Estimation of specific surface areas of coarse-grained materials with grain-size curves represented by two-parameter lognormal distributions

M. Mbonimpa[A]
Department of Applied Sciences, Université du Québec en Abitibi-Témiscamingue (UQAT), Rouyn-Noranda, Québec, Canada
M. Aubertin[1,A], R.P. Chapuis[1,A], A. Maqsoud[2,A], B. Bussière[2,A]
[1]*Department of Civil, Geological and Mining Engineering, École Polytechnique de Montréal, Montreal, Québec, Canada.*
[2]*Department of Applied Sciences, Université du Québec en Abitibi-Témiscamingue (UQAT), Rouyn-Noranda, Québec, Canada*
[A]*Industrial NSERC Polytechnique-UQAT Chair, Environment and Mine Wastes Management.*

ABSTRACT
The specific surface area (SSA) is an important characteristic of soils and other particulate media. The value of SSA can, for instance, be used to assess the hydraulic conductivity and moisture retention curve. Various methods have been proposed to evaluate the SSA of granular materials from the grain size curves. In this paper, a new approach is presented to estimate SSA for S-shaped grain-size distributions that can be represented by two-parameter lognormal distributions, using the equivalent mean diameter D_H. Calculated values from this method are compared with those obtained from existing analytical equations that rely on grain size curve parameters. The introduction of the proposed relationship in the modified Kovács (MK) model developed to predict the water retention curve is also discussed in a preliminary manner.

RÉSUMÉ
La surface spécifique (SS) est une caractéristique importante des sols et autres matériaux particulaires. La valeur de SS peut par exemple être utilisée pour estimer la conductivité hydraulique et la courbe de rétention d'eau. Diverses méthodes ont été proposées pour évaluaer la SS des matériaux granulaires à partir des courbes granulométriques. Dans cet article, une nouvelle approche est présentée pour estimer la SS de matériaux dont les courbes granulométriques en forme de S peuvent être représentées par des distributions log-normales à deux paramètres, en utilisant le concept du diamètre moyen équivalent D_H . Les valeurs calculées ici sont comparées à celles obtenues avec d'autres équations analytiques existantes basées sur les paramètres de la courbe granulométrique. L'introduction de la relation proposée dans le modèle de Kovács modifié (MK) développé pour prédire la courbe de rétention d'eau est analysée de façon préliminaire.

1 INTRODUCTION

Many physical, mechanical, and chemical properties of soils and other particulate materials are related to surface phenomena that occur at the interface between a fluid (liquid or gas) and the solid grains. Some of these properties have been correlated to the specific surface area (SSA) of the solid phase, which is assumed to correspond to the interstitial surface area of the voids in the porous media. In hydrogeology and geotechnique for instance, the SSA is sometimes used to predict the hydraulic conductivity and moisture retention curve of soils and similar materials such as tailings (e.g. Chapuis and Montour, 1992; Aubertin et al. 1996, 1998; Mbonimpa et al. 2002; Chapuis et Aubertin, 2003).

The value of the specific surface area can be related to parameters A_s, M_s, V_s, and V_t which represent the total surface area of particles, and their mass, volume of solids and total volume, respectively. Three distinct specific surface areas can be defined: a solid mass-based value $S_m=A_s/M_s$ [L^2/M], a solid particle volume-based value $S_s=A_s/V_s$ [L^2/L^3], and a total volume-based value $S_v=A_s/V_t$ [L^2/L^3]. These three SSA expressions are interrelated in the following manner:

$$S_m = \frac{S_v}{\rho_s} = \frac{S_s}{(1-n)\cdot\rho_s} \qquad [1]$$

where ρ_s [M/L^3] is the density of solid grain and n is total porosity of the medium. In this paper, the SSA value will be defined from the mass-based (S_m) expression.

The value of S_m can be directly measured, using various methods having different ranges of applicability (e.g., Lowell and Shields 1984; Igwe 1991; Arnepalli et al. 2008). Methods based on physisorption isotherms, such as the well known BET method, are particularly useful but these results must be interpreted with great care. In fact, none of the existing methods provide absolute values of the SSA. As these measurement techniques require the use of fairly expensive equipment with time-consuming procedures, it is often useful to evaluate the value of SSA (S_m) indirectly, from basic material parameters. For fine-grained plastic soils, correlations between S_m and Atterberg limits are believed to be appropriate, as they appear to give more reliable estimates than those based on grain-size distribution (GSD) or clay fraction (e.g., Locat et al. 1984; Mbonimpa et al. 2002; Chapuis and Aubertin 2003; Aubertin et al. 2005; Dolinar et al. 2007). For coarse-grained materials, many options exist to estimate S_m. For instance, the GSD can be used with a particle shape parameter, leading the following equation (Kovács 1981):

$$S_m = \frac{\alpha}{\rho_s D_H} \quad [2]$$

where α [-] is a shape factor ($6 \le \alpha \le 18; \alpha = 6$ for spherical particles) and D_H [L] is an equivalent mean particle diameter. In the following, the influence of particle shape is not explicitly considered (see Discussion below). The value of D_H is defined as the diameter of a spherical particle for an homogeneous mix (single size) with the same specific surface as that of the full grain size distribution.

