Reliability Based Analysis of Stability in Homogeneous Earth Dams by Means of Neural Network and Monte Carlo Simulation

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ABSTRACT
In this paper, probabilistic behavior of effective parameters on stability of earthfill dams is studied based on the reliability concept. A large set of deterministic factors of safety (FS) are calculated and used for training a neural network. This emulator is utilized to calculate FS for a large number of inputs created based on the probability density functions, expected values and standard deviations of dam parameters. Then, the reliability is determined by Monte Carlo Simulation and a sensitivity analysis is performed. The results of this analysis indicate that the height and cohesion of dam have the most significant effect on reliability.

RÉSUMÉ
Dans ce comportement de papier et probabiliste des paramètres efficaces sur la stabilité des barrages d'earthfill a été étudié a basé sur le concept de fiabilité. Beaucoup de facteurs de la sûreté déterministes (FS) ont été calculés et employés pour former un réseau neurologique. Cet émulateur a été utilisé pour calculer le FS pour un grand nombre d'entrées produites basées sur leurs fonctions de densité de probabilité, valeurs prévues et écarts type. Puis, la fiabilité a été déterminée par la simulation de Monte Carlo. En conclusion, une analyse de sensibilité a montré que la taille et la cohésion du barrage exercent la plupart d'effet sur la fiabilité.

1 INTRODUCTION

Slope stability analysis of earthfill dams is a practical topic in soil mechanics and foundation engineering that aims to study the behavior of these structures and their stability. Conventionally the calculation of factor of safety (FS) is based on deterministic approaches, such as the limit equilibrium method and the numerical method (Duncan 1996). The most important benefit of limit equilibrium methods is that their application is convenient. These methods require assumptions for statically solving the indeterminate equilibrium equations and lead to an over estimation for factor of safety.

Numerical methods in stability analysis of earthfill dams are discussed by many researchers (Griffiths and Lane, 1999). In numerical methods, the geometry of slope of earthfill dam is modeled carefully and its stability is analyzed. These methods are computationally extensive and require significant computing power, especially during the optimization process.

Artificial neural networks (ANN) can be used to address the above issues as they are able to learn complex behaviors with desirable degree of accuracy (Chau 2007). Therefore, they can be employed as a substitute of numerical modelling software in slope stability analysis (Ni et al. 1996; Saravut et al. 2002).

Conventional methods have a deterministic view in slope stability analysis, thus, all inputs and FS of stability of earth dam are considered as deterministic parameters. However, many of these inputs such as the angle of internal friction and cohesion of dam body and foundation, the geometry of earth dam and foundation, and the earthquake coefficient are probabilistic parameters.

In this paper, the reliability of slope stability of homogeneous earthfill dams after construction and before impoundment has been determined, based on the probability density function of the input parameters rather than deterministic calculation of FS. Since the limit state function is numerical, Monte Carlo Simulation is used to determine the reliability based on comparison between FS in probabilistic and failure conditions. Finally, a sensitivity analysis is performed to study the effects of earth dam characteristics on the reliability of slope stability.

2 NUMERICAL METHODS IN STABILITY ANALYSIS OF EARTHFILL DAMS

Numerical methods in analysis of earthfill dams were presented by and Clough and Woodward (1967) by means of nonlinear stress-strain behavior in finite element method.

FS presented in limit equilibrium methods such as Bishop's method (1955) is the ratio of available shear strength to driving shear stress. In Duncan's theory (1996), the minimum shear strength of soil to reach the failure conditions is obtained from division of actual soil strength by FS. Based on this definition, FS is considered as a strength reduction factor that can be defined as the ratio of actual shear strength of soil to minimum strength required to prevent the failure, so the reduced soil strength in slope failure conditions is obtained in numerical methods according to Eq. 1 and Eq. 2.

$$c_m = c / FS \quad [1]$$

$$\phi_m = \phi / FS \quad [2]$$

Where, c_m and ϕ_m are the reduced shear strength parameters and based on them, slope is analyzed with a nonlinear elasto-plastic model. If the slope is stable, then the value of FS will increase and if it is not stable, the value of FS will decrease. Slope stability analysis is repeated until convergence of FS to a fixed value of c and ϕ to c_m and ϕ_m in limit equilibrium conditions.

This method is the well-known Strength Reduction or c-ϕ Reduction method and has been used by many researchers such as Dawson et al. (1999).

