A numerical study of the effects of calibration chamber boundary conditions on pressuremeter test results in sand

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ABSTRACT
A numerical finite difference model of pressuremeter test is used in this paper to study the effects of different boundary conditions on pressuremeter test results in a calibration chamber of sand. Results show that for small chamber to probe diameter ratio, boundary conditions have major effect on pressuremeter results. It is also shown that for dense sands the effect of boundary condition is much more significant than loose sands.

RÉSUMÉ
Un modèle numérique de différence finie pressiomètre test est utilisé dans le présent document pour étudier les effets de différentes conditions aux limites sur pressiomètre les résultats des tests dans une chambre d'étalonnage de sable. Les résultats montrent que pour la petite chambre de sonde diamètre, des conditions limites ont des incidences majeures sur les résultats pressiomètre. Il est également montré que pour les sables bitumineux dense l'effet de la condition à la limite est bien plus importante que les sables lâches.

1 INTRODUCTION

Since it is very difficult to obtain undisturbed samples from sand, application and interpretation of in-situ tests to determine characteristics of soil is very popular. The pressuremeter test, especially self-boring pressuremeter test, is believed to be one of the best in-situ tests available to obtain some of the soil properties, such as soil moduli, friction angle, and cohesion coefficient.

Gibson and Anderson (1961) proposed a method to determine friction angle of soil from pressuremeter test assuming zero volume change of the soil. Vésic (1972) developed another method to include volume change of the soil in the analysis. Hughes et al. (1977) developed an analytical method to determine friction and dilation angle of soil using pressuremeter test results.

Later, Carter et al. (1987) proposed a closed form solution for the problem in cohesive frictional soils. Yu and Houlsby (1991) have proposed a solution for the problem of cavity expansion in a dilatant soil. Salgado and Randolph (2001) developed an algorithm using various friction and dilation angles to study the cavity expansion problem.

The method suggested by Hughes et al. (1977) is believed to be one of the best methods to interpret the results of pressuremeter test. They proposed a relationship between applied radial effective stress (σ'_r) and radial strain (ε_r) based on small strain theory:

$$\log(\varepsilon_r + \frac{c}{2}) = \frac{n+1}{1-N}\log(\sigma'_r - u_0) + \text{constant} \qquad [1]$$

where:

$$N = \frac{1-\sin\phi'}{1+\sin\phi'} \qquad [2]$$

and

$$n = \frac{1-\sin\psi}{1+\sin\psi} \qquad [3]$$

in which ϕ' is the soil friction angle, ψ is the soil dilation angle, and *c* is an experimental constant which is the intercept of the volumetric strain versus engineering shear strain plot.

According to Eq. 1, in logarithmic scale, probe pressure curve against radial strain is a straight line with a slope defined as below:

$$s = \frac{1-N}{n+1} = \frac{(1+\sin\psi)\sin\phi'}{1+\sin\phi'} \qquad [4]$$

To calibrate in-situ test devices, some chambers have been developed which are called calibration chambers. In these chambers a soil sample with known properties is tested by means of an in-situ device such as pressuremeter (Bellotti et al. (1982, 1989); Ghionna et al. (1990); Manassero (1989)). Relations can be found between pressuremeter test results and soil properties in the chamber. Performing a pressuremeter test in the field, one can interpret soil properties based on the relationships obtained in the chamber. However, calibration chamber has finite size, and the boundary condition imposed on the sample may influence the pressuremeter test results in the chamber. Thus, there may be some differences between the results of pressuremeter test in the field and in the calibration chamber. This has been reported by several researchers, such as Fahey (1980) and Schnaid and Houlsby (1991). For example, the fixity of the lateral boundary condition may induce larger lateral stresses than the in-situ horizontal stress. On the other hand, a constant stress on the boundaries may result in larger displacements on the lateral boundaries compared to field testing of sand. Therefore, many attempts have been performed to examine and formulate the effect of calibration chamber boundary condition on the test results.

A boundary may be fixed, that is, it cannot have any displacement, or a constant stress may be applied to it. Since there are top and bottom boundary and lateral boundary in a chamber, four types of boundary conditions may exist. Figure 1 and Table 1 describe these four boundary condition types of a chamber.