S-shaped grain size curves of soils may be described with various types of functions. A lognormal distribution seems more suitable than other functions, such as the normal distribution (Kézdi 1964; Wagner and Ding 1994). This holds true also for grinding materials, such as mine tailings (Bethea et al. 1995). Not surprisingly, several studies have relied on the use of a lognormal distribution to describe the GSD and pore-size distribution of granular soils (Kosugi 1994, Shirazi and Boersma 1984; Buchan et al. 1993, Chan and Govindaraju 2004). It should be recalled however that this distribution is only appropriate for S-shaped GSD, and that it is unsuitable for multimodal or gap-graded GSD (e.g., Fredlund et al. 2000).

This paper presents a theoretical approach to estimate the SSA of coarse-grained materials having a grain size curve represented by a two-parameter lognormal distribution (2PLND). The proposed relationship is based on the use of an equivalent mean diameter D_H. A relationship is developed to express D_H from commonly used parameters, i.e., the effective diameter D_{10} and the coefficient of uniformity C_U, using the ratio $\beta = D_H / D_{10}$. The proposed equation for β is compared with existing analytical equations. A preliminary evaluation is made to assess the impact of using the proposed β relationship in the modified Kovács (MK) model that has been developed to predict the water retention curve of soils with a GSD described with a 2PLND.

2 EXISTING METHODS TO ESTIMATE D_H FROM GRAIN-SIZE DISTRIBUTION

For coarse-grained (granular) materials, the value of D_H can be estimated from the grain-size curve by segmenting it into different sizes with average diameters D_i and mass percentages $p_{m,i}$ (%), and applying a relationship of the following type (Chapuis and Légaré, 1992):

$$D_H = 100\left(\sum_i \frac{p_{m,i}}{D_i}\right)^{-1} \quad [3]$$

Various options exist for segmenting the curve, and there is no consensus on whether the average diameter D_i for segment i represents the arithmetic ($D_{i\text{-}a}$), geometric ($D_{i\text{-}g}$) or harmonic ($D_{i\text{-}h}$) mean diameter of the corresponding grain size. These mean diameters are defined below.

$$D_{i-a} = \frac{D_{i<} + D_{i>}}{2} \quad [4]$$

$$D_{i-g} = \sqrt{D_{i<} \times D_{i>}} \quad [5]$$

$$D_{i-h} = \frac{2}{\frac{1}{D_{i<}} + \frac{1}{D_{i>}}} \quad [6]$$

In all cases, it can be shown that $D_{i\text{-}h} < D_{i\text{-}g} < D_{i\text{-}a}$.

Several authors (Hazen (in Beyer 1964); Huissman and Wood 1974; Moll 1980; Kovács 1981; Aubertin et al. 1998) have attempted to correlate D_H with the so-called effective diameter D_{10}, commonly used in geotechnique and hydrogeology. The general form of this relationship can be expressed as follows:

$$D_H = \beta D_{10} \quad [7]$$

where β [–] is a proportionality coefficient. In many applications, it was found that β depends on the coefficient of uniformity C_U (= D_{60}/D_{10} where D_{10} and D_{60} are the diameters corresponding to 10% and 60% passing on the cumulative grain-size distribution curve, respectively). Some existing expressions for $\beta = f(C_U)$, most of them empirically derived, are given below.

According to Hazen (in Beyer 1964), the coefficient β takes the values given in Table 1, with respect to C_U. In this table, parameter β is not well defined for C_U >10.

Table 1. Proportionality factor β between D_H and D_{10} after Hazen (in Beyer 1964).

C_U	1.0 – 1.9	2.0 – 2.9	3.0 – 4.9	5.0 – 9.9	> 10
β=D_H/D_{10}	1.0 – 1.6	1.6 – 1.9	1.9 – 2.2	2.2 – 2.5	> 2.5
Mean β	1.4	1.8	2.1	2.3	>2.5

For relatively uniform materials with C_U < 2, Huisman and Wood (1974) proposed the following empirical equation for β:

$$\beta = 1 + 2\log(C_U) \quad [8]$$

A theoretical investigation conducted by Moll (1980) on soils with grain size curves that can be represented by a normal distribution have led to:

$$\beta = 0.75 + \sqrt{0.54C_U - 0.48} \quad [9]$$

Kovács (1981) presented the relationship between the ratio D_H/D_{10} and C_U graphically for C_U up to about 50. The data obtained by digitizing the published results show a well defined trend for C_U less than about 10, but show relative scattering for C_U higher than about 10. These data are not presented here, but are used later for comparison purposes with the relationship developed in this paper (see Figure 5).

In the modified Kozeny–Carman model developed by Aubertin et al. (1996) (based in part on previous work conducted by Pavchich, cited in Goldin and Rasskazov, 1992), which serves to predict the saturated hydraulic conductivity of granular materials, the following parameter β is used (see also Mbonimpa et al. 2002):

$$\beta = C_U^{1/6} \quad [10]$$

Finally, using D_{10} and D_H data presented by Kovács (1981) for materials with $C_U \le 50$, Aubertin et al. (1998) derived the following empirical relationship:

$$\beta = 1 + 1.17\log(C_U) \quad [11]$$

It can be observed that eq. 11 takes the same general form as eq. 8, although these 2 equations have been developed for different ranges of C_U. Eq. 11 is used to estimate the equivalent capillary rise h_{co}, which is a reference parameter in the modified Kovács (MK) model developed to predict the water retention curve of granular materials (Aubertin et al. 2003), and in other associated model developments (Mbonimpa et al. 2006; Maqsoud et al. 2006).

