Plugging wellbore fractures: Limit equilibrium of a Bingham drilling mud cake in a tensile crack

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ABSTRACT
A theoretical model has been used to study invasion of a mud filter cake into a pre-existing or a drilling-induced crack at the edge of a wellbore. The presented solution allows to evaluate whether or not the mud cake could effectively plug the fracture, preventing fracture propagation and associated uncontrollable loss of wellbore drilling mud to it.

RÉSUMÉ
Un modèle théorique a été utilisé pour étudier l’invasion d’un cake de filtration de boue dans une fissure en angle de puit de forage pré-existante ou provoquée par forage. La solution présentée nous permet d’évaluer si le cake de filtration de boue pourrait à la fois boucher la ligne de fracture de façon effective et prévenir la propagation de la fracture et la perte incontrollable de boue de forage du puit qui l’accompagne.

1 INTRODUCTION

Successful drilling of a hydrocarbon well relies on the proper selection and use of drilling muds in order to maintain the wellbore mud pressure (weight) low enough to prevent circulation loss (loss of mud to the fractured and/or porous formation) and high enough to support the uncased wellbore against the shear failure (Bourgoyne et al. 1986). This mud pressure “window” can be very narrow or non-existent for practical implementation when, for example, drilling through a depleted reservoir. Reduced mud weight in order to avoid loss of circulation when drilling through a depleted reservoir with lower than normal pore pressure, may result in the shear failure of the wellbore. The solution to this problem is to maintain high enough weight of the mud in order to prevent the wellbore failure while using special mud composition/properties in order to minimize or control the imminent loss of circulation (e.g., Aston et al. 2003; Van Oort et al., 2003). One such approach to control drilling fluid loss to a natural or drilling-induced wellbore fractures is the use of mud filter cakes with engineered properties (e.g., yield stress) to “plug” fractures. In a successful scenario, the pressurized mud cake invades a portion of a pre-existing fracture and then comes to rest as the result of equilibrium between the shear stress generated between the mud cake and fracture walls and the mud pressure in the wellbore. Such equilibrium state, when exists, is the result of intricate coupling of various competing mechanisms. On one hand, larger mud-invaded region in the fracture (ℓ_f) is expected to generate larger shear resistance (proportional to ℓ_f) to mud flow into the fracture driven by pressure at the wellbore, and, therefore, would favour the cessation of the flow. On the other hand, larger ℓ_f can also imply higher net-loading on the fracture, which would result in widening of the fracture and, possibly, in initiating fracture propagation. The latter mechanism would serve to promote the flow and further mud-invasion. Yet another mechanism involved is related to the presence of the reservoir fluid in the fracture, which is displaced by the invading mud and is potentially pressurized by it to further increase the net-loading on the fracture and, possibly, initiate fracture propagation. The latter effect is expected to be particularly important in tight, low-permeability formations such as shales, where pressurization of the reservoir fluid in the fracture can not be effectively off-set by the leak-off.

This paper presents a mathematical model to study invasion of mud cake into a drilling-induced planar fracture at the edge of a wellbore perpendicular to the minimum in situ principal stress. The model assumes a planar edge-crack geometry loaded by the wellbore hoop stress, variable mud pressure along the invaded region adjacent to the wellbore, and uniform pore-fluid pressure along the rest of the crack (Figure 1). It is assumed that the invading mud freely displaces the pore-fluid in the crack without mixing with it. The case corresponding to sufficiently permeable formation (the pore-fluid pressure in the crack tip region ahead of the mud front is equal to a given ambient reservoir pressure, $p_{tip} = p_{amb}$) will be considered. The case of an impermeable formation (the pore-fluid pressure in the non-invaded part of the crack is one of the problem unknowns) and the case with transient leak-off of the pore-fluid from the crack into the formation is a subject of future work. The mud flow and its cessation is modeled using the lubrication theory and mud rheology characterized by a yield stress and post-yielding viscous behaviour. The changes in crack width and the condition for the initiation of fracture propagation are modeled under premises of the Linear Elastic Fracture Mechanics (LEFM).