2.1 The Method Available in CA2 Program

CA2 is an academic program for analysis of two dimensional problems in soil mechanics based on finite difference methods. In this program, stability analysis of slopes is carried out with a circular or arbitrary slip surface. In it's stability analysis, CA2 verifies system's equilibrium by a stress-deformation analysis for linear or nonlinear behaviors. The factor of safety against slope failure is obtained from Eq. 3 according to Figure 1 (US Army Corps of Engineers 2003).

$$FS = (\Sigma \tau_r . \Delta L_i) / (\Sigma \tau . \Delta L_i) \quad [3]$$

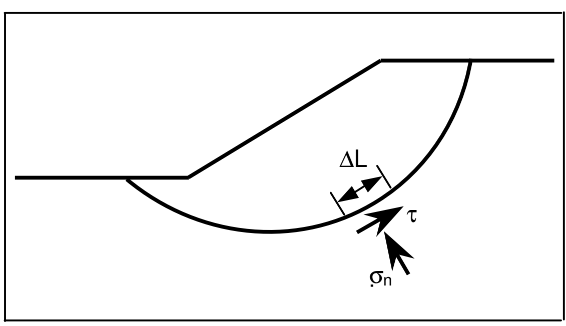

Figure 1. Existing shear stress of each element obtained from numerical analysis for calculation of FS

τ_r is the shear strength of each element of slip surface according to Eq. 4, ΔL_i is the length of slip surface elements and τ is the existing shear stress in each element obtained from numerical analysis.

$$\tau_r = \sigma_n \tan \phi + c \quad [4]$$

The advantage of this method in comparison to current limit equilibrium methods is that it obtains system equilibrium from the stress domain calculated from the stress-deformation analysis without virtual and simplification assumptions. Also, the FS can be determined for each arbitrary slip surface from mentioned stress domain in addition to use of deformation domain (such as the settlement of ground around the excavation area).

3 ARTIFICIAL NEURAL NETWORK

Artificial Neural Networks (ANN) are able to perform operations like biological neural systems. Training of an ANN is the modification of network's parameters such that it can show a desirable behavior against the external excitations. A multi-layer feed forward (MLFF) neural network has been used in this research. These kinds of networks consist of a number of processing units that can be divided into input, hidden and output layers (Figure 2). In every unit, activation function has an effect on the weighted sum of inputs and specifies the output, and is defined as sigmoid, sine, hyperbolic tangent function and etc. The effect of every unit on the next units depends on its activation content. Manner and pattern of relations between the units play an important role in the response of the system. Network training could be defined as: creating new units, creating new connections, elimination of some connections or weight correction of the existing connections.

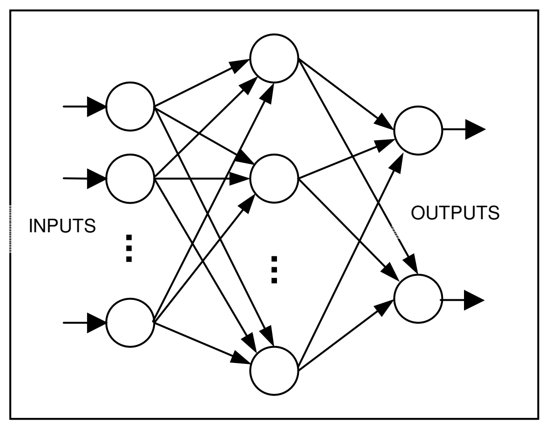

Figure 2. A multi-layer feed forward neural network with three layers

Network training is performed based on generalized δ rule (Rumelhart and McClelland 1980). In order to obtain a better training procedure, coefficients such as learning ratio and Momentum term (Jogataie 1995) are defined to control the changes made in the weights on each step. A neural network with one input layer, a hidden layer, an output layer and sigmoid activation function is able to learn every non-linear behavior (Chau 2007). In the mean time, in these kinds of networks the number of nodes in hidden layer is considered as system's degree of freedom (Zhang and Foschi 2004) and is obtained by trial an error (Chau 2007).

While training, it is possible that the neural network gets stuck in a local minimum and the level of error remains steady. Therefore getting an acceptable neural network solution can be difficult even by using generalized δ rule. The precision of an emulator, i.e. a trained neural network, depends on: the number of nodes in the hidden layer, type of connections of network, learning rate, momentum term, activation function, and number of input-output pairs. For each level of precision, an appropriate neural network can be obtained.