The authors experience with equations 10 and 11 tend to indicate that these may not represent adequately the actual value of parameter β (and hence of SSA) for granular materials having a relatively large C_U.

It is therefore useful to develop a more general method for estimating β. For this purpose, S-shaped grain-size curves represented by lognormal distributions were used. The different approaches to estimate the proportionality factor β are compared below.

3 TWO-PARAMETER LOGNORMAL DISTRIBUTION (2PLND)

3.1 Definitions and characteristics

A positive random variable D with $0 \le D < \infty$ is lognormally distributed if Y = lnD is normally distributed with mean μ and variance σ^2 (σ is the standard deviation). The general expressions for these (and other) distribution functions can be found in statistical textbooks (e.g. Bernhardt 1990; Krishnamoorthy 2006). When the GSD of a soil is represented by such a two-parameter (μ and σ) lognormal distribution (2PLND), it is typically assumed that $\mu = D_{50}$ (where D_{50} is the diameter corresponding to 50% passing on the cumulative GSD curve) and the standard deviation σ = S. The probability density function (PDF) of grains finer than diameter D is then defined from these two parameters (D_{50} and S) as follows (DIN 66164; Bernhardt 1990; Wagner and Ding 1994):

$$F(D) = \frac{1}{S\sqrt{2\pi}} \int_{-\infty}^{\ln\frac{D}{D50}} \exp\left\{-\frac{1}{2}\left(\frac{\ln\frac{D}{D_{50}}}{S}\right)^2\right\} d\left(\ln\frac{D}{D_{50}}\right) \quad [12]$$

In a double logarithmic scale, a 2PLND grain size curve is represented by a linear curve with slope S. In many applications, parameter S is approximated by the following relationships:

$$S = \frac{1}{2}\ln\frac{D_{84}}{D_{16}} \quad [13a]$$

$$S = \ln\frac{D_{84}}{D_{50}} \quad [13b]$$

$$S = \ln\frac{D_{50}}{D_{16}} \quad [13c]$$

Parameters D_{16} and D_{84} are diameters corresponding to 16% and 84% passing on the cumulative grain-size distribution curve fully described by a 2PLND, respectively.

The cumulative distribution function (CDF in %) of the lognormal particle size distribution is given by (Wagner and Ding 1994):

$$F(D \ge D_{50}) = 50 + 50\,\mathrm{erf}\left(\frac{\ln\frac{D}{D_{50}}}{S\sqrt{2}}\right) \quad [14]$$

$$F(D \le D_{50}) = 50 - 50\,\mathrm{erf}\left(-\frac{\ln\frac{D}{D_{50}}}{S\sqrt{2}}\right) \quad [15]$$

where erf is the error function. Equations 14 and 15 can also be expressed with the complementary error function erfc (erfc (x) = 1 – erf(x)). Figure 1 illustrates typical GSD curves in a semi-log plane represented by 2PLNDs for D_{50} = 0.1 mm and different S values (S=0.5, S=1.0 and S =2.0).

Based on eqs. 14 and 15, diameter D_n (corresponding to $F(D_n)$%) on the cumulative grain-size distribution curve can be expressed as follows:

$$D_n(\ge D_{50}) = D_{50}\exp\left[S\sqrt{2}\,\mathrm{inverf}\left(\frac{F(D_n) - 50}{50}\right)\right] \quad [16]$$

$$D_n(\le D_{50}) = D_{50}\exp\left[-S\sqrt{2}\,\mathrm{inverf}\left(\frac{50 - F(D_n)}{50}\right)\right] \quad [17]$$

where inverf is the inverse of the error function erf.

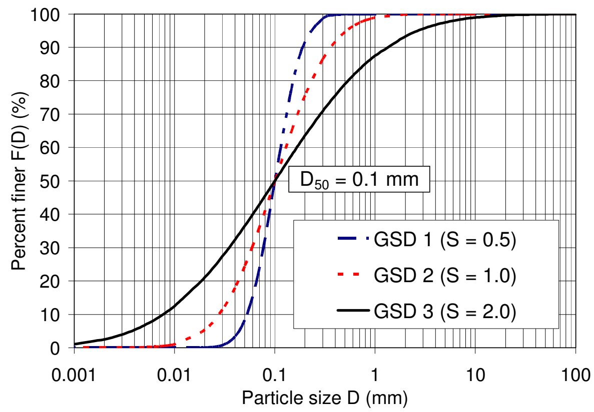

Figure 1. Typical 2PLND curves in a semi-log plane.

Diameters D_{10} and D_{60} can be calculated using $F(D_n)$ =10% in eqs. 17 and $F(D_n)$ = 60% in eq. 16; the arguments x of the inverf function then become 0.8 and 0.2, respectively. The coefficient of uniformity C_U (=D_{60}/D_{10}) with respect to S can thus be expressed as follows:

$$C_U = \frac{\exp\left[S\sqrt{2}\,\mathrm{inverf}(0.2)\right]}{\exp\left[S\sqrt{2}\,\mathrm{inverf}(0.8)\right]} = \exp(1.535S) \qquad [18]$$

This equation shows that GSDs with the same S value but with different D_{50} values will have the same C_U. Figure 2 shows the relationship between S and C_U in an arithmetic scale. When a logarithmic scale is used for C_U, the relationship is represented by a straight line in the semi-log plot (see embedded object in Figure 2).