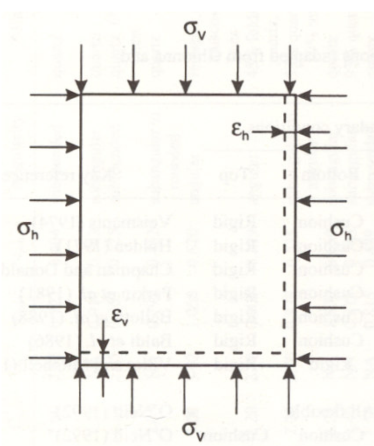

Figure 1 - Illustration of calibration chamber boundaries

Table 1- Definition of calibration chamber boundary conditions

Boundary Condition Type	Top and Bottom Boundary Condition	Lateral Boundary Condition
BC1	σ_v[1] = Constant	σ_h[2] = Constant
BC2	ε_v[3] = 0	ε_h[4] = 0
BC3	σ_v = Constant	$\varepsilon_h = 0$
BC4	$\varepsilon_v = 0$	σ_h = Constant

[1] σ_v= vertical stress [2] σ_h= horizontal stress
[3] ε_v= vertical strain [4] ε_h= horizontal strain

Schnaid and Houlsby (1991), using both calibration chamber test results and numerical modeling, show that for BC1, pressuremeter limit pressure is increased as chamber to probe diameter ratio increases. Furthermore, from their results it can be concluded that for different chamber to probe diameter ratios, as chamber to probe diameter ratio increases, pressuremeter limit pressures converge to a constant value.

In this study the effect of different boundary conditions on pressuremeter results is discussed, and the numerical analyses performed in this study are compared with other results. The continuum based FLAC program is used for all numerical analyses involved in this study.

2 MODEL SPECIFICATION

Pressuremeter test model is a typical axisymmetric cavity expansion problem. Finite length of pressuremeter may cause some irregularities in the results of the test (Ajalloeian and Yu 1998). However, since the main goal of this paper is to study boundary condition effect and pressuremeter geometry (length) effect is present for all cases studied here, this effect is ignored and it is assumed that the cavity is infinitely long. This assumption can simplify the modeling to a great extent.

In the numerical analysis performed in this study, an axisymmetric model is used. A schematic of the model showing the four boundary conditions is illustrated in Figure 2. These boundary conditions are consistent with the definition provided in Table 1.

Figure 2 also shows the mesh used for numerical analysis. The length (height) of the pressure meter is L, the cavity radius is r_c, and the chamber radius is R_c. In order to explore the effect of cavity expansion in the soil, the zones near the cavity are much smaller than the far ones.

In all of the analyses performed in this study, Mohr-Coulomb model is used as the constitutive law governing the problem. An initial vertical stress of 100 kPa and an initial horizontal stress of 50 kPa are used, i.e. coefficient of earth pressure at rest is equal to 0.5.

The Mohr-Coulomb elastic properties are defined in (5) and (6), meaning that a stress-dependent stiffness is used for the model as proposed by Ahmadi et al. (2005) for sandy soils.

$$G = K_G P_A \left(\frac{\sigma'_m}{P_A} \right)^n \quad [5]$$

$$B = K_B P_A \left(\frac{\sigma'_m}{P_A} \right)^m \quad [6]$$

In these equations:
G: Soil shear modulus
B: Soil bulk modulus
K_G: Shear modulus number, a function of relative density
K_B: Bulk modulus number, a function of relative density
P_a: Atmospheric pressure (1 kg/cm^2 = 98.1 kPa)
σ'_m: Mean effective stress
n, m: empirical numbers (with a range of 0.2 to 0.7, 0.5 is assumed here)