The neural network is implemented in FORTRAN programming language and its learning algorithm is based on generalized δ rule. In order to eliminate the local minimum effects in this software, the following techniques have been used (Tahamouli and Habibi 2008):

I- Randomized selection of the input-output pairs

At every training cycle for the input-output pairs, a randomized arrangement is selected and the error back propagation is performed. The reason of randomized arranged selection is to avoid the similarity of each cycle with its previous one, and also to reduce the probability of neural network halting in a local minimum.

II- Automatic node generation in the hidden layer and weight freezing

If the error level does not decrease and remains steady in several training cycles, the program automatically generates a node in hidden layer and chooses random weights for its connections. After adding the new node, for first few cycles the new connections will be updated and the rest of the connections are kept fixed. After a few cycles training of the whole network is resumed.

III- Simultaneous Use of different activation functions

The program can use sigmoid, linear, sine, and parabola as activation functions to have a better output. Most of the nodes in the network have a sigmoid activation function. However, the activation function in a few nodes is selected as linear, sine and etc. Having nodes with different activation functions is shown to improve learning properties of neural networks.

Small number of input-output pairs in the training set decreases the output accuracy of an ANN in operation phase, and a large number of pairs without a proper distribution on inputs' n-dimensional domain, also reduces the accuracy of the network in addition to taking too much CPU time. Therefore, optimum number and distribution of training pairs should be selected to decrease the training time and increase the accuracy of ANN. Hence, Hypercube method has been utilized according to Eq. 5 (Yun and Bahng 2000).

$$\text{The Number of Training Pairs} = 2^M + 2 \times M + 1 \qquad [5]$$

M is the dimension of inputs' domain.

4 RELIABILITY

The Reliability (R_e) is defined as the system's operation probability. If P_f is the probability of system's failure, then the reliability can be defined according to Eq. 6 (Ranganathan, R. 1990).

$$R_e = 1 - P_f \qquad [6]$$

The Limit State Function (LSF) is defined according to Eq. 7 to calculate the reliability.

$$LSF = R - S \qquad [7]$$

R is the system's resistance (capacity or strength) and S is the external action (load or load effect). So, based on Figure 3 the reliability is calculated from Eq. 8 with a definite probability density function (PDF) for R and S ($F_s(s)$ and $F_r(r)$).

$$R_e = \int dR_e = \int_{-\infty}^{+\infty} F_s(s) \left[\int_{-\infty}^{+\infty} F_r(r)\, dr \right] ds \qquad [8]$$

The hatched portion shown in Figure 3 is an indicative measure of the probability of failure. The proximity of PDFs for R and S leads to lower reliability of the system.

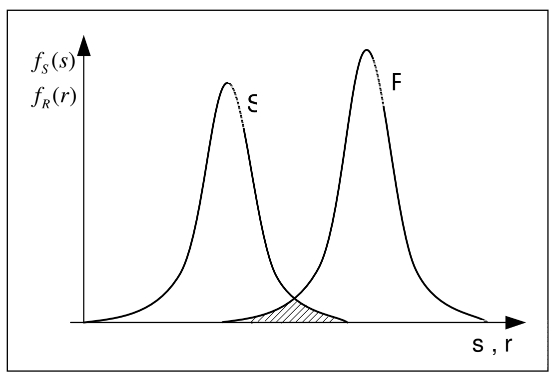

Figure 3. Probability of failure for random variations of S and R

It must be noted that the integrals in Eq. 8 are to be evaluated numerically. Except for a few cases, the closed form solutions are not available. In many cases, it is not possible to calculate the reliability according to Eq. 8 because the R and S should be calculated by numerical methods, therefore defining an explicit LSF will be impossible. Different methods have been used by researchers to calculate the reliability. In some cases, "First Order Reliability Method (FORM)" or "Second Order

Reliability Method (SORM)" is used (Deng 2005). For complex systems, some methods are more applicable such as "Sum of Disjoint Product" (Yeh 2007), "Inclusion / Exclusion" (Ramírez-Márquez and Jiang 2006), or "Monte Carlo Simulation (MCS)" (Zio et al. 2004). The MCS method is illustrated in the following.

4.1 Monte Carlo Simulation

In this method, a large number of values (N) are generated for the set of input parameters based on their PDF. Then, LSF is calculated according to Eq. 7 for each set of input values. LSF < 0 means that the failure has occurred in the system.

Finally, the reliability is computed according to Eq. 9, considering n for number of occurred failures after calculation of LSF for all input sets.

$$R_e = 1 - P_f = 1 - (n / N) \quad [9]$$

The number of generated input sets (or sample size) is important in MCS, i.e. a bigger sample size leads to more confident results. Some equations have been presented about minimum required sample size (Nowak and Collins 2000; Ranganathan 1990). In order to guarantee the accuracy of our results, the sample size selected in our simulations is much higher than the minimums suggested in these references.

