# A general method for using recovery data for pumping tests in complex hydrogeological settings



Christopher J. Neville S.S. Papadopulos & Associates, Inc., Waterloo, Ontario Garth van der Kamp Environment Canada, National Hydrology Research Centre, Saskatoon, Saskatchewan

## ABSTRACT

Collection of water-level recovery data is a common practice for pumping tests. The resulting data can provide some of the most useful information from the tests, but are rarely used to their full value. van der Kamp (1989) proposed a general method for the interpretation of recovery data that is easy to use and applicable for simple or complex hydrogeology, depending only on the principle of superposition. No other assumptions about the properties and geometry of the formations are required. The method can greatly increase the value of pumping tests by extending their effective duration for as long as significant residual drawdowns can be measured.

## RÉSUMÉ

La collection de données de rétablissement de niveau d'eau est une pratique commune pour des essais de pompage. Les données en résultant peuvent fournir une grande partie des informations les plus utiles des essais, mais sont rarement employées à leur pleine valeur. van der Kamp (1989) a proposé une méthode générale pour l'interprétation des données de rétablissement qui est facile à utiliser et qui est applicable pour hydrogéologie simple ou complexe, dépendant seulement du principe de la superposition. Aucune autre assomption au sujet des propriétés et géométrie des formations sont exigées. La méthode peut considérablement augmenter la valeur des essais de pompages en prolongeant la durée efficace des essais, aussi long que des abaissements résiduels significatifs peuvent être mesurés.

#### 1 INTRODUCTION

Guidance documents for conducting pumping tests typically require that water levels be monitored for a specified time following the end of pumping. In our experience, frequently nothing is done with the recovery data after they have been collected, plotted, and included in the appendix to a report. In most cases, the cursory treatment of recovery data represents a genuine loss. Recovery data frequently provide some of the most reliable information from pumping tests.

The traditional approach to interpreting recovery data has involved the application of the Theis model of aquifer response with the Cooper-Jacob approximation of the Theis well function (Cooper and Jacob, 1946). It assumes an ideal, confined aquifer of infinite extent, which is rarely encountered in practice, even approximately. The Cooper-Jacob straight-line analysis has a particularly simple implementation for recovery analysis:

$$s = \frac{Q}{4\pi T} 2.303 \log_{10} \left\{ \frac{t}{t - t_{off}} \right\}$$
[1]

In Equation [1], *s* is the drawdown, *Q* is the pumping rate (assumed constant during pumping), *T* is the transmissivity, *t* is the elapsed time since the start of pumping, and  $t_{off}$  is the duration of pumping. Equation [1] can be used directly to estimate the transmissivity from the slope of the semi-log plot. Apart from the assumption of an ideal confined aquifer, this approach essentially breaks the pumping test up into two independently analyzed portions, the pumping period and the recovery period. These may or may not give comparable results for the transmissivity of the aquifer, depending on how well the assumption of an ideal aquifer is met, even though they apply to the same well-aquifer system.

van der Kamp (1989) introduced a different approach for working with recovery data. The approach is based only on the principle of superposition and does not require other assumptions about the hydraulic properties and geometry of the aquifer and adjacent formations. The approach provides a straightforward and useful extension of existing methods. It allows consideration of the pumping and recovery periods together, essentially extending the effective duration of the pumping test to as long as measurable drawdown persists. Our experience suggests that van der Kamp's approach has been largely overlooked. As far as we are aware, it has not been implemented in any of the widely used interpretation packages. This is an important oversight and this note has been prepared in part to renew interest in this approach.

## 2 DEVELOPMENT OF THE GENERAL THEORY

For a general linear conceptual model, the drawdown s(r,t) caused by pumping at a variable rate Q(t) can be written as:

$$s(r,t) = \int_{0}^{t} Q(\tau) G(r,t-\tau) d\tau \qquad [2]$$

Equation [2] is a general statement of the principle of superposition, and is referred to as a convolution integral. The term G(r,t) represents the drawdown at a distance r caused by pumping for an instant at time t = 0, and is frequently referred to as the Green's function for a particular problem. van der Kamp's method considers an arbitrary pumping history represented by a set of discrete steps, as shown in Figure 1.