The limiting equilibrium states of the mud cake in a stationary crack, when the cake is at its plastic threshold, are studied. These states correspond to either onset or cessation of the transient mud flow, and, as such, allow to predict conditions for the initiation of the fracture propagation. The ensuing continuous propagation driven by viscoplastic mud in limit equilibrium, although not a part of this paper, can be addressed by adopting the methods developed by Garagash (2006) for different inlet boundary condition (mud flux, instead of the mud pressure, specified at the crack inlet).

2 MATHEMATICAL MODEL

We consider a pre-existing crack of length ℓ at the edge of a wellbore in elastic permeable rock. The crack plane is perpendicular to the direction of the minimum in-situ stress, Figure 1a.

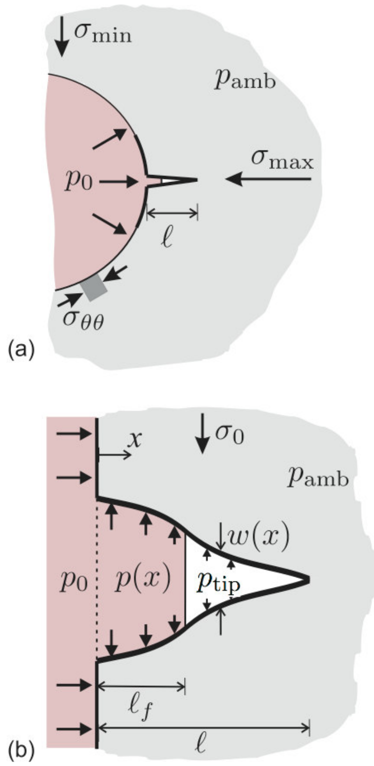

Figure 1. (a) Crack at the edge of a wellbore partially invaded by the mud cake. (b) Magnification of the crack.

The fluid (mud cake) invades a priori unknown part of the crack, $x \le \ell_f$, from the wellbore under a given mud pressure $p(0,t) = p_0(t)$. When the crack is *partially* filled with mud $(\ell_f < \ell)$ the tip region ahead of the wellbore fluid front is filled with the reservoir pore fluid at ambient pressure, $p_{tip}(t) = p_{amb}$. Otherwise $(\ell_f = \ell)$, the tip pressure is an unknown of the problem. Under condition when the length of the crack is much smaller than the wellbore radius, the approximation of a crack at the edge of a half-plane is considered, Figure 1b, where the remote confining stress

$$\sigma_0 = 3\sigma_{\min} - \sigma_{\max} - p_0 \qquad [1]$$

is equivalent to the Lame's hoop stress $\sigma_{\theta\theta}$ at the wellbore edge (Figure 1a).

2.1 Mud Cake Equilibrium Considerations

Mud-cake is considered as a viscoplastic fluid with yield stress τ_o. Fluid flow in the crack channel is characterized by the maximum and minimum (zero) shear stress values at the crack walls, $z = \pm w/2$, and the crack symmetry plane, z = 0, respectively (axis z is perpendicular to the crack plane). Consequently, fluid flow (if any) takes place along the part of the crack channel, $w_o/2 < |z| < w/2$, where the shear stress exceeds the plastic threshold, while the inner core $|z| \le w_o/2$ with $\tau(z) \le \tau_o$ is translated as a rigid body with the outer flow. Integrating equilibrium equations across the rigid core allows to relate the core thickness to the local pressure gradient (e.g, Economides and Nolte 2000)

$$w_o(x) = \frac{2\tau_o}{-dp/dx} \qquad [2]$$

The flow of the viscoplastic fluid in the crack channel takes place when the local crack width exceeds the rigid core width given by Eq. 2. On the other hand, the flow cessation or, alternatively, mobilization corresponds to the state of fluid *limit equilibrium*, when the width of rigid core just coincides with the crack width, $w(x) = w_o(x)$.