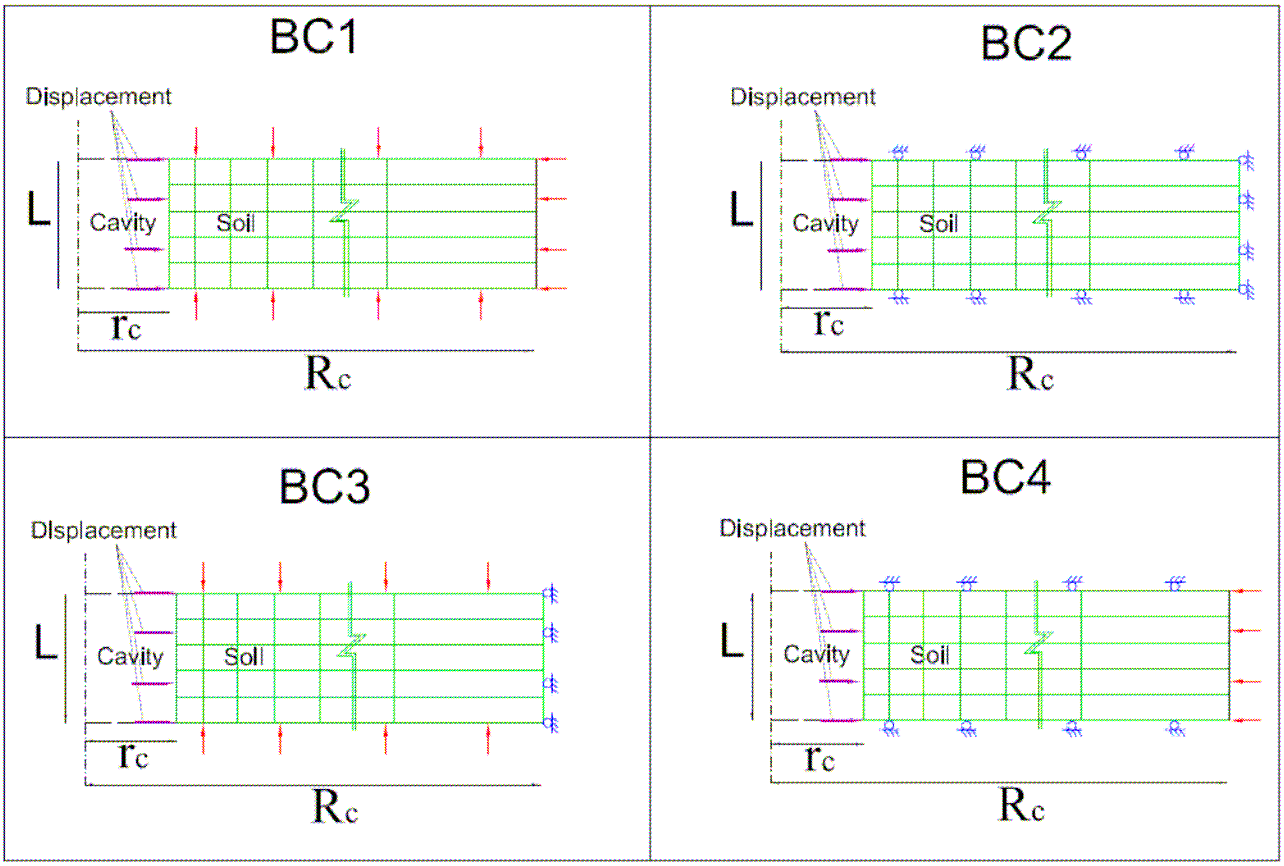

Figure 2 – Schematic of the boundary conditions

Proposed values for shear modulus number and bulk modulus number for different relative densities of a typical sandy soil are presented in Table 2 (Ahmadi (2000)):

Table 2 – Proposed values for shear modulus number and bulk modulus number (Ahmadi (2000))

Sand state	D_r	K_G	K_B
Loose	45 %	195	325
Medium dense	65 %	230	385
Dense	85 %	290	480

3 VERIFICATION OF THE MODEL

To verify the model, comparison is made with the results obtained by Hughes et al. (1977). Therefore their assumption of small strain theory was implemented in the analysis. In order to ensure that the lateral boundary conditions do not affect the results of the model, a chamber to cavity diameter ratio (R_D = R_c/r_c) of 500 is used. At this ratio, lateral boundary condition of the soil is so far that radial strain of the farthest zone from cavity will be almost zero and therefore for constant lateral stress boundary condition the theoretical criterion is met, which states that at infinity radial displacement of soil is zero (Briaud 1992). To compare with results of Hughes et al. (1977) method, results are plotted as radial effective stress induced by expansion of cavity against radial strain in the first zone, in logarithmic scale (a typical plot is illustrated in Figure 3).

