Resistance factors for design of deep foundations in undrained soils

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ABSTRACT
The design of a pile involves two limit states: a serviceability limit state, which generally involves settlement or differential settlement limits, and an ultimate limit state. This paper proposes a Load and Resistance Factor Design (LRFD) approach for the ultimate limit state design of cohesive deep foundations. The load factors used are as specified by the National Building Code of Canada. The resistance factors required to achieve a certain acceptable failure probability are estimated as a function of the spatial variability of the soil. A mathematical theory was developed in this paper to analytically estimate the failure probability of deep foundations. The spatially random soil field was modeled using random field theory. The analytical results are validated by the simulation and then used to estimate failure probabilities and required resistance factors.

RÉSUMÉ
La conception d'un tas implique deux états de limite : un état de limite d'aptitude à l'usage, qui implique généralement le règlement ou les limites de règlement différentielles, et un état de limite ultime. Ce papier propose qu'un Chargement et la Résistance Factorise la Conception (LRFD) l'approche pour la limite ultime conception de fondations profondes cohésives. Le chargement factorise utilisé est comme spécifié par le Code de Bâtiment National de Canada. La résistance factorise a exigé atteindre une certaine probabilité d'échec acceptable est estimée comme une fonction de la variabilité spatiale du sol. Une théorie mathématique a été développée dans ce papier analytiquement pour estimer la probabilité d'échec de fondations profondes. Le champ spatialement de sol fait au hasard a été modelé l'utilisation théorie de champ faite au hasard. Les résultats analytiques sont validés par la simulation et ont estimé alors les probabilités d'échec et les facteurs de résistance exigés.

1 INTRODUCTION

Deep foundations, or piles, can fail either through excessive settlement, which is generally a serviceability limit state (SLS), or through a punching shear failure where the load applied to the pile exceeds the shear supporting strength of the surrounding soil (Fenton and Griffiths, 2007). The soil supports the pile through a combination of end-bearing, and friction and/or cohesion between the soil and the pile sides. In this paper, only cohesion resistance is considered, as would typically be found in an undrained soil, and end-bearing is ignored.

In piles designed to support loads in undrained soils, geotechnical engineers try to find the effective perimeter, p, and length, H, required to avoid a cohesive resistance failure. In this paper, it is assumed that the pile type is already know, so that p is known and the design involves determining H. To find H, ultimate limit state (ULS) conditions are checked using separate factors on loads and on ultimate geotechnical resistance. This leads to the load and resistance factor design (LRFD) methodology, collectively referred to as Limit States Design (LSD) in Canada, which requires that the factored geotechnical resistance at each limit state exceed the factored load. At the ultimate limit state, the design requirement is

$$\varphi_g \hat{R}_u \geq I\left(\alpha_T \hat{L}_T\right) \qquad [1]$$

where φ_g is the geotechnical resistance factor, $\hat{R}_u$ is the characteristic (design) ultimate geotechnical resistance, I is an importance factor, α_T is the total load factor and $\hat{L}_T$ is the total characteristic load, also referred to as the design load. The resistance factor, φ_g, is typically less than 1.0 and accounts for uncertainties in geotechnical parameters (Allen, 2005). The load factor, α_T, is typically greater than 1.0 and accounts for uncertainty in loads.

The ultimate geotechnical resistance, $\hat{R}_u$, is determined using characteristic soil properties, in this case the characteristic soil cohesion, $\hat{c}$. The importance factor, I, reflects the severity of the failure consequences and maybe larger than 1.0 for important structures, such as hospitals, whose failure consequences are severe and whose probabilities of failure should be much less than typical structures. Typical structures are usually designed using $I=1$, which will be assumed in this paper. Structures with low failure consequences may have $I<1$ (NRC, 2006).

In this paper only dead and live loads will be considered, so that the total factored load is obtained from

$$\alpha_T \hat{L}_T = \alpha_D \hat{L}_D + \alpha_L \hat{L}_L \qquad [2]$$

where $\hat{L}_L$ is the characteristic live load, $\hat{L}_D$ is the characteristic dead load, and α_L and α_D are the live and dead load factors, respectively. The load factors used in this paper are as given by the National Building Code of Canada: $\alpha_L = 1.5$ and $\alpha_D = 1.25$ (NRC, 2005).