2.2 Crack Opening

Crack opening is related to the net normal loading $p(x) - \sigma_0$ on the crack walls by an integral equation of the linear elasticity theory, which, for an edge crack, can be written in the following form

$$w(x) = \int_0^{\ell} G\left(\frac{x}{\ell}, \frac{s}{\ell}\right) \frac{p(s) - \sigma_0}{E'} ds, \qquad [3]$$

where E' is the plane strain elastic modulus. Kernel G is defined by the following expression

$$G(\xi, \eta) = \frac{8}{\pi} \int_{\max\{\xi,\eta\}}^{1} \frac{f(\xi/\zeta)}{\sqrt{\zeta^2 - \xi^2}} \frac{f(\eta/\zeta)}{\sqrt{\zeta^2 - \eta^2}} \zeta d\zeta \qquad [4]$$

in terms of the "configurational" function for an edge crack, $f(\xi) \approx 1.297 - 0.297\xi^{5/4}$ (Tada et al. 2000).

Integrating Eq. 3 by parts leads to the form

$$\frac{w(x)}{\ell} = \int_0^{\ell_f} J\left(\frac{x}{\ell}, \frac{s}{\ell}\right) \frac{1}{E'} \frac{dp}{ds} ds + \frac{p_0 - \sigma_0}{E'} J\left(\frac{x}{\ell}, 0\right), \qquad [5]$$

where $J(\xi, \eta) = \int_{\eta}^{1} G(\xi, \eta)\, d\eta$. In Eq. 5, the upper integration limit has been set at the mud cake front position since the pressure gradient is zero in the tip region ahead of it.

2.3 Crack Propagation Condition

Asymptotics of deformation and stress near the crack tip loaded by normal tractions can be expressed in terms of the stress intensity factor (SIF) defined by (e.g., Rice 1968)

$$K_I = \sqrt{\pi l}\,\frac{2}{\pi}\int_0^l \left(p(x)-\sigma_0\right)\frac{f(x/l)}{\sqrt{l^2-x^2}}\,dx \qquad [6]$$

where f is the previously defined configurational function for an edge crack.

Adopting the Linear Elastic Fracture Mechanics (LEFM) theory, initiation of crack propagation requires that the stress intensity factor reach the critical value K_{Ic} (rock fracture toughness parameter), $K_I = K_{Ic}$. The latter "breakdown condition" can be recast in terms of the rock "tensile strength" σ_c for an edge crack as follows

$$K_I / (1.121\sqrt{\pi l}) = \sigma_c, \qquad [7]$$

where $\sigma_c \equiv K_{Ic} / (1.121\sqrt{\pi l})$. In view of Eq. 6, the left hand side of Eq. 7 is a particular average of the net pressure along the crack. Tensile strength is dependent on the crack length and, thus, it can not be regarded as a rock material parameter. In spite of this, the advantage of the "tensile strength notation" is realized in a special case when the net pressure is uniform in the crack, i.e. $p(x)-\sigma_0$ is constant along the crack (and equal to the average defined by the left hand side of Eq. 7). Two well-known limiting breakdown scenarios corresponding to the uniform net pressure correspond to: (i) the crack filled with reservoir pore fluid under ambient conditions, $p_{amb}-\sigma_0=\sigma_c$, (Hubert and Willis 1957); and (ii) the crack filled by the equilibrated wellbore fluid with uniform net pressure distribution, $p_0-\sigma_0=\sigma_c$, (Fairhurst and Haimson 1967). In application to the problem at hand, the case (i) corresponds to the situation when wellbore mud does not enter the crack $(p_0 \le p_{amb})$ which is critically loaded by ambient reservoir pressure. The case (ii) does not apply to mud cake plugs, since the latter maintain a pressure gradient (Eq. 2), which contradicts the net pressure uniformity assumption of (ii).

3 SOLUTION FOR A PLUG IN LIMIT EQUILIBRIUM

Under conditions of the limit equilibrium, when the non-deforming mud cake core just coincides with the crack width, $w(x)=w_o(x)$, Eqs. 2 and 5 are solved for the net pressure $p(x)-\sigma_0$ distribution in mud cake plug $(x \le \ell_f)$, and length of the plug ℓ_f as a function of the inlet $p_0-\sigma_0$ and tip $p_{tip}-\sigma_0$ values of the net pressure.

