Modelling water flow and transport of contaminants from mine wastes stored in open pits within fractured rock



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ABSTRACT

The flow of water and transport of contaminants from mining wastes disposed of in fractured rock is largely controlled by the presence of discontinuities. Numerical simulations have been carried out to assess the influence of joints on underground flow in and around surface mine openings. The unsaturated flow modelling was performed with HydroGeosphere, a 3D numerical flow and transport model (Therrien et al. 2005). A parametric analysis was conducted by simulating various 2D cases using experimentally obtained material properties and controlled boundary conditions. The effects of material hydraulic properties (i.e. the water retention curve and hydraulic conductivity function), fracture network characteristics, variable recharge rates and saturated hydraulic conductivity of the joints are investigated. The parametric analysis illustrates how water and contaminants may migrate around waste disposal areas created in fractured rock at mine sites.

RÉSUMÉ

L'écoulement de l'eau et le transport de contaminants dans les rejets miniers entreposés dans les massifs rocheux fracturés sont fortement influencés par la présence de discontinuités. Des simulations numériques ont été réalisées pour étudier l'influence des joints sur l'écoulement souterrain autour d'une mine à ciel ouvert. La simulation de l'écoulement non saturé a été effectuée à l'aide du modèle numérique 3D HydroGeosphere (Therrien et al. 2005). L'analyse paramétrique est réalisée en deux dimensions pour différents cas, en utilisant des propriétés des matériaux obtenues expérimentalement et en imposant différentes conditions aux limites. Les effets des propriétés hydriques des matériaux (i.e. la courbe de rétention d'eau et la fonction de conductivité hydraulique), des caractéristiques du réseau de fractures, des taux de recharge en surface et de la conductivité hydraulique saturée des joints ont été analysés. Cette étude paramétrique permet de mieux comprendre l'écoulement de l'eau et le transport de contaminants autour des sites d'enfouissement des rejets miniers crées dans des massifs fracturés.

1 INTRODUCTION

The mining industry contributes significantly to the economy of many countries throughout the world. Mining operations also generate significant volumes of solid wastes that must be managed properly to limit their environmental impact. Over the last few years, the mining industry and various governmental and research organisations have spent significant effort to develop tools and geo-environmental methods to help improve solid and liquid waste management at mine sites, and to develop more effective rehabilitation strategies.

Quantitative studies of groundwater flow and solute transport in fractured media are now commonly conducted. These studies have been motivated by a variety of practical issues including locating and evaluating water supplies and geothermal and petroleum energy resources.

In addition, the search for safe storage of hazardous wastes, where the primary concern is the release of

contaminants to the ecosphere through interconnected conductive discontinuities, has led to studies of lowpermeability rock formations (e.g. Xu and Hu 2005). Mining wastes are potential sources of contaminants that are sometimes disposed of in fractured rock masses. If the fracture network is sufficiently interconnected, water flow and contaminant transport can be significant. Discontinuities can form potential pathways for rapid transport of contaminants to underlying aquifers and any assessment of potential contaminant migration through mining wastes must therefore account for discontinuities. Describing the movement of solutes or chemicals in such domains is still a major challenge.

Numerical codes are very useful and powerful tools to solve environmental problems. They can help us to predict water flow network and contaminant migration.

This paper presents the results of numerical simulations conducted with the 3D code HydroGeosphere (Therrien et al. 2005) for water flow and contaminant transport from mining wastes disposed of in open pits in fractured rock. This model is based on the existing

Frac3DVS and HydroSphere models, which solves variably-saturated and multi-component transport in discretely fractured porous media. First, a brief litterature review is presented. Then, results for two cases with different types of waste material are presented and discussed. The effect of an orthogonal fracture network is also investigated. Finally, a scenario with a variable surface recharge is presented. The results presented here are complementary to those presented by Ben Abdelghani et al. (2007); more details on the calculations are presented in Ben Abdelghani (2009).