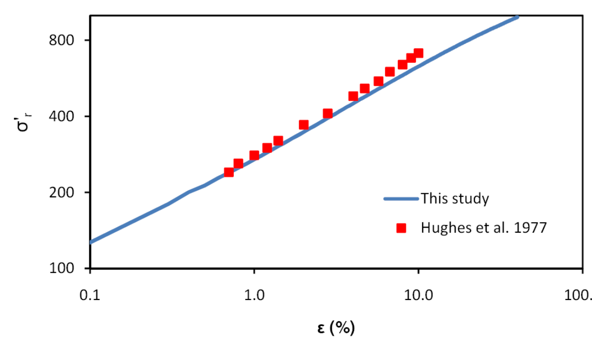

Figure 3 – Comparison of the model and results of a pressuremeter test in the Wash sand (Hughes et al. 1977)

In Figure 3, the properties of the sample of the Wash sand described by Hughes et al. (1977) are implemented in the numerical model of pressuremeter test. Radial effective stress resulted from numerical modeling is plotted against radial strain in logarithmic scale, to provide a means to compare the numerical model and Hughes el al. (1977) experimental data. Referring to Figure 3, there is a good agreement between predictions in this study and the experimental results on Wash sand reported by Hughes et al. (1977).

To study the performance of the model used in different states (different friction and dilation angles), in this study the slope of the curve of radial effective stress versus radial strain is obtained by both Hughes et al. (1977) method (s_1) and the present model (s_2). The range of friction angle is from 30^0 to 42^0. Friction angle of the soil in critical state (ϕ_{crit}) is assumed to be 33^0. Therefore, according to Bolton (1986) (Eq. 7) the range of dilation angle (ψ) is between -3.75^0 and 11.75^0.

$$\phi' - \phi_{crit} = 0.8\psi \quad [7]$$

in this equation, a positive value for dilation angle (ψ) means expansion, and a negative value means contraction of the sandy soil.

In Table 3, the magnitudes of the slopes obtained by both methods are presented and the error of the model with reference to Hughes et al. (1977) method is calculated by Eq. 8:

$$\text{Error} = \left|\frac{s_2 - s_1}{s_1}\right| \quad [8]$$

Table 3 – Comparison of model using small strains to Hughes et al. (1977) model

ϕ (^{0})	30	33	36	39	42
s_1 (Hughes et al.,1977)	0.312	0.353	0.394	0.437	0.479
s_2 (This study)	0.303	0.344	0.382	0.422	0.463
Error (%)	3	2	3	3	3

Table 3 indicates that the maximum error of the model with reference to Hughes et al. (1977) method is about 3% which is mostly due to the assumption of empirical coefficient c in their model. Magnitude of the difference shows that the numerical model agrees well with Hughes et al. (1977) solution.

However, it can be argued that the small strain assumption is not quite appropriate for simulating pressuremeter test. During pressuremeter testing, soil strains around the cavity are usually large (Hughes el al. (1977), Baguelin et al. (1978)) and hence, modeling based on small strain theory may have some shortcomings.

Table 4 shows the predicted slopes of the curves of radial effective stress against radial strain (in logarithmic scale) obtained by numerical analysis implementing large strain theory and compares them with slopes obtained by Hughes et al. (1977) method.

Table 4 – Comparison of model using large strains to Hughes et al. (1977) model

ϕ (^{0})	30	33	36	39	42
s_1 (Hughes et al.,1977)	0.312	0.353	0.394	0.437	0.479
s_2 (This study)	0.265	0.302	0.336	0.370	0.409
Error (%)	18	17	17	18	17

Table 4 shows that by implementing a large strain analysis, the maximum difference between predicted values of the slope of the curves in this study and those by Hughes et al. (1977) is 18%.