3.1 Scaling

In order to facilitate the solution, it is convenient to introduce the nondimensional plug length ξ_f, net pressure Π, and crack opening Ω as follows

$$\xi_f = \frac{\ell_f}{\ell}, \quad \Pi = \frac{p-\sigma_0}{p_*}, \quad \Omega = \frac{w}{w_*} \qquad [8]$$

where p_* and w_* are the characteristic values of the net pressure and crack opening defined as a small fraction $\varepsilon = \sqrt{2\tau_o/E'}$ of the modulus E' and the crack length ℓ, respectively,

$$p_* = \varepsilon E' = \sqrt{2\tau_o E'}, \quad w_* = \varepsilon\ell = \ell\sqrt{2\tau_o/E'} \qquad [9]$$

Normalized solution (Eq. 8) is a function of normalized coordinate $\xi = x/\ell$ along the crack (distributions Π and Ω only) and of the values of the normalized pressure at the crack inlet and the tip,

$$\Pi_0 = \frac{p_0-\sigma_0}{p_*} = \frac{2p_0-(3\sigma_{\min}-\sigma_{\max})}{p_*},$$
$$\Pi_{tip} = \frac{p_{tip}-\sigma_0}{p_*} = \frac{p_{tip}+p_0-(3\sigma_{\min}-\sigma_{\max})}{p_*}, \qquad [10]$$

respectively, governed by normalized Eqs. 2 and 5:

$$\Omega = \frac{1}{-d\Pi/d\xi} \qquad [11]$$

$$\Omega(\xi) = \int_0^{\xi_f} J(\xi,\eta)\frac{d\Pi}{d\eta}d\eta + \Pi_0 J(\xi,0) \qquad [12]$$

and the tip boundary condition $\Pi(\xi_f \le \xi \le 1) = \Pi_{tip}$.

Normalized form of the breakdown condition, Eq. 7, states

$$\mathrm{K} = \mathrm{K}_c \qquad [13]$$
$$\mathrm{K} \equiv \frac{K_I}{1.121 p_*\sqrt{\pi l}} = 0.568\int_0^1 \frac{\Pi(\xi)f(\xi)}{\sqrt{1-\xi^2}}\,d\xi,$$
$$\mathrm{K}_c \equiv \frac{\sigma_c}{p_*} = \frac{K_{Ic}}{1.121 p_*\sqrt{\pi l}},$$

where K is the normalized SIF (Eq. 6) and K_c is the normalized toughness (tensile strength), respectively.

3.2 Numerical Method

Numerical method to solve the system of Eqs. 11-12 relies on the discretization of the mud plug extent into a set of elements (intervals) and adopting a piecewise constant pressure gradient approximation over this set. This approximation allows explicit evaluation of the elasticity integral for the opening, Eq. 12. Substituting resulting approximations for the opening and pressure gradient in Eq. 11 and evaluating it at the midpoints of the elements' set yields an algebraic system of equation in terms of the unknown values of the pressure gradient. This equation set is solved using the Newton's iterative method. Full details of the numerical method are given by Garagash (2009).

3.3 Solution Map and Loading Trajectory

Figure 2 shows the contour plot of the normalized length ξ_f of the plug and the normalized SIF K in the space of the normalized problem parameters: the inlet and tip values of the net pressure. Parametric regimes (I) and (II) correspond to a fully-open and partially-open crack (along its length), respectively, invaded by mud cake. (Zero SIF corresponds to smoothly closing crack walls at the tip – corresponding contour line separates the two regimes in Figure 2). Regime (III) corresponds to a closed crack (net pressure in the crack is negative) and regime (IV) corresponds to an open crack (net pressure in the crack is positive) which is not invaded by mud (inlet mud pressure is less than the reservoir ambient value). No limit equilibrium mud cake plugs exist in parametric regime (V) in a sense that it can not be reached in the space of parameters of Figure 2 by a continuous, quasi-static loading trajectory, as discussed further.