2 WATER FLOW AND CONTAMINANT TRANSPORT IN FRACTURED ROCK

2.1 Water Flow in a Single Fracture

Water flow in fractures is described by the "cubic law" which is an analytical solution of the Navier-Stokes equation for laminar and steady state water flow between two surfaces. It can be written as (Witherspoon and al. 1981; Tsang 1984):

$$Q_{f} = V_{f} A_{sec} = -[\rho_{w}g b^{3} w \Delta h / 12\mu_{w}L]$$
[1]

where
$$A_{sec} = b w$$
 [2]

and with the following parameter definitions:

 $\begin{array}{l} Q_{f}: \mbox{fracture discharge } (m^{3}/s) \\ V_{f}: \mbox{mean water flow velocity in fracture } (m/s) \\ A_{sec}: \mbox{area of fracture perpendicular to water flow } (m^{2}) \\ b: \mbox{fracture aperture } (m) \\ w: \mbox{fracture width transverse to water flow } (m) \\ L: \mbox{fracture length parallel to water flow } (m) \\ \Delta h: \mbox{hydraulic head difference along the flow direction } (m) \\ \rho_{w}: \mbox{water density } (kg/m^{3}) \\ g: \mbox{gravity acceleration } (m^{2}/s) \\ \mu_{w}: \mbox{water dynamic viscosity } (kg/m/s) \\ \end{array}$

Equation (1) was validated for raminar flow in microfractures composed of planar surfaces. This equation can be modified with additional parameters to take into account influence factors such as surface rugosity, tortuosity, and the Reynolds number (e.g. Gale 1990; Indraratna and Ranjith 2001).

For transient and partially saturated water flow conditions, Eq.1 can be used with the continuity equation of flow discharge to determine the equation of partially saturated water flow in fractures (Wang et Narasimhan 1993). Under these conditions, it is important to correctly define the unsaturated hydraulic functions of the fracture and of all materials modelled in the selected numerical code.

2.2 Contaminant Transport

Contaminant transport in fractured rock is an important but difficult aspect to consider due to the complexity of fracture networks and the role of fractures affecting contaminant migration.

For most reactive and non-reactive contaminants, the principal modes of transport are advection and hydrodynamic dispersion which includes molecular diffusion and mechanical dispersion. Molecular diffusion is often negligible when the porous medium is impervious. Advection is the contaminant migration by water flow in response to a hydraulic gradient. Mechanical dispersion is the process of contaminant migration due to a concentration gradient and tortuosity of the medium.

In order to describe contaminant transport in a discretely-fractured porous medium, two equations are needed, one for the porous matrix and one for the fractures. Three-dimensional transport in a variably-saturated porous matrix is described by the following equation (Bear 1972 cited in Therrien and Sudicky 1996):

$$\theta_{s}S_{w}\frac{\partial c}{\partial t} + q_{i}\frac{\partial c}{\partial x_{i}} - \frac{\partial}{\partial x_{i}}(\theta_{s}S_{w}D_{ij}\frac{\partial c}{\partial x_{j}}) + \theta_{s}S_{w}c = 0$$
^[3]

i,j = 1,2,3

where C is the concentration, D_{ij} is the hydrodynamic dispersion coefficient, q_i is the fluid flux, θ_s is the porosity and S_w is water saturation. The hydrodynamic dispersion coefficient D_{ij} is given by (Bear 1972):

$$\theta_{s}S_{w}D_{j} = (\alpha_{l} - \alpha_{l})\frac{q_{l}q_{j}}{|q|} + \alpha_{l}|q|\delta_{j} + \theta_{s}S_{w}\mathcal{D}_{0}\delta_{j} = 0$$

$$[4]$$

where α_{l} and α_{t} are the longitudinal and transverse dispersivities, respectively; $|\mathbf{q}|$ is the magnitude of the Darcy flux; τ is the matrix tortuosity; D_{0} is the free solution diffusion coefficient; and δ ij is the Kronecker delta. The product τD_{0} represents an effective diffusion coefficient for the matrix (D_{e}). Typical values for D_{e} under saturated conditions in soils range between 1x10⁻⁹ and 2x10⁻⁹ m²/s (Sharma and Reddy 2004). The tortuosity coefficient usually varies between 0.01 and 0.5 (Freeze and Cherry 1979).

Other equations similar to [3] and [4] can be written to describe contaminant transport in a variably-saturated fracture.