The characteristic loads, $\hat{L}_D$ and $\hat{L}_L$, are obtained by applying bias factors to the means of the load distribution: $\hat{L}_D = 1.18\mu_D$ (Allen, 1975) and $\hat{L}_L = 1.41\mu_L$ (Becker, 1996), where μ_D and μ_L are the means of the maximum lifetime dead and live loads, respectively.

While $\hat{R}_u$ is the characteristic ultimate resistance used in design, the actual ultimate resistance of a pile, R_u, due to the actual cohesion, c, between the pile surface and its surrounding soil is given by (Das, 2000),

$$R_u = \int_0^H p\tau(z)dz \qquad [3]$$

where p is the effective perimeter length of the pile section and $\tau(z)$ is the ultimate shear stress acting on surface of the pile at depth z. Several methods are available for obtaining the cohesive resistance of piles in undrained soils. One of the presently accepted procedures, the α - method, is described by Das (2000). According to the α - method, the unit surface shear resistance in undrained soils can be represented by the equation,

$$\tau(z) = \alpha c(z) \qquad [4]$$

where $c(z)$ is the soil cohesion at depth z and α is an empirical adhesion factor, typically in the range from 0.5 to 1 (CGS, 2006). For a normally consolidated clay, with cohesion less than 50 kPa, Das (2000) suggests an adhesion factor of 1.0, which will be assumed here. Combining Eq's 3 and 4, the cohesive ultimate resistance of a pile with length H and effective perimeter length p is given by

$$R_u = p\alpha\int_0^H c(z)dz \qquad [5]$$

where now $c(z)$ is the average cohesion around the pile perimeter at depth z.

In order to determine the resistance factor required in Eq. 1, the failure probability of the pile must be estimated. This probability will depend on the load distribution, the load factors selected, and the resistance distribution. The resistance distribution is discussed in Section 2 and the load distribution is discussed in Section 3. Section 4 develops the mathematical framework and simulation for the failure probability estimate, and illustrates how the theoretical estimates agree with simulation.

The paper considers four maximum lifetime failure probabilities of a single pile: $10^{-2}, 10^{-3}, 10^{-4}$, and 10^{-5}. Meyerhof (1995) suggests that a typical lifetime failure probability for a foundation is around 10^{-4} and so these numbers range on the low side of that suggested by Meyerhof. However, foundations are normally supported by more than a single pile, and multiple piles provide at least some system redundancy which serves to reduce the failure probability. If it is assumed that Meyerhof's estimate is for the entire foundation system, then the required failure probability for a single pile would be less than the system failure probability of 10^{-4}. Although more research is required to determine the failure levels appropriate for redundant pile systems, the National Cooperative Highway Research Program (NCHRP) reports (Barker et al., 1991, and Paikowsky, 2004) are based on a lifetime failure probability of about 10^{-3} for an individual pile which suggests that NCHRP is considering pile redundancy.

Some of the failure probabilities considered herein, i.e $10^{-3}, 10^{-4}$, and 10^{-5}, might be appropriate for designs involving low (e.g. storage facilities), medium (typical structures), and high (e.g. hospitals and schools) failure consequence structures, respectively. The resistance factors required to achieve these target probabilities will be recommended in Section 5.

2 THE RANDOM SOIL MODEL

The soil cohesion, c, is assumed to be lognormally distributed with mean, μ_c, standard deviation σ_c and spatial correlation length, θ. The lognormal distribution is selected because it is commonly used to represent non-negative soil properties and has a simple relationship with the normal distribution – that is, $\ln c$ is normally distributed, with parameters $\mu_{\ln c}$ and $\sigma_{\ln c}$. The correlation coefficient between the log cohesion at some point x_1 and a second point x_2 is specified by a correlation function, $\rho_{\ln c}(t)$, where $t = x_1 - x_2$ is the distance between the two points. In this study, a simple exponentially decaying (Markovian) correlation function will be assumed. The Markov correlation function has the form