5 STABILITY ANALYSIS OF EARTHFILL DAMS BY MEANS OF NEURAL NETWORK

In this paper, the MLFF Neural Network is used to predict the FS of slope stability for homogeneous earthfill dams in the case of after construction and pseudo static analysis (Heidari and Hassanlou Rad 2008). The input parameters of ANN have been selected as the effective factors on FS as shown in Figure 4. H is the height of earthfill dam, L is the width of dam's crest, D is the depth of dam's foundation, k is the earthquake coefficient, a_1 and a_2 are the cotangent of angle of dam's slopes in upstream and downstream, c_1 and c_2 are the cohesion of dam's body and foundation respectively, ϕ_1 and ϕ_2 are the angle of internal friction of dam and foundation, and γ_1 and γ_2 are the unit weight of dam and foundation. The range of variation of each parameter is presented in Table 1.

A large number of deterministic slope stability analyses (5151 analyses) using CA2 program are used to train the ANN to learn the FS values. The training set includes 4121 stability analyses (based on Eq. 5) and the testing set consists of 1030 analyses. For training set, selection of different values of input parameters and their distribution is based on the hypercube method. For testing set, we have randomly selected different values within the ranges of parameters in Table 1.

As mentioned before, the ANN with three layers and enough neurons in hidden layer can estimate every complex nonlinear function. So, in this research, 13 neurons are obtained as optimum number of neurons in hidden layer for the ANN with 12 input neurons and the FS as one output of it.

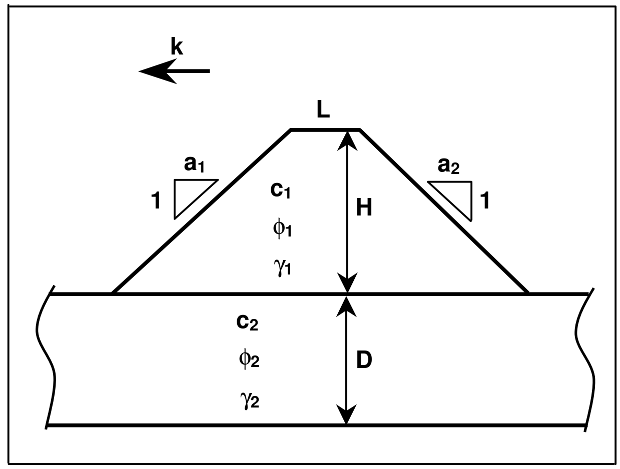

Figure 4. Input parameters of neural network as the effective factors on FS

Table 1. The range of variation of each input parameter of neural network

Input Parameters	Minimum	Maximum
H (m)	4	30
D (m)	0	20
a_1	2	5
a_2	2	4
L (m)	3	13
c_1 (kN / m^2)	30	100
ϕ_1 (deg.)	0	32
γ_1 (kN / m^3)	15	22
c_2 (kN / m^2)	40	380
ϕ_2 (deg.)	18	38
γ_2 (kN / m^3)	18	28
K	0	0.2

Figure 5 shows the ability of ANN in FS prediction with a high correlation factor between ANN outputs and target values. The target values are Factors of Safety calculated with CA2 program.

6 RELIABILITY ANALYSIS OF SLOPE STABILITY

The trained ANN (emulator) or CA2 program can determine the value of FS for every set of inputs. Whereas this FS has a deterministic nature, it can only determine the stability or failure occurrence. However, the effective parameters on FS have a probabilistic nature and the failure probability of an earth dam can be calculated based on their PDF. For this purpose, PDF, expected value (μ) and standard deviation (σ) should be determined for input parameters.

In this paper, normal distribution is selected for PDF of soil characteristics of dam's body and foundation (Malkawi et al. 2000; Al-Homoud and Tahtamoni 2000; Liang et al. 1999). Same distribution is assumed for earthquake coefficient and the geometry of dam and foundation. Table 2 shows the standard deviation of input parameters where, these values have been considered based on the construction conditions in Iran.