2.3 The HydroGeosphere Code

HydroGeosphere is a 3D numerical model describing fully integrated subsurface and surface flow and solute transport (Therrien and al. 2005). The model originates from FRAC3DVS with the governing equations for flow and transport derived from a continuum approach.

A unique feature in HydroGeosphere is that when the flow of water is simulated in a fully-integrated mode, water derived from rainfall inputs is allowed to partition into components such as overland and stream flow, evaporation, infiltration, recharge, and subsurface discharge into surface water features such as lakes and streams in a natural, physically-based fashion. That is, the fully-coupled numerical solution approach allows the simultaneous solution of both the surface and variablysaturated flow regimes at each time step. This approach also permits dissolved solutes to be naturally exchanged between the surface and subsurface flow domains such that solute concentrations are also solved simultaneously at each time step in both regimes. This makes HydroGeosphere a unique tool to simulate the movement of water and solutes with watersheds in a realistic, physically-based manner (Therrien and al. 2005).

The finite element method is used to spatially discretize the flow and transport equations. The solution takes into account advective flow and transport and molecular diffusion (with dispersion) in both the fractures and the porous matrix.

3 SIMULATION OF A CONCEPTUAL OPEN PIT MODEL

Figure 1 presents the conceptual 2D model of an open pit filled with mining wastes simulated with the HydroGeosphere code.

The open pit is symmetric about the axis at x = 0 m. It has a depth of 100 m and the wall slope is 68 ° (from the horizontal axis). The lower limit of the model is 200 m below the pit base. The left and right limits of the model are located respectively at – 400 m and + 400 m from the origin. Two types of mine wastes are considered: waste rock resulting from mining operations and tailings from the milling process.



Figure 1. Open pit conceptual model.

Material Hydraulic Characteristics

The water retention curves representing the variation of the degree of saturation versus suction for the mill tailings, waste rock, and intact rock are shown in Figure 2 (semi-log graph).

Table 1 lists the different hydraulic parameters of these materials. These functions are defined by using experimental data from Aubertin et al. (2005) for the mine waste rock, from Cifuentes (2005) for the tailings and from Wang and Narasimhan (1985) for the rock matrix.



Figure 2. Material water retention curves.

Table 1 Material hydraulic parameters

Parameters	Waste Rock	Mill Tailings	Intact Rock
Porosity	0.34	0.43	0.02
Air entry value (m)	0.3	3.5	35
Saturated hydraulic conductivity Ksat(m/s)	1 x 10⁻⁵	1 x 10 ⁻⁸	3.2 x10 ⁻⁸
Residual volumetric water content	0.03	0.1	0.0015

Figure 2 shows that as suction increases, the rock mass remains at a higher degree of saturation than the tailings and waste rock, due to its low porosity and higher air entry value (AEV).

4 RESULTS AND DISCUSSION

4.1 Open Pit Filled with Waste Rock

For all simulations, the initial water table is fixed at an elevation of 220 m (20 m above the pit bottom). To generate a regional gradient, a decreasing hydraulic head between 220 m and 210 m is imposed at the base of the model for x between - 400 m and + 400 m (which gives a regional gradient of 0.0125). The left and right boundaries are pervious with fixed hydraulic heads of 220 m and 210 m respectively. The base of the model is assumed impervious. A constant recharge rate of 1.5 mm/d is fixed at the surface for 10 days followed by a period of 10 days without rain in alternation for a period of 20 years. To simulate contaminant migration, a constant unit concentration is fixed within the open pit, and is initially set to zero within the surrounding host rock. The contaminant is assumed inorganic and non-reactive with a free diffusion coefficient of 2 x 10⁻⁹ m²/s. All transport model parameters are summarized in Table 2. These parameters are introduced in the HydroGeosphere code files. The results for two cases are presented here: a homogeneous rock mass, and a rock mass with an orthogonal fracture network. All simulations are made under unsaturated and transient flow conditions.

The Gridbuilder V.5.6 code (McLaren 2005) is used to generate the mesh network and the Tecplot code (Amtec, Inc.) is used for data extraction and visualization. This simulation has generated 8128 nodes and 7906 elements.