$$\rho_{\ln c}(t) = \exp\left(\frac{-2|t|}{\theta}\right) \qquad [6]$$

3 THE RANDOM LOAD MODEL

In this paper only live and dead loads are considered, as is typically the case in code development. Assuming that the total load is equal to the sum of the maximum lifetime live load, L_L, and the static dead load, L_D, i.e,

$$L_T = L_L + L_D \qquad [7]$$

and that the total load L_T is still at least approximately lognormally distributed (Fenton et al., 2008), then the parameters of the total load distribution can be obtained from

$$\begin{aligned}\sigma_{\ln T}^2 &= \ln(1+v_T^2)\\ \mu_{\ln T} &= \ln(\mu_T) - \frac{1}{2}\sigma_{\ln T}^2\end{aligned} \qquad [8]$$

where $\mu_T = \mu_L + \mu_D$ is the sum of the mean (maximum lifetime) live and (static) dead loads, and v_T is the coefficient of variation of the total load defined by (assuming that dead and live loads are independent),

$$v_T = \frac{\sqrt{\sigma_L^2 + \sigma_D^2}}{\mu_L + \mu_D} \qquad [9]$$

The design problem considered in this study involves a pile supporting loads having means and standard deviations which are shown in Table 1.

Table 1. Load distribution parameters

Parameter	Values
μ_L	20 kN
μ_D	60 kN
σ_L	6 kN
σ_D	9 kN
μ_T	80 kN
σ_T	10.82 kN
$\mu_{\ln T}$	4.4
$\sigma_{\ln T}$	0.14

4 ANALYTICAL AND SIMULATION RESULTS

In this section, the background to an analytical solution to the failure probability of an individual pile placed in a spatially varying undrained soil is presented. Although space limitations in this paper prohibit a complete solution, the basic ideas are presented, from which a solution can be formulated. The authors will be publishing their complete analytical solution shortly. In this paper, it was felt that the complete description of the simulation used to validate the analytical solution would be more valuable to the reader. That description follows the analytical formulation.

In order to determine the probability of failure of a pile, the soil must first be modeled as a spatially varying random field. In general, the cohesion will vary in three dimensions, but there is little advantage in considering the 3rd dimension since piles are essentially one-dimensional and only the 2nd dimension is needed to provide distance from the pile location. Hence, this study considers a two-dimensional random field in which the pile is place vertically at a certain position and soil samples are take vertically at some possibly different position (as in a CPT or STP sounding).

The analytical approximation to the probability of pile failure in undrained soils can now be explained as follows. When the soil properties are spatially variable, as they are in reality, the hypothesis made in this paper is that Eq. 5 can be replaced by

$$R_u = pH\alpha\bar{c} \qquad [10]$$

where $\bar{c}$ is the equivalent cohesion as 'seen' by the pile over its entire length. It is assumed here that $\bar{c}$ is the arithmetic average of the spatially variable cohesion over the pile length H,

$$\bar{c} = \frac{1}{H}\int_0^H c(z)dz = \frac{1}{n}\sum_{i=1}^{n} c_i \qquad [11]$$

Again, $c(z)$ is interpreted as an average cohesion around the pile perimeter at depth z. If the pile is broken up into a series of elements (as will be done in the simulation), the average is determined using the sum at the right of Eq. 11, where c_i is the local average of $c(z)$ over the i^{th} element, for $i = 1, 2, ..., n$.

To design the pile, Eq. 10 is replaced by

$$\hat{R}_u = pH\alpha\hat{c} \qquad [12]$$

where $\hat{c}$ is the characteristic cohesion which is commonly estimated from a set of m observations $\hat{c}_1, \hat{c}_2, ..., \hat{c}_m$ of soil cohesion taken at the site. In this paper, an arithmetic average of the observations will be used to define the characteristic cohesion,

$$\hat{c} = \frac{1}{m}\sum_{j=1}^{m} \hat{c}_j \qquad [13]$$

This is an estimate of the mean cohesion and no bias factor is applied to obtain the characteristic value.