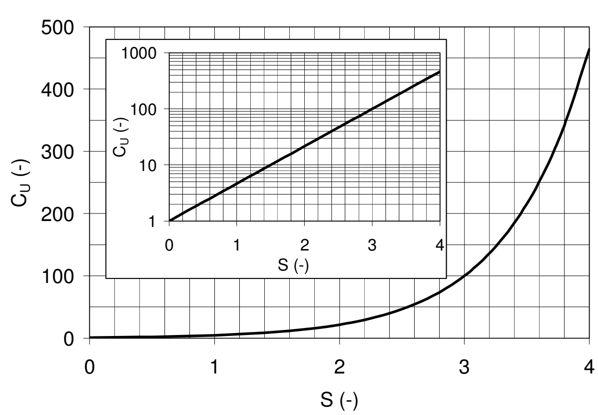

Figure 2. Relationship between C_U and S in arithmetic scale and semi-log scale (embedded view).

3.2 Estimation of D_H and β for a 2PLND

Considering a continuous distribution function F(D), the effective diameter D_H can be defined by the following equation (Sedran and de Larrard 1994):

$$D_H = \frac{100}{\int_{D_{min}}^{D_{max}} \frac{\delta F(D)}{D}} \qquad [19]$$

For a GSD represented by a 2PLND, the theoretically derived equivalent mean diameter D_H is given by eq. 20 (DIN 66164; Bernhardt 1990):

$$D_H = \frac{D_{50}}{\exp\left(\frac{S^2}{2}\right)} \qquad [20]$$

Results from eq. 20 have been compared with those of eq. 19, using the MAPLE Code (Maplesoft 2004), and also from the segments method defined by eq. 3. For this purpose, various GSDs represented by 2PLND were created by fixing their mean diameters D_{50} and standard deviations S. It was observed that using eq. 3 with the geometric mean $D_{i\text{-}g}$ for each segment i produced values D_H closer to those calculated with eq. 20, i.e. $D_H(D_{i\text{-}g}) \approx D_{H\text{-}2PLND}$.

Using D_{50} =D_{16} exp(S), (see eq. 13), eq. 20 can also be expressed in terms of D_{16}, as follows:

$$D_H = D_{16}\ \exp\left(S - \frac{S^2}{2}\right) \qquad [21]$$

The exponential function (eq. 21) for the ratio D_H/D_{16} is represented graphically in Fig. 3. This figure indicates that $D_H = D_{16}$ for S = 2, $D_H < D_{16}$ for S > 2 and $D_H > D_{16}$ for S < 2. The maximum D_H/D_{16} value is about 1.65 and corresponds to S = 1. According to eq. 18, S = 2 corresponds to a C_U value of about 22. In other words, it can be stated that $D_H < D_{16}$ for C_U > 22, and $D_H > D_{16}$ when C_U < 22.

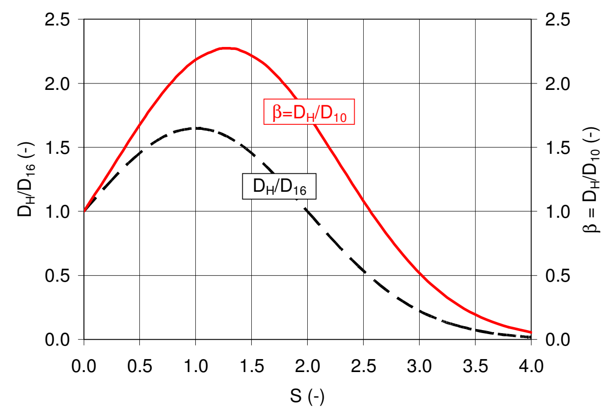

Figure 3. Relationship between the ratio D_H/D_{16} the value of S (based on eq. 21) and between β and S (based on eq. (22) for GSDs represented by a 2PLND

Diameters D_{10} and D_{15} are more commonly used than D_{16}. As D_{16} is close to D_{15}, it can be assumed that the ratio D_H/D_{15} and D_H/D_{16} may also be very close. Based on eq. 17, which can be used to calculate a selected small size diameter (such as D_{10}), and on eq. 20, one can express the proportionality factor β (= D_H/D_{10}; see eq. 7) as follow:

$$\beta = \exp(1.28S - \frac{S^2}{2}) \quad [22]$$

This new relationship is also represented graphically in Fig. 3; it is seen that the shape is similar to that of the D_H/D_{16} ratio (eq. 21). It can also be seen that $\beta = 1$ (or $D_H = D_{10}$) for S = 2.56 (and also for S =1, corresponding to a single size distribution); $\beta < 1$ (or $D_H < D_{10}$) for $S > 2.56$; and $\beta > 1$ (or $D_H > D_{10}$) when $S < 2.56$. According to eq. 18, a value S = 2.56 corresponds to a C_U value of about 50. In other words, $D_H < D_{10}$ for $C_U > 50$, and $D_H > D_{10}$ when $C_U < 50$. The maximum D_H/D_{10} value is about 2.3, and it corresponds to S =1.28 (or to a C_U of about 7).

Combining equations 18 and 22 leads to the following function:

$$\beta = \frac{C_U^{0.83}}{\exp\left[\frac{(\ln C_U)^2}{4.712}\right]} \quad [23]$$

Figure 4 illustrates this variation of β with respect to C_U (in a semi-log plot). It can be seen that β increases with C_U up to $\beta = 2.3$ (for a C_U of about 7), and then decreases as C_U is increased; its value is equal to 1 for a C_U of about 50. This type of variation is not taken into account in existing equations applied to estimate the value of β.