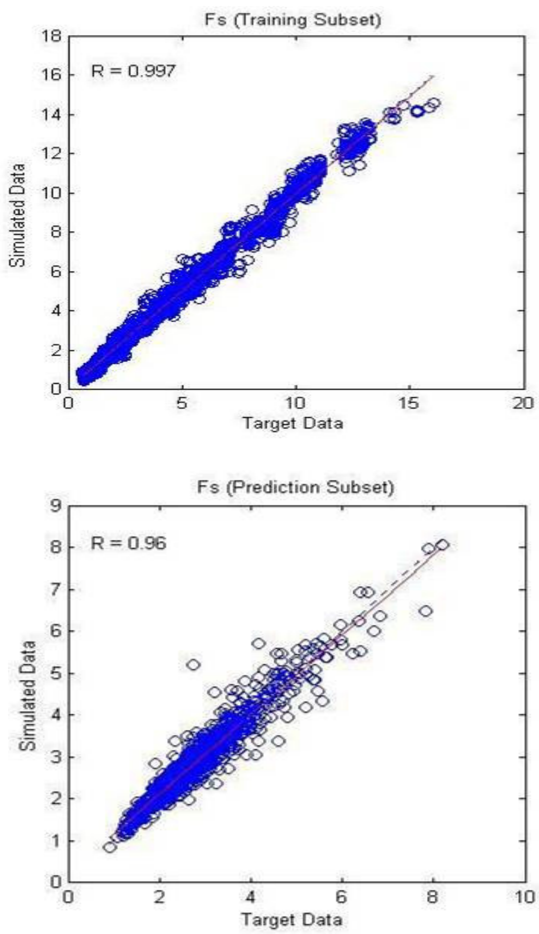

Figure 5. The well trained ANN in prediction of FS

The MCS is employed to see the probabilistic nature of FS. For this purpose, a large number of combinations of input sets are generated based on PDF, μ and σ of input parameters for each set. The numbers of these combinations are selected as mentioned in part 4.1 to reach statistically reliable results. Then, the FS of each combination is calculated by means of the emulator and the result is a PDF for FS instead of a deterministic FS. This PDF is shown in Figure 6 for a selected input set with FS = 1.1.

Table 2. The coefficient of standard deviation of effective parameters on FS

Inputs	H	D	a_1	a_2	L	c_1	ϕ_1	γ_1	c_2	ϕ_2	γ_2	k
σ / μ (%)	1	10	1	1	1	20	10	5	30	15	10	30

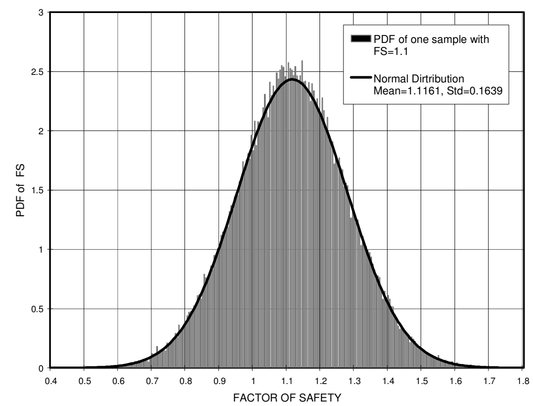

Figure 6. Probability density function obtained for FS = 1.1

As shown in Figure 6, PDF for FS is normal distribution and the probability of failure not occurring is equal to R_e = P (FS $\geq$ 1) = 0.758 in expected value of 1.1 for FS.

It is notable that, in deterministic approach we can find the different sets of inputs with the same value of FS, but the obtained reliability will be different for each set. Hence, in this work we have generated a large number of deterministic input sets that will produce the same FS. Each of the selected input sets is considered as mean of a normal distribution and for each input combination a large number of sets are generated based on PDF, μ, and σ and the reliability is calculated using MCS to observe the reliability variations for each FS. Figure 7 shows the variations of reliability related to FS.

The following results are obtained from Figure 7:

- Increase in FS values leads to decrease in the range of variations of reliability and consequently, increase of reliance to deterministic FS
- Probability of failure exists even for large values of FS such as FS = 1.8 (in the worst case scenario)
- For FS = 1.1 the reliability changes between 0.61 and 1 with the expected value of 0.82 according to Figure 7. It means that a high probability of failure (18%) exists for earthfill dams. This is in contrast with some standard codes where the proposed value of FS is about 1.1 in pseudo-static analysis conditions.

Even in best construction conditions soil and geometry properties are different from the values assumed in deterministic analysis and acceptable tolerance values can be determined based on a statistical analysis and reliability concept.