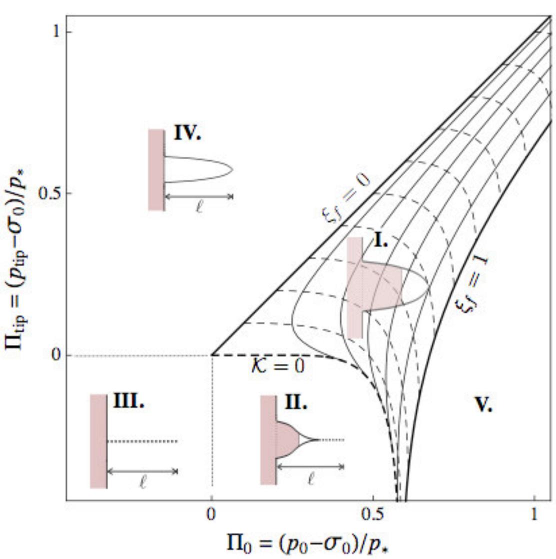

Figure 2. Map of limit equilibrium solutions in the parametric space of the inlet and tip net pressure. Solid lines - plug length contours ξ_f ={0, 0.1, 0.3, 0.5, 0.7, 0.9, 1}; and dashed lines - normalized stress intensity factor contours K ={0, 0.1, 0.2,…, 0.9, 1}.

When the crack is not fully-filled with mud cake, tip pressure is equal to the (constant) reservoir ambient value, $p_{tip} = p_{amb}$. Consequently, in view of Eq. 1, loading trajectory due to, say, increasing mud pressure in the wellbore, p_0, corresponds to a line with slope ½ in the parametric space of Figure 2

$$\xi_f < 1: \quad \Pi_{tip} = (\Pi_{amb} + \Pi_0)/2 \qquad [14]$$

where constant

$$\Pi_{amb} = \left(2p_{amb} - (3\sigma_{\min} - \sigma_{\max})\right)/p_* \qquad [15]$$

corresponds to the intercept of the loading trajectory with $\Pi_{tip} = \Pi_0$ line. (Two such trajectories are illustrated in Figure 3). Once the crack is fully-filled with the mud cake (when loading trajectory first intersection with the $\xi_f = 1$ line on Figure 2), loading trajectory follows the latter line of full-saturation, corresponding to further pressurization and inflation of the fully mud-filled crack.

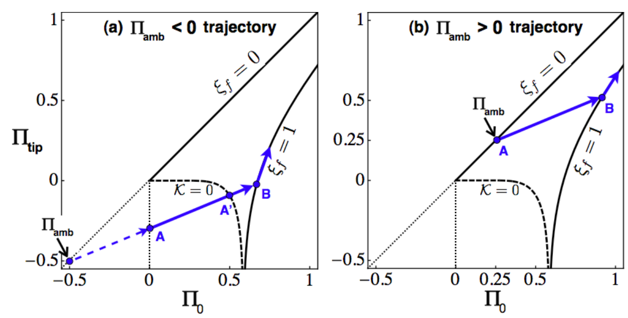

Figure 3. Examples of (a) $\Pi_{amb} < 0$ and (b) $\Pi_{amb} > 0$ loading trajectories in the parametric space of Figure 2. State A – incipient mud plug $(\xi_f = 0)$; State A' – the first state of a fully-open crack $(\mathrm{K} = 0)$; State B – the first state of the full mud plug $(\xi_f = 1)$.

Figure 3 shows two representative loading trajectories. Positive Π_{amb} trajectory corresponds to gradual filling of the initially *open* crack with mud followed by further crack pressurization along the full-saturation line $\xi_f = 1$. Negative Π_{amb} trajectory corresponds to filling of initially *closed* crack (once the inlet net pressure raised to exceed the zero value) , which remains partially closed (Region II on Figure 2) until the trajectory reaches the $\mathrm{K} = 0$ line. Past that point, the gradual filling of now fully-open crack (Region I on Figure 2) continues until the line of full-saturation is reached and followed along.

Figures 4 and 5 show the evolution of the net pressure and the crack opening profiles for the two loading trajectories depicted in Figure 3a and 3b, respectively.