The required minimum design pile length, H, is obtained by using Eq. 12 in Eq. 1,

$$H = \frac{I(\alpha_c L_c)}{\varphi_g p\alpha\hat{c}} \qquad [14]$$

By further substituting Eq. 14 into Eq. 10, the ultimate resistance becomes

$$R_u = \frac{I(\alpha_T \hat{L}_T)\bar{c}}{\varphi_g \hat{c}} \qquad [15]$$

The design philosophy in this study is as follows: find the required length, H, such that the probability that the actual load, L_T, exceeds the actual ultimate resistance, R_u, is less than some small acceptable failure probability, p_m. The actual failure probability, p_f, is

$$p_f = P\left[L_T > R_u\right] \qquad [16]$$

and a successful design methodology will have $p_f \le p_m$.

Substituting Eq. 15 into Eq. 16 and collecting random terms to the left of the inequality leads to

$$p_f = P\left[\frac{L_T\hat{c}}{\bar{c}} > \frac{I(\alpha_T \hat{L}_T)}{\varphi_g}\right] \qquad [17]$$

The computation of the probability in Eq. 17 now involves the determination of the distribution of $L_T\hat{c}/\bar{c}$. If the random load, L_T, and cohesion values, $\hat{c}$ and $\bar{c}$, are all assumed to be lognormally distributed, which is a reasonable assumption (Fenton and Griffiths, 2008, and Fenton et al., 2008), then the term $L_T\hat{c}/\bar{c}$ will also be lognormally distributed and its parameters can be determined by considering the individual distributions of L_T, $\hat{c}$, and $\bar{c}$. As mentioned above, space in this paper does not permit the full mathematical description. The interested reader is referred to an upcoming paper by the authors. The simulation approach to computing p_f is simper and will be described in detail shortly. Once the probability of failure is computed via Eq. 17, it can be compared to the maximum acceptable failure probability, p_m. If p_f exceeds p_m, then the resistance factor needs to be reduced.

The probability in Eq. 17 can also be estimated via simulation, and this is used to validate the analytical computation. Simulation essentially proceeds by carrying out a series of hypothetical designs on a series of simulated soil fields and checking to see what fraction of the designs fail. In detail, the steps involved in the simulation are as follows;

1. The cohesion, c, of a soil mass is simulated as a spatially variable random field using the Local Average Subdivision (LAS) method (Fenton and Vanmarcke, 1990). Cohesion is assumed to be lognormally distributed with mean $\mu_c = 50$ kPa and coefficient of variation, v_c, ranging from 0.1 to 0.5. The correlation length (see Eq. 5) is varied from 0 to 50 m.
2. The simulated soil is sampled at m locations along a vertical line through the soil at some distance, r, from the pile. These virtually sampled soil properties are used to estimate the characteristic cohesion, $\hat{c}$, according to Eq. 13. Three sampling distances are considered. The first is at $r = 0$ m which means that the samples are taken at the pile location. In this case, the uncertainty about the pile resistance only arises if the pile extends below the sampling depth. Typically, probabilities of failure when $r = 0$ m are very small. The other two sample distances considered are $r = 4.5$ m and $r = 9$ m, corresponding to reducing understanding of the soil conditions at the pile location. These rather arbitrary distances were based on preliminary random field simulations, which happened to involve fields 9 m in width. However, it is really the ratio, r/θ, which governs the failure probability.
3. The length of the pile required by the design, H, is computed using Eq. 14.
4. The equivalent cohesion, $\bar{c}$, is computed according to Eq. 11 by sampling the soil along the sides of the pile.
5. The 'true' ultimate pile resistance, R_u, is computed using Eq. 10.
6. Dead and live loads, L_D and L_L, are simulated as independent lognormally distributed random variables and then added to produce the actual total load on the pile, $L_T = L_D + L_L$. The means and coefficients of variation of the dead and live loads are assumed to be $\mu_D = 60$ kN, $v_D = 0.15$ and $\mu_L = 20$ kN, $v_L = 0.3$, respectively.
7. The ultimate resistance factor, R_u, and total load, L_T, are compared. If $L_T > R_u$, then the pile, as designed, is assumed to have failed.
8. The entire process from step 1 to step 7 is repeated n_{sim} times. If n_f of these repetitions result in a pile failure, then an estimate of the probability of failure is

$$p_f \simeq \frac{n_f}{n_{sim}} \qquad [18]$$

9. Repeating steps 1 through 8 using various values of φ_g in the design step allows plots of failure probability vs. resistance factor, for the various sampling distances, coefficient of variation of the cohesion, and correlation length, to be produced.