It seems that a reliability based design will be suitable instead of deterministic design, because, the reliability concept considers the probabilistic nature of input parameters. In reliability based design, standard deviation of inputs directly affects the results, we can limit the standard deviation by exact control of construction conditions which reduces the variations of reliability and hence, leads to more confident values of FS.

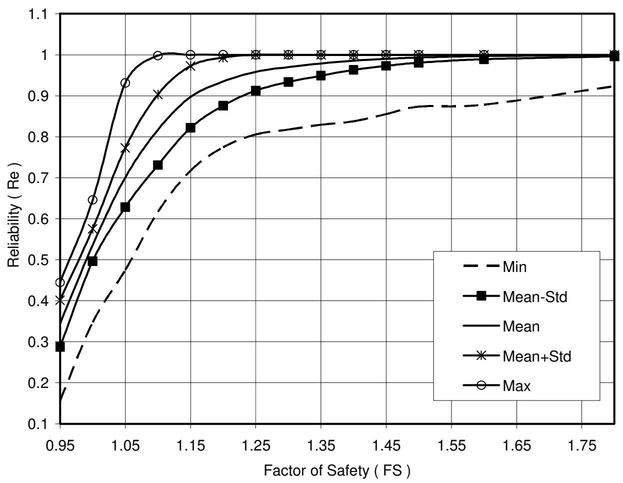

Figure 7. The range of variations of reliability related to FS

7 SENSITIVITY ANALYSIS

There are high variations of reliability in different values of FS especially in the range of FS between 1 and 1.4 where, the input parameters don't have the same effect on these variations. It is important to know which parameters have more effect on reliability to increase the value of reliability especially during an optimization process. Therefore, a sensitivity analysis of reliability related to input parameters is carried out. In this research, the sensitivity value of reliability related to inputs has been evaluated by calculation of Importance Measure (IM) (Zio et al. 2004), which it is defined as the ratio of variations of reliability to variations of input parameters. For example, Eq. 10 shows the IM value of height (H) of earthfill dam.

$$IM_H = \eth R_e / \eth H \qquad [10]$$

Based on direct relation between the reliability and FS (Figure 7), Eq. 11 has been utilized instead of Eq. 10 for dam's height (H).

$$IM_H = \eth FS / \eth H \qquad [11]$$

Then the IM values are obtained for different normalized inputs in [0.1, 0.9] domain. It can be concluded that the effect of H, c_1, ϕ_1, a_1 (a_2 for other side) and k is more than the other parameters. Figure 8 shows the variations of IM related to the normalized mentioned inputs.

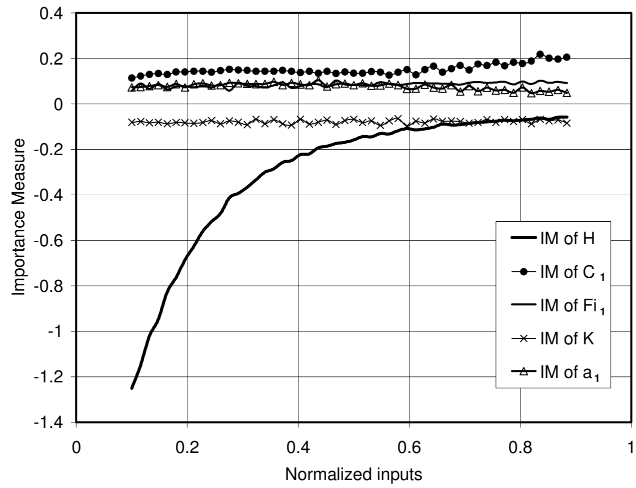

Figure 8. Variations of IM related to more effective inputs

As observed in Figure 8, H and k have a decreasing effect on reliability and c_1, ϕ_1 and a_1 have an increasing effect on it. Also, in small values of earthfill dam's height, H has the greatest effect on reliability and if the height of dam is relatively high (22m < H < 30 m), then c_1 has the maximum effect on reliability.

8 CONCLUSIONS

In this paper, the stability of homogeneous earthfill dams is studied based on the Reliability concept and using an artificial neural network. In some situations where the deviations of soil parameters and dam geometry are significant, deterministic analysis and relying on factors of safety published in standard codes does not seem proper and may result in instability in dams.

It is also indicated that the variations of input parameters based on their probabilistic nature has a considerable effect on the calculated deterministic FS, and it seems that it is necessary to study the stability of earthfill dams with a probabilistic approach.

Finally, by performing a sensitivity analysis, it is shown that the height and soil cohesion of body of dam have the most important effect on the reliability of the stability of homogeneous earthfill dams.

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