Comparison of Table 3 and Table 4 indicates that the differences between predicted values of the slope of the curves in this study and those by Hughes et al. (1977) are much larger when using large strain analysis than when small strain analysis is implemented. Since all predicted values of the slopes by large strain theory are smaller than the values predicted by small strain analysis, it can be concluded that during the numerical analysis, stiffness of the soil decreases more rapidly in large strain analysis. This has been investigated by observing the soil stiffness in both small strain and large strain analyses carried out in this study. The comparison of the stiffness of the soil in these analyses proves that this conclusion is correct.

4 BOUNDARY CONDITION EFFECT

To study the effect of boundary condition on pressuremeter test results, three series of analyses lave been performed based on sand relative densities. These series of analyses correspond to: a) dense sand with friction angle of 42^0 and dilation angle of 11.25^0; b) medium dense sand with friction angle of 36^0 and dilation angle of 3.75^0; c) loose sand with friction angle of 30^0 and dilation angle of -3.75^0. For all three sand models, friction angle in critical state is assumed to be 33^0. All four boundary conditions are modelled in the numerical analyses. Chamber to probe diameter ratios ($R_D = R_c/r_c$) ranging from 10 to 500 are examined.

Limit pressure is chosen as a measure to compare the numerical results for different boundary conditions. Limit pressure is the pressure of the probe when the cavity volume becomes twice as large as its initial volume.

In Figure 4, the results of pressuremeter numerical model are illustrated as limit pressure versus chamber to probe diameter ratio for different boundary conditions.

Figure 4(a) illustrates the results of the numerical model for dense sand. It is observed that for BC1 boundary condition, the limit pressure increases as chamber to probe diameter ratio increases. For a chamber to probe diameter ratio of more than about 80, increase of this ratio does not affect limit pressure and the model result stays constant.

The trend of BC4 boundary condition is very similar to the trend of BC1 boundary condition. For all chamber to probe diameter ratios, limit pressure for BC1 and BC4 boundary conditions are almost the same.

However, the trends of BC2 and BC3 boundary conditions are quite the opposite. For these two boundary conditions, limit pressure decreases as chamber to probe diameter ratio increases. For a chamber to probe diameter ratio of more than about 80, decrease of limit pressure stops and it will be almost constant as chamber to probe diameter ratio increases.

It is seen that for all boundary conditions after a chamber to probe diameter ratio, the limit pressure curves for different boundary conditions converge to a value for chamber to probe diameter ratio greater than 80. This value is the same for all boundary conditions. This implies that for large chamber to probe diameter ratios, effects of different boundary conditions are so small that it can be neglected and all boundary conditions represent the same condition, which from now on in this paper, is referred to "the field" condition. However, the limit pressure of the field condition is different for different sand states, i.e. for dense sand the field limit pressure is 1091 kPa, while for medium dense and loose sand the field limit pressures are 672 and 425 kPa, respectively.

Referring to Figure 4, it can be seen that for medium dense (Figure 4(b)) and loose sand (Figure 4(c)), the trends observed for dense sand are also present. However, some differences can be found.

As the soil becomes looser, the limit pressure for the field condition becomes smaller. In addition, the difference between the limit pressures of different boundary conditions for a specific chamber to probe diameter ratio decreases. This will cause the curves to converge in smaller chamber to probe diameter ratios. For example, for dense sand the limit pressure of different boundary conditions become almost the same for chamber to probe diameter ratio of about 80. This ratio is about 60 for medium dense sand and about 40 for loose sand. Therefore, it can be concluded that as the soil becomes looser, the field condition may be simulated by smaller chambers.

Considering the definition of boundary conditions, Figure 4 indicates that the most important feature of a boundary condition concerning pressuremeter test results is its lateral boundary. As stated before, the trend of BC1 and BC4 is alike while trend of BC2 resembles the trend of BC3. As illustrated in Figure 1 and stated in Table 1, constant stress is applied at lateral boundaries of BC1 and BC4 boundary conditions, while for BC2 and BC3 boundary conditions lateral boundaries are fixed.

Figure 4 indicates that fixed lateral boundary condition (in which lateral displacement of the soil is prevented) leads to limit pressures larger than the field limit pressure for small chamber to probe diameter ratios, while having constant pressure applied to lateral boundaries (BC1 and BC4 boundary conditions) results in smaller limit pressures than the field limit pressure in small chamber to probe diameter ratios.

