- Holtz, R. D. 1991. Stress distribution and settlement of shallow foundations. Chapter (5), Foundation engineering handbook, Edited by Fang, H. Y., Springer Berlin Heidelberg.
- Hussein, M. G. 2004. Evaluation of some elastic properties of cohesionless soils, M.Sc. thesis submitted to Al-Azhar University, Cairo, Egypt.
- Indian Standards code of practice IS: 9214-1979. Method of Determination of Modulus of Subgrade reaction (K-Value) of soils in field
- Kulhawy, F.H. and Mayne, P.W. 1990. Manual on estimating soil properties for foundation design. Report EL-6800, Electric Power Research Inst., Palo Alto, 306 p.
- Lin, P. S., Yang, L. W., and Juang, C. H. 1998. Subgrade reaction and load-settlement characteristics of gravelly cobble deposits by plate-load tests, Canadian Geotechnical Journal, 35: 801-810.
- Soderman, L. G., et al. 1968. Field and laboratory studies of modulus of elasticity of a clay till. Highway Research Board, HRR No. 243: 1-11.
- Terzaghi, K. 1955. Evaluation of coefficient of subgrade reaction, Geotechnique, 5(4): 297- 326.
- U.S. Army Corps Engineers Manual 1988. Site investigation, TM 5-809-1/AFM 88-3, Chapter 15(Publications).
- Ulrich, E. J. 1988. Suggested analysis and design procedures for combined footings and mats, ACI Structural Journal, 85(3): 304-324.
- Vesic, A. B. 1961. Beams on elastic subgrade and Winkler's hypothesis. Proc. 5<sup>th</sup> Int. Conf. on Soil Mechanics and Foundation Engineering, Paris:845-850.
- Vlasov, V. Z and Leontiev, N. N. 1966. Beams, plates and shells on elastic foundation. Israel Program of Scientific Translations, NTIS N67-14238.
- Murthy, V.N.S. 2002. Geotechnical engineering: principles and practices of soil mechanics and foundation engineering. Marcel Dekker Inc., New York.
- Nascimento, V. and Simoe A. 1957. Relation between C.B.R. and Modulus of Strength. Proc. 4th Int. Conf. on Soil Mechanics and Foundation Engineering, London: 166-168.
- Recordon E. 1957. Determination of soil characteristics necessary for foundation calculations on elastic soils. Proceedings of the Fourth International Conference on Soil Mechanics and Foundation Engineering, London, 1: 414–418.

- Schultze, E and K. J. Melzer 1965. The determination of the density and the modulus of compressibility of noncohesive soils by soundings. In Proceedings of the Sixth International Conference on Soil Mechanics and Foundation Engineering, 1, Montreal, 354.
- Schultze, E., and Sherif, G. 1973. Prediction of settlements from evaluated settlement observations for sand. Proc., 8th Int. Conf. Soil Mechanics and Foundation Engineering, 1(3): 225-230.
- Scott, R. F. 1981. Foundation analysis, Prentice-Hall, Inc.
- Singh, A. (1967). Soil engineering in theory and practice, by Asia publishing house.