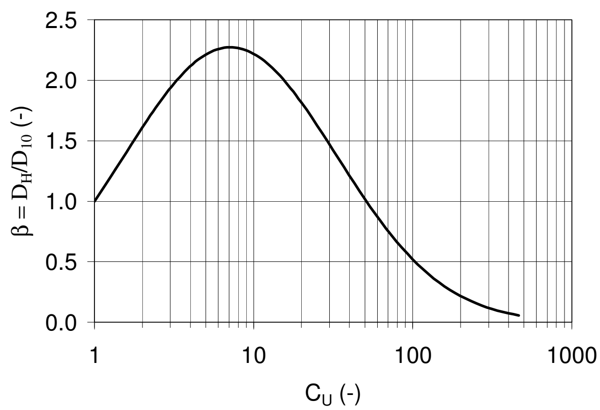

Figure 4. Relationship between β and C_U (semi-logarithmic plot).

3.3 Comparison of the various equations for β.

Figure 5 compares the values of β obtained from the equations presented in section 2 and from the newly proposed equation 23 (for C_U up to 50); in this representation, all the equations are assumed valid for the entire range shown in the figure ($1 \leq C_U \leq 50$). It is seen that the β function obtained for 2PLND (eq. 23) agrees well with the values obtained by Hazen (given in Table 1) for $C_U \leq 8$, with the values given by Kovacs for $C_U \leq 5$ as well as with the functions of Huisman and Wood (eq. 8) and Moll (eq. 9) Eq. 10 tends to underestimate the β value for C_U <20 and to overestimate it for C_U >20. The relationship proposed by Aubertin et al. (1998) (eq. 11) somewhat underestimates the β value for C_U < 11 and overestimates it for C_U >11. None of the existing β functions follows the tendency of eq. 23, as these all steadily increase with C_U. For a given GSD, underestimating β leads to an underestimation of D_H (see eq. 7), and to an overestimation of S_m (see eq. 2). Overestimating β leads to inverse results.

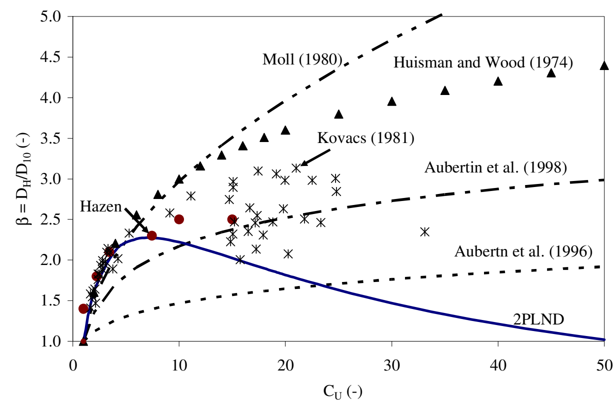

Figure 5. Comparison between the various functions to calculate the value of β.

4 PRELIMINARY USE OF THE PROPOSED EQUATION WITH THE MK MODEL

The modified Kovács (MK) model can be used to describe, and in some instances predict, the water retention curve (WRC) for coarse- and fine-grained materials (Aubertin et al. 2003). The MK model considers that water is retained in porous media by capillary forces responsible for capillary saturation S_c and by adhesive forces that cause saturation by adhesion S_a. The volumetric water content θ_w can be obtained from the MK model as follows:

$$\theta_w = n\left[1 - \langle 1 - S_a \rangle (1 - S_c)\right] \quad [24]$$

where $\langle\ \rangle$ are the Macauley brackets ($\langle y \rangle = 0.5(y+|y|)$). The relationship between degrees of saturation S_a and S_c and the matric suction head ψ (cm of water) can be calculated using the equivalent capillary rise h_{co} (cm of water) with the following equations:

$$S_c = 1 - \left[(h_{co}/\psi)^2 + 1\right]^m \exp\left[-m\,(h_{co}/\psi)^2\right] \quad [25]$$

$$S_a = a_c\left(1-\frac{\ln(1+\psi/\psi_r)}{\ln(1+\psi_0/\psi_r)}\right)\frac{\left(h_{co}/\psi_n\right)^{2/3}}{e^{1/3}\left(\psi/\psi_n\right)^{1/6}} \qquad [26]$$

where m (–) is the pore size distribution parameter, a_c (–) is the adhesion coefficient, ψ_n (cm) is a normalization parameter introduced for unit consistency (ψ_n =1 cm of water) and ψ_0 (cm of water) is the suction head corresponding to complete dryness ($\psi_0 = 10^7$ cm of water). The parameter ψ_r (cm of water) is the suction at residual water content θ_r. Its value has also been related to the equivalent capillary rise h_{co}:

$$\psi_r = 0.86\, h_{co}^{1.2} \qquad [27]$$

The parameter h_{co} is defined as the water rise corresponding to an idealized system of regular channels having a diameter expressed as the equivalent hydraulic pore diameter of the media. For granular materials, h_{co} is defined using the equivalent mean diameter D_H (cm) as follows:

$$h_{co,G} = \frac{0.75}{eD_H} \qquad [28]$$

With the MK model (Aubertin et al. 2003), D_H (cm) is defined using eq. 11. Measured WRC were fitted using the MK model to obtain optimal parameter fit for m (in eq. 25) and a_c (in eq. 26), which were then used to develop general relationships for predictive purposes. For coarse-grained materials, the observed trends indicated that m ≈ $1/C_U$ and a_c ≈ 0.01. These parameters were, in most cases, derived for materials with $1.3 \le C_U \le 15$.