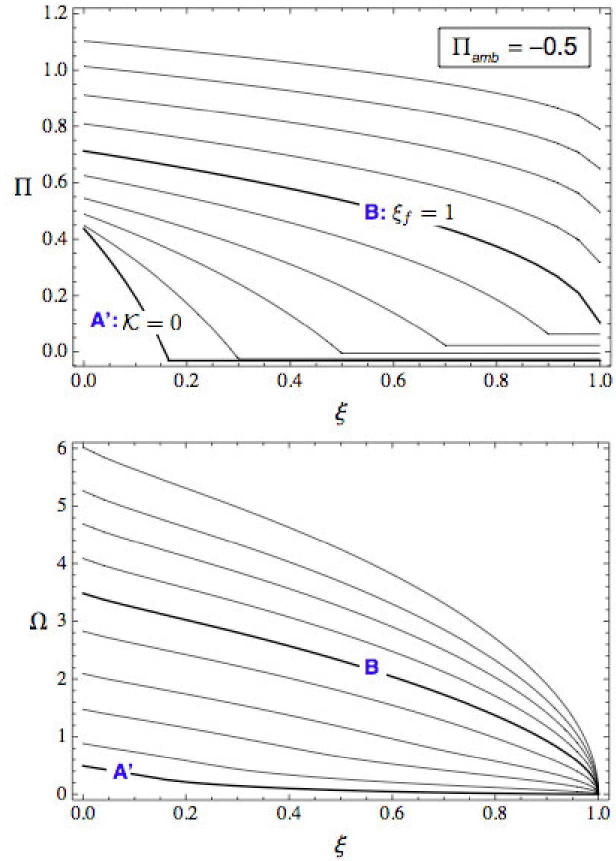

Figure 4. Example of the evolution of the normalized net-pressure and crack opening profiles along a $\Pi_{amb} < 0$ loading trajectory (Figure 3a).

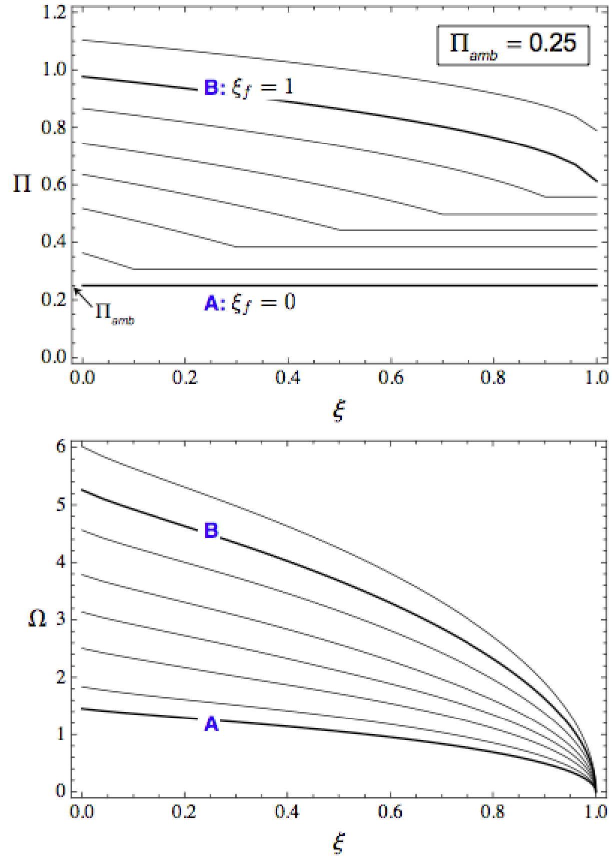

Figure 5. Example of the evolution of the normalized net-pressure and crack opening profiles along a $\Pi_{amb} > 0$ loading trajectory (Figure 3b).