The analytically estimated failure probabilities can be superimposed on the simulation-based failure probability plots, allowing a direct comparison of the methods. Figures 1, 2 and 3 illustrate the agreement between failure probabilities estimated via simulation and those computed analytically using Eq. 17. Given all the approximations made in the theory, the agreement is considered to be excellent, allowing the resistance factors to be computed analytically with reasonable confidence even at probability levels which the simulation cannot estimate – the simulation involved only 2000 realizations and so cannot properly resolve probabilities less than about 0.001.

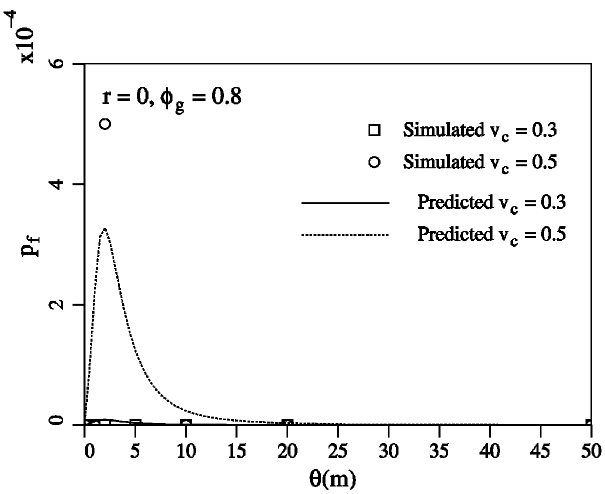

Figure1. Failure probabilities when the soil has been sampled at the pile location ($r = 0$ m).

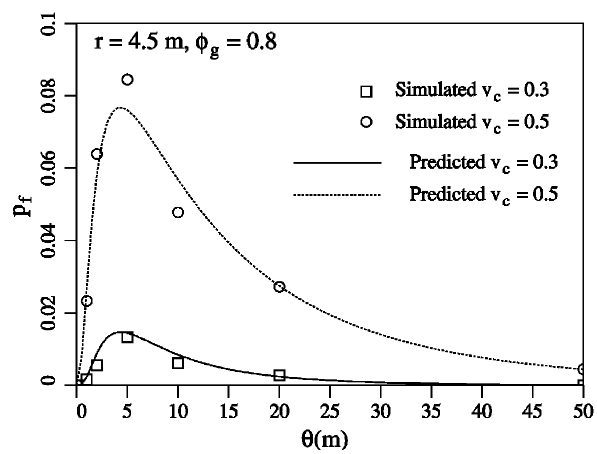

Figure 2. Failure probabilities when the soil has been sampled at $r = 4.5$ m from the pile centerline.

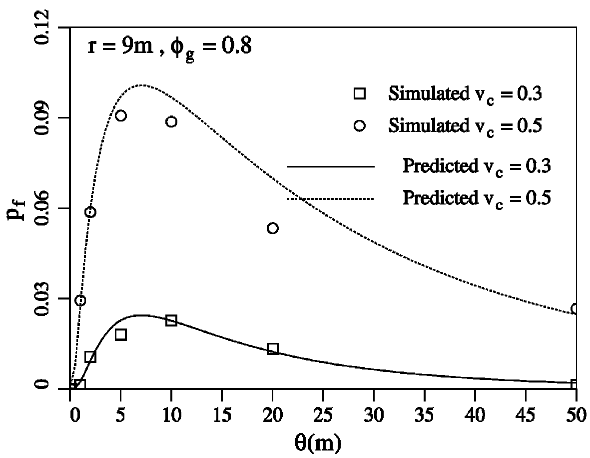

Figure 3. Failure probabilities when the soil has been sampled at $r = 9$ m from the pile centerline.