For GSD curves represented by 2PLND, the results presented above tend to indicate that eq. 11 may not be accurately representing the value of β (and hence D_H and h_{co}).

A preliminary investigation was conducted on the effect of using eq. 23 (instead of eq. 11) on the value of parameters m and a_c. For this purpose, 11 soils taken from the GRIZZLY database (Haverkamp et al. 1997) and 8 soils taken from the UNSODA database (Leij et al. 1996; Nemes et al. 2001) were analysed. The coefficient of uniformity ranges between 1.5 and 10 for these soils. The GSD curves of the selected soils are deemed to be S-shaped. Each curve was fitted using the 2PLND equation by adjusting parameter S, with D_{50} obtained from measured data (on the experimental curves). Figure 6 illustrates typical results for 3 soils, with the measured data and GSD curve fitting. It can be mentioned that the fitting exercise may be performed using S calculated from eqs. 13a to 13c. These equations however lead to different values when the GSD is not fully described by a 2PLD. As the fine fraction of a GSD has more impact on the SSA than the coarse, the fitting parameter may be controlled by the fine branch of the GSD, i.e. parameter S may be calculated from eq. 13c using D_{16} and D_{50}. This aspect is not investigated in this paper.

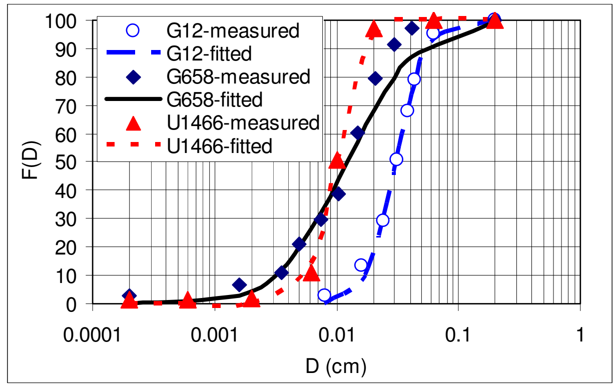

Figure 6. Measured GSD curves fitted with the 2PLND (by adjusting parameter S, and using D_{50} obtained from measured data)

The coefficients of uniformity C_U obtained from the measured grain-size distribution curves were used in eqs. 11 and 23 to calculate β and hence D_H (using D_{10}). Figure 7 compares the β values obtained using the two equations. As all C_U values considered here are lower than 11, it is seen that equation 23 overestimates β when compared with eq. 11 (see also Figure 5). For each of the 19 soils, measured WRC was then fitted to the MK model equations by adjusting parameters m and a_c using the β values obtained from eqs. 11 and 23. Figures 8 and 9 compare the adjusted MK model parameters m and a_c, respectively, for β obtained from eqs. 11 and 23.

The results indicate that applying eq. 11 leads to smaller m values than those obtained using eq. 23; parameter a_c values appear to be less sensitive to this difference. A more extensive analysis on this aspect is underway, and will be presented elsewhere; it can be expected that modifying the definition of β and D_H will affect the predictive capabilities of the MK model, particularly for materials having a grain-size distribution with a large C_U value.

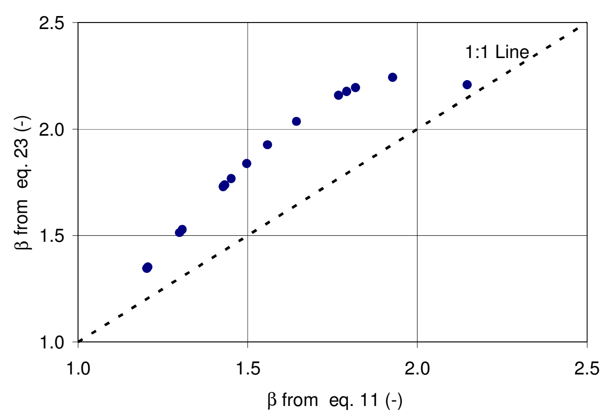

Figure 7. Comparison between β values obtained from eqs. 11 and 23 using C_U values from the measured GSD curves.

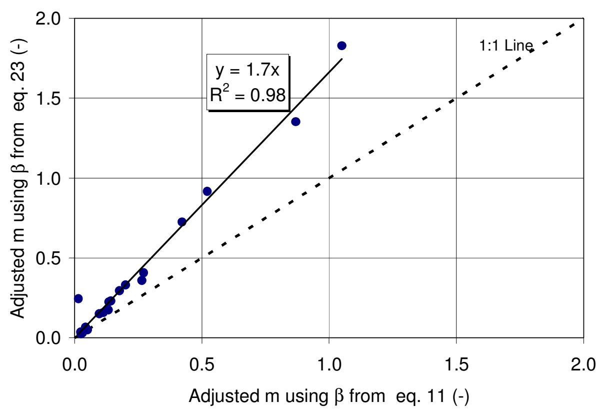

Figure 8. Comparison between the adjusted MK model parameter m when β is obtained from eqs. 11 and 23, using C_U values from the measured GSD curves.