3.4 Breakdown Conditions

Toughness or tensile strength based criterion, Eq. 13, for the initiation of crack propagation can be readily evaluated for the equilibrium states of the pressurized fluid in the crack based on Figure 2. Namely, given a specific value of the dimensionless toughness or, equivalently, normalized tensile strength, K_c, the corresponding $\mathrm{K} = \mathrm{K}_c$ curve in Fig. 2 provides the set of critical normalized values of the inlet and tip net pressure to initiate propagation of the crack. The latter values are not independent, but rather related via a fixed loading trajectory parameterized by number Π_{amb}, Eq. 15. In view of that, the SIF solution is recast on Figure 6 as a function of number $\tfrac{1}{2}\Pi_{amb} = (p_{amb} - \tfrac{1}{2}(3\sigma_{\min} - \sigma_{\max})) / p_*$ (which characterize the ambient pore pressure and in situ stress) and normalized pressure difference (overbalance) between the wellbore mud pressure and ambient reservoir pore pressure), $\tfrac{1}{2}(\Pi_0 - \Pi_{amb}) = (p_0 - p_{amb}) / p_*$. A loading trajectory in the space of Figure 6 is given by a vertical line corresponding to a specified ambient state. The crack is fully-filled with the mud at and above the $\xi_f = 1$ line.

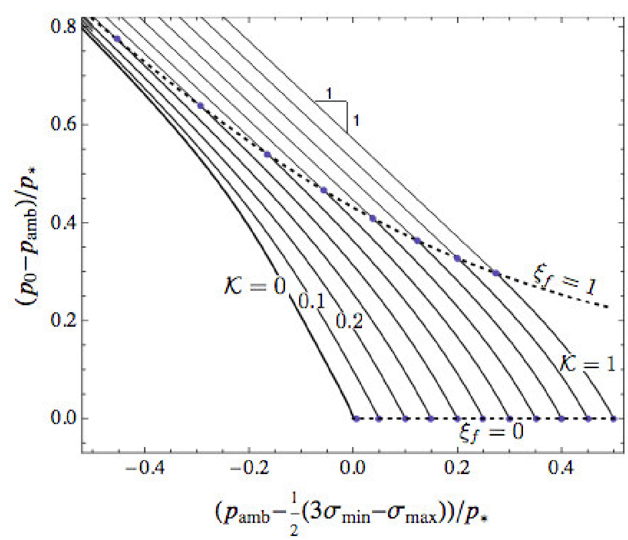

Figure 6. Contours of the normalized stress intensity factor $\mathrm{K} = K_I / (1.121 p_* \sqrt{\pi \ell})$ in the space of the ambient pressure measure $p_{amb} - \tfrac{1}{2}(3\sigma_{\min} - \sigma_{\max})$ and mud

pressure overbalance $p_0 - p_{amb}$. Dashed lines show the incipient mud plug ($\xi_f = 0$), and the full mud plug ($\xi_f = 1$).

If, for a given ambient pressure and in situ stress conditions (value along the horizontal axis of Figure 6), the wellbore pressure (vertical axis of Figure 6) is increased to reach the breakdown curve $\mathrm{K} = \mathrm{K}_c$, the crack propagates indefinitely (i.e., it can not be stopped) as long as the wellbore pressure is maintained at or above the breakdown level. To show that, consider the case when the wellbore pressure is maintained at the constant (breakdown) value during the crack propagation, which is assumed to be slow enough so that the mud maintains the limit plastic equilibrium (which normalized solution is identical to that for the breakdown state). Then, the normalized stress intensity factor K is constant, while the normalized toughness K_c decreases with growing crack length, Eq. [13], therefore resulting in unstable propagation. (In terms of dimensional quantities, the SIF K_I is increasing function of the length during propagation, while the dimensional rock toughness K_{Ic} is material constant). The identified unstable propagation is an artefact of the assumed limit plastic mud equilibrium. In this case, the mud shear stress will exceed the plastic threshold, resulting in a viscoplastic transient mud flow in the propagating fracture. The latter corresponds to more effective pressure dissipation/redistribution along the fracture which lowers the normalized SIF (from its limit equilibrium value) to maintain the stable propagation condition, $\mathrm{K} = \mathrm{K}_c$. It is worthwhile to mention that stable fracture propagation with the mud in the limit plastic equilibrium is possible when the wellbore pressure is decreased past the breakdown, such as in the case of a crack driven by a (non-Newtonian) fluid injected at the crack inlet at a constant volumetric rate (Adachi and Detournay 2002; Garagash 2006).