Figure 1. Discrete representation of an arbitrary pumping history

The equivalent constant-rate drawdown,  $s_1$ , is defined as the drawdown that would be observed at time *t* if the pumping rate had remained constant at a rate  $Q_1$ . For an arbitrary step pumping history, it follows from Equation [2] that the equivalent constant-rate drawdown is given by:

$$\begin{bmatrix} (Q_2 - Q_1) \\ Q_1 \end{bmatrix} s_1(r, t - ts_2) + \dots + \frac{(Q_{NP} - Q_{NP-1})}{Q_1} s_1(r, t - ts_{NP}) \end{bmatrix}$$
[3]

In principle it is possible to reduce the drawdown data from any pumping test with varying pumping rates to the equivalent drawdown that would have been observed if the pumping rate had remained constant. This general principle depends only on the principle of superposition, and assumes only mathematical "linearity" of the equations that govern the flow. Linearity in turn means that the hydraulic properties of the formations do not change and that the boundary conditions remain constant (e.g., no dewatering of the formations).

#### 3 APPLICATION FOR PUMPING AT A CONSTANT RATE FOLLOWED BY RECOVERY

Although the general form of the van der Kamp (1989) algorithm appears to be relatively complicated, it is particularly simple for the analysis of recovery following pumping at a constant rate. This is by far the most common pumping test practice. For this case, during the recovery period NP = 2,  $t_{s2} = t_{off}$ , and  $Q_2 = 0$ , and van der Kamp's general form reduces to:

$$s_1(r,t) = s(r,t) + s_1(r,t-t_{off})$$
[4]

This result can be interpreted directly: if pumping had continued, the drawdown at any time *t* would be equal to the actual drawdown at time *t* plus the drawdown observed at time *t*-*t*<sub>off</sub>. Note that Equation [4] is not just limited to a recovery period that has the same duration as pumping. It can be applied for as long as the measured drawdown s(r,t) is significant compared to the possible errors of measurement and uncertainties in what the water level would have been in the absence of pumping.

To illustrate the method, an idealized case of a well that penetrates the full thickness of an ideal confined aquifer is considered. The following parameter values are assumed: transmissivity,  $T = 10^{-4} \text{ m}^2/\text{sec}$ ; storativity,  $S = 10^{-4}$ ; pumping rate,  $Q = 1.7 \times 10^{-3} \text{ m}^3/\text{sec}$ ; duration of pumping,  $t_{off} = 250$  seconds; and radial distance, r = 10 m.

The calculated drawdown history is plotted in Figure 2.



Figure 2. Calculated drawdown during pumping and recovery

To demonstrate the van der Kamp approach, the drawdown that would have been observed after 400 seconds if pumping had continued at a constant rate is calculated. The equivalent constant-rate drawdown at t = 400 seconds is given by:

$$s_1(t = 400 \text{ s}) = s(t = 400 \text{ s}) + s_1(t - t_{off} = 150 \text{ s})$$
 [5]

At t = 400 seconds, the observed drawdown plotted in Figure 2 is 1.33 m. At t = 150 seconds, the well is still pumping; therefore  $s_1 = s$  and the drawdown estimated from Figure 2 is  $s_1(t = 150 \text{ s}) = 15.98 \text{ m}$ . The equivalent drawdown is therefore:

$$s_1 (t = 400 \text{ s}) = 1.33 \text{ m} + 15.98 \text{ m} = 17.31 \text{ m}$$
 [6]

The calculation is illustrated in Figure 3.



Figure 3. Calculation of equivalent constant-rate drawdown at 400 seconds

The results of applying the van der Kamp method for all of the results of the example are plotted in Figure 4.



Figure 4. Actual drawdown and equivalent constant-rate drawdown

A simple but widely applicable illustration of the potential utility of the method can be drawn from the above example. Consider a pumping test with recovery data taken for the same time after pumping as the duration of pumping. The standard 24-hour test with 24 hours of recovery is a case in point. The residual drawdown after 24 hours of recovery is equal to the additional drawdown that would have occurred between 24 and 48 hours if pumping had continued. The one data point obtained after 24 hours of recovery already doubles the effective length of the pumping test, especially if further analysis is based on methods making use of semi-log or log-log plots of drawdown versus time. Numerous "24-24" pumping test analyses could make good use of this simplest of calculations. Other data points can also be calculated, as illustrated in Figures 3 and 4.

An additional advantage of making full use of the recovery data is that "noise" introduced into the drawdown data by irregularities of the pumping rate is much reduced during the recovery phase.

#### 4 CASE STUDY

The utility of the van der Kamp approach is demonstrated by using the recovery data to extend the effective duration of a pumping test conducted in a confined buried-channel aquifer near Estevan, Saskatchewan. The test was conducted in 1984 and was reported in van der Kamp (1985; 1989). The aquifer is described in Walton (1970), van der Kamp and Maathuis (2002), and Maathuis and van der Kamp (2003). This is a complex semiconfined channel aquifer, involving complicating factors such as several intersecting channels, partial blockages, lateral inflow from surrounding formations and unknown regional permeability of the overlying glacial till aquitard. No simple analytical aquifer model could be expected to apply and the numerical model that was developed was highly unconstrained.