Table 2 Model parameters used in transport simulation

Parameter	Value
Tortuosity	0.1
Matrix longitudinal dispersivity	0.1 m
Matrix transverse dispersivity	0.01 m
Fracture longitudinal dispersivity	0.5 m
Fracture transverse dispersivity	0.05 m

Case 1: Homogeneous rock

Figures 3 and 4 show simulated profiles of suction and degree of saturation as a function of time and distance within a horizontal section at y = 280 m.

Figure 3 shows that the initial suction distribution is linear and varies between - 60 m and - 70 m for x varying between - 400 m and + 400 m. With time and due to precipitation, the suction decreases greatly, especially within the rock mass. The variation of suction remains linear in the open pit. Due to the regional gradient, values are slightly higher at the left model boundary compared to the right boundary.



Figure 3. Simulated water suction profiles at y = 280 m; open pit filled with waste rock; homogeneous rock mass.



Figure 4. Simulated degree of saturation profiles at y = 280 m; open pit filled with waste rock; homogeneous rock mass.

As shown in Figure 4, the low porosity rock mass remains at a high degree of saturation with S_r values between 90 and 100 %, whereas wastes remain at a lower degree of saturation (near their residual water content). We can also notice an increasing of degree of saturation in the waste with time, with values varying from 28 % to 38 %. This is due to a progressive filling of the open pit.

Figure 5 shows results of contaminant concentration evolution with time. This figure shows that concentration remains constant and equals unity inside open pit (which is the imposed condition). Contaminant migration is slowest at early simulation times. As time increases and due to the effect of precipitation, migration becomes more significant especially with depth. Due to the low value of the regional gradient, the contaminant outlet is quite symmetric.



Figure 5. Simulated contaminant concentration levels; open pit filled with waste rock; homogeneous rock mass.

Case 2: Effect of discontinuities

Here, an orthogonal fracture network is introduced into the rock mass (Figure 6). All fractures have an aperture of 0.3 mm. Figures 7 and 8 show simulated profiles of suction and degree of saturation as a function of time and distance. It can be seen from Figures 7 and 8 that fractures have a significant effect on the water flow network. Figure 7 shows that variations of suction are less important here in comparison with the homogeneous rock mass. Also, as shown by Figure 8, the variation of degree of saturation is very low so S_r is almost constant. We can therefore say that introducing fractures in the model causes a desaturation effect in the pit. This is due to significant water flow through the fracture network thus water does not have enough time to accumulate.



Figure 6. Open pit with orthogonal fracture network (not to scale).



Figure 7. Simulated water suction profiles at y = 280 m; open pit filled with waste rock; fractured rock mass.



Figure 8. Simulated degree of saturation profiles at y = 280 m; open pit filled with waste rock; fractured rock mass.

For contaminant migration, the resulting concentration evolution with time is shown in Figure 9. This figure shows a significant difference in contaminant migration in comparison with case 1 (homogeneous rock mass). Migration is more important here, and contaminant transport occurs primarily through the fracture network. In contrast to case 1, contaminants migrate more rapidly along the regional gradient direction. So, if a surface water source exists near the open pit, the risk of contamination at this source will increase significantly with the existence of a such fracture network in the rock mass.



Figure 9 Simulated contaminant concentration levels; open pit filled with waste rock; homogeneous rock mass.

Case 3: Effect of variable surface recharge rate

Here, the recharge rate imposed on the surface varies with time. Data from the Latulipe meteorological station located in northern Quebec was used to establish the recharge rate, as shown in Figure 10 (taken from Cifuentes 2005).

As can be seen in Figure 13, the first period of precipitation starts at day 120, a maximum is reached at day 273 and the last period of precipitation is at day 303. The surface is considered frozen the rest of the year so there is no infiltration. For our simulations, we consider a repetitive cyclic period of 1 day of precipitation followed by 2 days without precipitation for each month, for a period of 2 years.



Figure 10. Precipitation distribution, Latulipe station, variable recharge case.

For this simulation, the initial water table is deeper being located at an elevation of 50 m. Hydraulic heads of 50 m and 40 m are imposed respectively at the left and right boundaries. The regional hydraulic gradient is therefore identical to the one imposed in cases 1 and 2. The rock mass is homogeneous and no fractures are present.