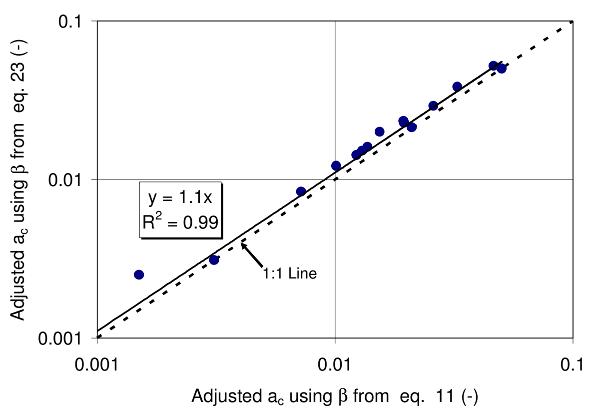

Figure 9. Comparison between the adjusted MK model parameter a_c when β is obtained from eqs. 11 and 23 using C_U values from the measured GSD curves.

5 DISCUSSION AND CONCLUSION

A new analytical relationship is proposed here to estimate the specific surface area, SSA, for materials having a grain-size curve represented by a two-parameter lognormal distribution, 2PLND. The latter function is defined from the mean diameter D_{50} and standard deviation S of the grain size curve. The new equation is expressed in terms of the equivalent mean diameter D_H and ratio $\beta = D_H/D_{10}$, with parameter β expressed with respect to the coefficient of uniformity C_U. Calculated β values are compared with values obtained from other existing analytical equations involving D_{10} and C_U. The effect of using the proposed equation for β with the MK model, developed to predict the water retention curve, is also investigated in a preliminary manner; results indicate that significant differences may be induced by changing the way the SSA is obtained. Further investigations are planned on these and related aspects.

The analysis presented above is based on the assumption that the two-parameter lognormal distribution (2PLND) is theoretically appropriate for S-shaped grain size curves with grain diameters D ranging from 0 to ∞ ($0 \leq D < \infty$). In reality, particle size distributions have a lower limit D_0 larger than 0 and an upper limit D_∞ << infinity. Such grain size distribution may be better described using a four-parameter lognormal distribution (4PLND) with a cumulative density function, CDF, that involves not only the mean diameter D_{50} and standard deviation S, but also the particle size limits themselves (D_0 and D_∞). Such a 4PLND can then be transformed into a 2PLND using a variable transformation; this aspect is also being investigated. As the bigger solid particles have little influence on the value of SSA, three-parameter lognormal distributions (3PLND) with parameters (D_0, D_{50} and S) are also being investigated. Additional work will also be performed to assess the possible application of the approach presented here to bi-modal (or possibly multi-modal) grain size distribution curves. Other corrections for particular CDF are also being considered to take into account complementary effects such as the shape of the grains and the influence of the fraction smaller that the D_{10}. These results will be presented elsewhere.

ACKNOWLEDGEMENTS

For this and related projects, the authors received financial support from NSERC and the participants of the Polytechnique-UQAT Industrial Chair on Environment and Mine Waste Management (www.polymtl.ca/enviro-geremi). We are also grateful for financial support from the Fondation de l'UQAT (FUQAT) in 2007–2008.

REFERENCES

Arnepalli, D.N., Shanthakumar, S., Hanumantha Rao, B., and Singh, D. N.2008. Comparison of methods for determining specific-surface area of fine-grained soils, *Geotechnical and Geological Engineering*, 26(2): 121-132.

Aubertin, M., Bussière, B., and Chapuis R. P. 1996. Hydraulic conductivity of homogenized tailings from rock mines. *Canadian Geotechnical Journal*, 33: 470-482.

Aubertin, M., Chapuis, R.P., and Mbonimpa, M. 2005. Discussion of "Goodbye, Hazen; Hello, Kozeny-Carman" by W. David Carrier III, Published in Journal of Geotechnical and Geoenvironmental Engineering ASCE (2003; 129(1): 1054-1056). *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 131(8): 1056-1057.

Aubertin, M., Ricard, J.F., and Chapuis, R.P. 1998. A predictive model for water retention curve: application to tailings from hard-rock mines, *Canadian Geotechnical Journal*, 35: 55-69.

Aubertin, M., Mbonimpa, M., Bussière, B., and Chapuis, R.P. 2003. A model to predict the water retention curve from basic geotechnical properties, *Canadian Geotechnical Journal*, 40(6): 1104-1122.

Bernhardt, C. 1990. *Granulometrie. Klassier- und Sedimentationsmethoden*. Deutscher Verlag für Grundstoffindustrie, Leipzig.

Bethea, R.M., Duran, B.S. and Boullion, T.L 1995. *Statistical methods for engineers and scientists*. 3rd Edition, revised and expanded. Marcel Dekker, Inc., New York, Basel, Hong Kong.

Beyer, W. 1964. Zur Bestimmung der Wasserdurchlässigkeit von Kiesen und Sanden aus der Kornverteilungskurve, *Wasserwirtschaft-Wasser-technik*, 14. Jahrgang. Heft 6.

Buchan, G.D., Grewal, K.S. and Robson, A.B. 1993. Improved models of particle-size distribution: An illustration of model comparison techniques, *Soil Science Society America Journal* (ASCE), 57: 901-908.

Chan, T.P. and Govindaraju, R.S. 2004. Estimating soil water retention curve from particle-size distribution data based on polydisperse sphere systems. *Vadose Zone Journal*, 3: 1443-1454.