The finings here are consistent with previous studies of several researchers. Schnaid and Houlsby (1991) using experimental data and numerical analysis, provided some chart to study the behaviour of pressuremeter test in BC1 calibration chamber. The trend they have found is very similar to the trend of BC1 described above.

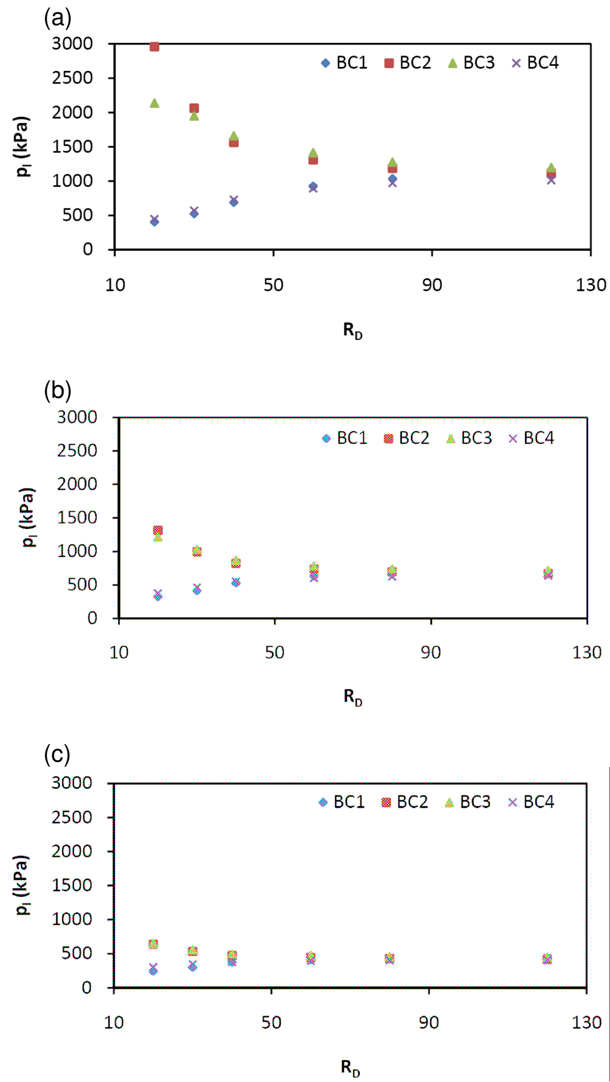

Figure 4 – Boundary condition effect of calibration chamber on pressuremeter test in (a) Dense sand, (b) Medium dense sand, (c) Loose sand

Moreover, Schnaid and Houlsby (1992) suggested that for cone pressuremeter test, chamber size has a relatively small influence on cone tip resistance to limit pressure ratio. This means that chamber size has a similar effect on both cone penetration test and pressuremeter test performed in a calibration chamber.

Comparing the results of this study (Figure4) to the results of Ahmadi and Robertson (2008), it can be seen that the trend of the curves for different boundary conditions is very similar, despite the fact that the former is regarded to calibration of pressuremeter test while the latter is concluded from a numerical study of cone penetration test in calibration chamber. The similarity of these results is consistent with Schnaid and Houlsby (1992) suggestion that for cone pressuremeter test, chamber size has a small influence on cone resistance to limit pressure ratio.

5 CONCLUSION

A numerical study of pressuremeter test in calibration chamber has been performed. It was shown that the model performs well compared to available methods.

After validation of the model, effect of boundary condition of calibration chamber on the results of the numerical model of pressuremeter test was studied for three sands with different relative densities: dense, medium dense and loose.

Results of the model imply that for all sand relative densities, the curves of limit pressure versus chamber to probe diameter ratio, have similar trends for different boundary conditions. For BC2 and BC3 boundary conditions, the trends are similar in which as the chamber to probe diameter ratio increases, limit pressure decreases until it reaches a constant value. For BC1 and BC4 boundary conditions the opposite holds true. However the constant limit pressure value is the same for all four types of boundary conditions.

It is also shown that for looser sands, the constant value of limit pressure is reached in smaller chamber to probe diameter ratios. This means that boundary conditions are less influential in loose sands.

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