Figures 11 and 12 show the simulated profiles of suction and degree of saturation as a function of time and distance.



Figure 11. Simulated water suction profiles at y = 280 m; open pit filled with waste rock; variable recharge case.



Figure 12. Simulated profiles of the degree of saturation profiles at y = 280 m; open pit filled with waste rock; variable recharge case.

Figure 11 shows that the suction variation is more significant here with the variable recharge rate (especially within the open pit) in comparison with case 1 (Figure 3). This is due to the short period without rain (2 days) relative to the period with rain. As shown by the results of Figure 12, the variation of the degree of saturation is more pronounced, especially in the rock mass.

4.2 Open Pit Filled with Tailings

In this case, the open pit is filled with mill tailings which have a saturated hydraulic conductivity of the same order of magnitude as the intact rock matrix, but lower than the waste rock. The same initial and boundary conditions used in case 1 of the open pit filled with waste rock are used here. The rock mass is homogeneous (no fractures are present here). The simulated results of suction and degree of saturation as a function of time and distance along a horizontal section at y = 280 m are shown in Figures 13 and 14.



Figure 13. Simulated water suction profiles at y = 280 m; open pit filled with tailings.



Figure 14. Simulated profiles of the degree of saturation profiles at y = 280 m; open pit filled with tailings.

From figures 13 and 14, we can notice that fluctuations are more significant here than for case 1 with the open pit filled with waste rock. Figure 13 shows a large decrease in suction within the open pit and rock mass. In contrast to case 1, the suction decrease is more significant in the tailings relative to the rock mass.

Figure 14 shows a marked increase in degree of saturation in tailings with time, with saturations near 58 % after 20 years. This value is greater than for the case with waste rock. The degree of saturation in the rock mass fluctuates between 93 % and 100%.

Contaminant concentration evolution with time is shown in Figure 15. With time, contaminant migration becomes more important and contaminant migrates from the open pit to the rock mass. In comparison with the waste rock, lateral migration is more important here. The contaminant plume shape is strongly oriented in the direction of regional gradient.

Introducing an orthogonal fracture network with a 0.3 mm aperture for all fractures (see Figure 6 for fracture network), we obtain concentration results as shown by Figure 16.



Figure 15 Simulated contaminant concentration levels; open pit filled with tailings; homogeneous rock mass.



Figure 16 Simulated contaminant concentration levels; open pit filled with tailings; fractured rock mass.

As was the case for the open pit with waste rock, fractures have a great influence on contaminant transport. In fact, we can see from Figure 16 that the contaminant migrates rapidly through the fracture network thus reaching greater distances than when the rock mass is homogeneous.

5 CONCLUSIONS

This study highlights several of the most important factors which affect water flow and contaminant transport from mine wastes stored in open pits in fractured rock. The 2D simulations with the 3D HydroGeosphere code have shown that: Water flow and contaminant transport are largely affected by the type of waste material in the open pit, the type and nature of initial and boundary conditions, and the nature of the rock mass (homogeneous or fractured).

For a homogeneous rock mass, water accumulates in the open pit due to water infiltration. This induces a suction decrease over time and an increasing degree of saturation within the open pit and rock mass. Suction and degree of saturation variations depend on the type of open pit filling material.

When a fracture network is present in the rock mass, water will flow preferentially through the fractures and will not have enough time to accumulate within the open pit when the water table is deep. This leads to a partial desaturation of the system and especially in the rock mass. Variations in the suction and degree of saturation over time become less important.

Fractures also have a great effect on contaminant transport. Contaminants migrate through the fracture network and can reach greater distances than through an unfractured porous medium. The contaminant plume is more affected by the regional gradient when fractures are present. Thus, the degree of fracturing is an important aspect to consider in any evaluation of contaminant transport in fractured rock.

A regional gradient can have a great effect on the water flow network and on contaminant transport. Contaminant concentrations are greater in the direction of the regional gradient. This difference in concentrations becomes more importance with time.

Imposing a variable rate of precipitation at the surface induces a different type of variation in the suction and degree of saturation.

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