Chapuis, R.P., and Aubertin, M. 2003. On the use of the Kozeny-Carman equation to predict the hydraulic conductivity of soils. *Canadian Geotechnical Journal,* 40(3): 616-628.

Chapuis, R.P., and Légaré, P.P. 1992. A simple method for determining the surface area of fine aggregates and fillers in bituminous mixtures. Effects of aggregates and mineral filler on asphalt mixture performance, *ASTM STP 1147*, pp. 177-186.

Chapuis, R.P. and Montour, I. 1992. Évaluation de l'équation de Kozeny-Carman pour prédire la conductivité hydraulique, *Proceedings, 45ième Conférence Canadienne de Géotechnique*, Toronto, Ontario, 78-1 à 78-10.

DIN 66144. Darstellung von Korn- (Teilchen-) größenverteilungen. Logarithmisches Normal-verteilungsnetz. Ausgabe März 1974, Deutsches Institut für Normung e.V.

Dolinar, B., Mišič, M. and Trauner, L. 2007. Correlation between surface area and Atterberg limits of fine-grained soils, *Clays and Clay Minerals*, 55(5): 519-523

Fredlund, M.D., Fredlund, D.G., and Wilson, G.W. 2000. An equation to represent grain-size distribution, *Canadian Geotechnical Journal*, 37(4): 817-827.

Haverkamp, R., Zammit, C., and Bouraoui, F. 1997. *Grizzly soil data bank, base de données de paramètres caractéristiques des sols*. LTHE, Grenoble.

Huisman, L. and Wood, W. E. 1974. *Slow Sand Filtration*, WHO-Geneva.

Goldin, A.L. and Rasskazov, L.N. 1992. *Design of Earth Dams, Geotechnica 2*, A.A. Balkema.

Igwe, G.J.I. 1991. Powder *Technology and Multiphase systems. Gas Permeametry and Surface Area Measurement*. Ellis Horwood, New York.

Kézdi, A. 1964. *Bodenmechanik. Band 1*, Gemeinschaftsauflage VEB Verlag für Bauwesen Berlin - Verlag der ungarischen Akademie der Wissenschaften Budapest.

Kosugi, K. 1994. Three-parameter lognormal distribution model for soil water retention, *Water Resources Research*, 30: 891-901.

Kovács, G. 1981. *Seepage hydraulics*, Elsevier Science Publishers, Amsterdam.

Krishnamoorthy. K. 2006. *Handbook of statistical distributions with applications*. Chapman &Hall/CRC. Boca Raton, Fl.

Leij, F.J., Alves, W.J., and van Genuchten, M. Th. 1996. *The UNSODA unsaturated soil hydraulic database*. EPA/600/R-96/095, 103p.

Locat, J.; Lefebvre, G.; and Ballivy, G. 1984. Mineralogy, chemistry and physical properties interrelationships of some sensitive clays from eastern Canada, *Canadian Geotechnical Journal*, 21(3): 530-540.

Lowell, S., and Shields, J.E. 1984. *Powder Surface Area and Porosity*. 2nd Edition. Chapman and Hall, London.

Maple soft 2004. *Maple 9.5*. Maplesoft, Waterloo Maple Inc. 2004.

Maqsoud, A., Bussière, B., Aubertin, M., Mbonimpa, M. 2006. Modification of the predictive MK model to integrate hysteresis of the water retention curve. Unsaturated Soils 2006: *Proceedings of the 4th International Conference on Unsaturated Soils*, April 2-6, 2006, Carefree, Arizona, G.A. Miller, C.E. Zapata, S.L. Houston, D.G. Fredlund (eds.). ASCE Geotechnical Special Publication, No. 147, pp. 2465-2476.

Mbonimpa, M., Aubertin, M., and Bussière, B. 2006. Predicting the unsaturated hydraulic conductivity of granular soils from basic geotechnical properties using the MK model and statistical models. *Canadian Geotechnical Journal*, 43(8): 773-787.

Mbonimpa, M., Aubertin, M., Chapuis, R.P., and Bussière, B. 2002. Practical pedotransfer functions for estimating the saturated hydraulic conductivity. *Geotechnical and Geological Engineering*, 20: 235-259.

Moll, H. G. 1980. Zur Charakterisierung von Korngemischen im Hinblick auf die Durchströmung mit Wasser, *Bohrtechnik, Brunnenbau und Rohrleitungsbau*, 31 (12): 517-520

Nemes, A., Schaap, M.G., Leij, F.J., and Wösten, J.H.M. 2001. Description of the unsaturated soil hydraulic database UNSODA version 2.0*, Journal of Hydrology*, 251: 151-162.

Sedran, T. and de Larrard, F. 1994. RENÉ-LCPC: un logiciel pour optimiser la granularité des matériaux de génie civil. Note technique, *Bulletin de Liaison des Laboratoires des Ponts et Chaussées*, N°194, Novembre-Décembre, 1994. Laboratoire Central des Ponts et Chaussées – Paris.

Skirazi, M.A. and Boersema, L. 1984. A unifying quantitative analysis of soil texture, *Soil Science Society of America Journal*, 48:142-147.

Wagner, L.E and Ding, D. 1994. Representing aggregate size distributions as modified lognormal distributions, *Transactions of the American Society of Agricultural Engineers*, 37(3): 815-821.