In the following we consider an example evaluation of the breakdown condition based on Figure 6. Consider the following set of the rock and fracture parameters: $E' = 3$ GPa, $K_{Ic} =$ 1 MPa√m, $\ell =$ 10 cm; and the mud cake yield strength $\tau_o = 0.01$ MPa. Then characteristic pressure and crack opening, Eq. 9, are $p_* =$ 7.75 MPa and $w_* = 0.26$ mm, respectively, and the normalized toughness is $\mathrm{K}_c = 0.2$. Using corresponding normalized SIF contour on Figure 6 and applying above value of the characteristic pressure, the minimum value of drilling overbalance, $p_0 - p_{amb}$, to cause fracture propagation can be determined for a given ambient state of the reservoir pore pressure and stress. Say, if reservoir conditions are characterized by $p_{amb} - \tfrac{1}{2}(3\sigma_{\min} - \sigma_{\max}) \le -1\,\mathrm{MPa}$ $(\tfrac{1}{2}\Pi_{amb} \le -0.13)$, then the minimum value of the pressure overbalance for the onset of crack propagation is 2.94 MPa $(= 0.38 p_*)$. Corresponding maximum fraction of the crack length invaded by the mud cake prior to propagation is 0.525.

CONCLUSIONS

Physical and corresponding mathematical model of the equilibrium of a viscoplastic mud cake in a partially invaded crack at the edge of a wellbore has been formulated. The scaling of the problem makes use of the characteristic values of pressure and crack opening defined as $p_* = \sqrt{2\tau_o E'}$, and $w_* = \ell\sqrt{2\tau_o / E'}$, respectively, to express the normalized net pressure of the mud cake, $(p - \sigma_0)/p_*$, the normalized crack opening w/w_*, the normalized extent of the mud cake plug $\xi_f = \ell_f / \ell$, and the normalized stress intensity factor $K_I / (1.121 p_* \sqrt{\pi \ell})$ at the crack tip as a function of two parameters: the normalized net fluid pressure at the inlet, $(p_0 - \sigma_0)/p_*$, and at the tip, $(p_{tip} - \sigma_0)/p_*$, of the crack, respectively. For a crack partially filled with mud cake ($\xi_f < 1$), the pressure at the tip is equal to the ambient pore pressure value p_{amb}, which defines a linear loading trajectory (as the inlet pressure is continuously increased) in the space of the inlet and tip values of the net pressure, parameterized by single number $(p_{amb} - \sigma_{amb})/p_*$. (Here $\sigma_0 = 3\sigma_{\min} - \sigma_{\max} - p_0$ is the wellbore hoop stress, and σ_{amb} is its value when the wellbore pressure is equal to the ambient pore pressure value, $p_0 = p_{amb}$). For a crack fully plugged with the mud-cake ($\xi_f = 1$), the net pressure at the tip is at or above the ambient level, and is given by the unique function of the inlet net-pressure, which defines the continuation of the loading trajectory.

The presented solution allows to evaluate the toughness or tensile strength based criterion for initiation of crack propagation, which may lead to uncontrollable loss of mud circulation in a well. Specifically, this investigation provides pertinent information on the breakdown pressure (wellbore pressure at the initiation of crack propagation) as a function of the rock ambient stress, ambient pore pressure, pre-existing crack length, and mud cake properties.

The theoretical framework suggested in this paper to model the mud cake invasion into a pre-existing wellbore crack under particular constraints on the crack geometry and fluid exchange between the crack and the permeable rock can be extended to other relevant cases. Some possible extensions include (i) the case of a longer crack, i.e. a crack which is not small compared to the wellbore radius, relevant to some extreme cases of mud losses associated with mud-driven fracture (Bratton et al. 2001); and (ii) different fracture geometry in response to different minimum in situ stress orientation (e.g., penny-shape crack when the minimum in situ stress is aligned with the well).

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