The aquifer was pumped at a constant rate for 41,520 minutes (about 29 days), and water levels following the end of pumping were monitored for an additional 249,000 minutes (173 days). Drawdowns at observation well 11L-84 during the pumping and recovery periods are shown in Figure 5 (data from Figure 3 of van der Kamp, 1989). For subsequent analysis, the original observations are supplemented with interpolated values indicated by the crosses. The interpolated drawdown observations, taken from van der Kamp (1989; Table 2), are smoothed slightly with respect to the original observations.



Figure 5. Raw drawdown data

Complete results obtained from applying van der Kamp's method are shown in Figure 6.



Figure 6. Equivalent constant-rate drawdowns

In this example, the use of recovery data lengthens the useful duration of the pumping test from one month to more than six months. The implications of this extension are best illustrated by plotting the raw drawdowns and the equivalent constant-rate drawdowns against the logarithm of time, as shown in Figure 7.



Figure 7. Semi-log plot raw and equivalent drawdown data

As shown in Figure 7, even after 29 days of pumping it is only possible to identify the beginning of the long-term trend of the drawdown. In contrast, the accelerating trend that is characteristic of a buried-channel aquifer is clearly evident in the equivalent constant-rate drawdowns. The drawdown at the end of pumping is 4.70 m. The equivalent constant-rate drawdown for the last recorded water level is 12.05 m.

The application of the van der Kamp analysis in this example is possible because significant drawdowns persisted more than six months beyond the end of pumping. The persistent drawdown reflects the conditions of the aquifer: the buried-channel aquifer is overlain by a thick aquitard of low conductivity, which allows only minimal recharge to the aquifer.

Subsequently the aquifer was pumped at a high rate for 6 years to supply cooling water for a coal-fired power plant (Maathuis and van der Kamp, 2003). The long-term drawdown due to such pumping was predicted on the basis of the 6 months of extrapolated drawdown illustrated in Figure 7, and the actual measured drawdown agreed closely with the prediction.

## 6 DISCUSSION

Robust and inexpensive pressure transducers have become widely available in recent years. These can be left securely in observation wells without requiring the continuous on-site presence of field staff. The collection of extended recovery data has therefore become easier and more economical. It may become standard practice to record water level data after the cessation of pumping for as long as it takes to attain full recovery. Such long-term monitoring of recovery has the additional advantage that it may allow a more robust estimate of changes of the "static" water level during the pumping and recovery period.

The authors' experience with pumping tests suggests that the general method for the analysis of recovery data described in van der Kamp (1989) could have enhanced the value of almost every pumping test that they have encountered, with only minor additional effort in data analysis. Full recognition and exploitation of the potential value of recovery data is therefore recommended to all practitioners.

## 7 REFERENCES

- Cooper, H.H., Jr., and Jacob, C.E., 1946: A generalized graphical method for evaluating formation constants and summarizing well-field history, *Trans. American Geophysical Union*, vol. 27, no. 4: 526-534.
- Maathuis, H., and van der Kamp, G., 2003: Groundwater resource evaluations of the Estevan Valley aquifer in southeastern Saskatchewan: A 40-year historical perspective, in *Proceedings of the 4<sup>th</sup> Joint IAH-CNC and CGS Conference*, Winnipeg, Manitoba: 4 p.
- Theis, C.V., 1935: The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage, *Trans. American Geophysical Union*, 16<sup>th</sup> Annual Meeting, Part 2: 519-524.
- van der Kamp, G., 1985: Yield estimates for the Estevan Valley aquifer system using a finite element model, Saskatchewan Research Council, Publication No. R-844-4-C-85.
- van der Kamp, G., 1989: Calculation of constant-rate drawdowns from stepped-rate pumping tests, *Ground Water*, vol. 27, no. 2: 175-183.
- van der Kamp, G., and H. Maathuis, 2002: The peculiar groundwater hydraulics of buried-channel aquifers, in *Ground and Water: Theory to Practice*, Proceedings of the 3<sup>rd</sup> Joint IAH-CNC and CGS Conference, Niagara Falls, Ontario: 695-698.
- Walton, W.C., 1970: Groundwater Resource Evaluation, McGraw-Hill Book Company, New York, New York.