The above figures show the comparison of failure probabilities estimated from simulation based on 2000 realizations and theoretical estimates using Eq. 17. Note the change in the vertical scales – the probability of failure is much lower when samples are taken at the pile location, as expected, and increases as the distance to the sample increases.

5 REQUIRED RESISTANCE FACTORS

As suggested Section 4, the determination of the required resistance factor φ_g involves deciding on a maximum acceptable probability of failure p_m. In this section, the value of φ_g required to achieve four maximum acceptable failure probability levels ($10^{-2}, 10^{-3}, 10^{-4}$, and 10^{-5}) will be investigated. The corresponding reliability indices of these four target probabilities are approximately 2.3, 3.1, 3.7, and 4.3, respectively.

Figures 4, 5, and 6 show the resistance factors required for the cases where the soil is sampled at the pile, at a distance of 4.5 m and at a distance of 9 m from the pile centerline, respectively, to achieve the first three maximum acceptable failure probabilities. The case where $p_m = 10^{-5}$ was omitted from the plots for the sake of brevity, but the results are similar and the resistance factors for this case are presented later. Figure 4 corresponds to sampling at the pile location, and in this case, the design conditions are so well understood that the resistance factor exceeds 1.0 when $p_m < 10^{-4}$ and so these plots are not shown. The authors do not recommend using $\varphi_g > 1.0$ since this analysis does not consider measurement errors.

The worse case occurs when the correlation length, θ is between about 1 and 10 m. This worst case is important, since the correlation length is very hard to estimate and will be unknown for most sites. In other words, in the absence of knowledge about the correlation length, the lowest resistance factor in these plots, at the worst case correlation length, should be used.

To explain why a worst case exists, the nature of the correlation length must be considered. The correlation length, θ, measures the distance within which soil properties are significantly correlated. Low values of θ lead to soil properties which vary rapidly in space, while high values mean that the soil properties vary only slowly with position. A large correlation length, of say $\theta = 50$ m, means that soil samples taken well within 50 m from the pile location will (e.g. at $r = 10$ m) will be quite representative of the soil properties at the pile location. In other words, lower failure probabilities are expected when the soil is sampled well within the distance θ from the pile. Interestingly, because our soil sample is generally some sort of average, when θ is very small (say, 0.01 m), then the sample will again accurately reflect the average conditions along the pile regardless of the sampling location, and in this case the failure probability is again low. At intermediate correlation lengths soil samples become imperfect estimators of conditions along the pile, and so the probability of failure increases, or conversely, the required resistance factor decreases. Thus, the minimum required resistance factor will occur at some correlation length between 0.0 and infinity.

The worst case correlation length occurs when θ is approximately equal to the distance from the pile to the sampling location. Notice in Figures 4, 5, and 6 that the worst case correlation length does show some increase as the distance to the sample location, r, increases.

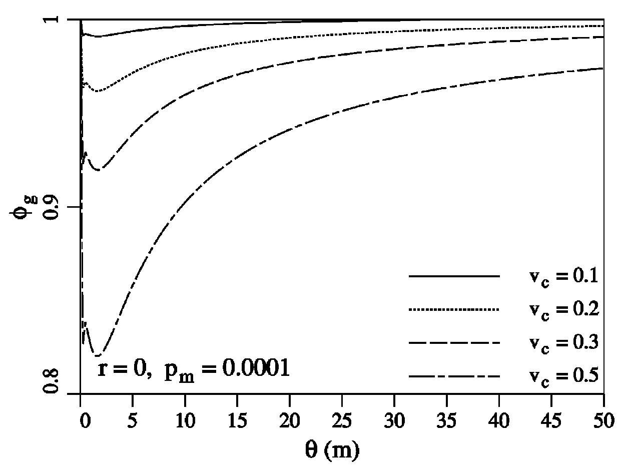

Figure 4. Resistance factor when the soil has been sampled at the pile location (note the reduced vertical scale).

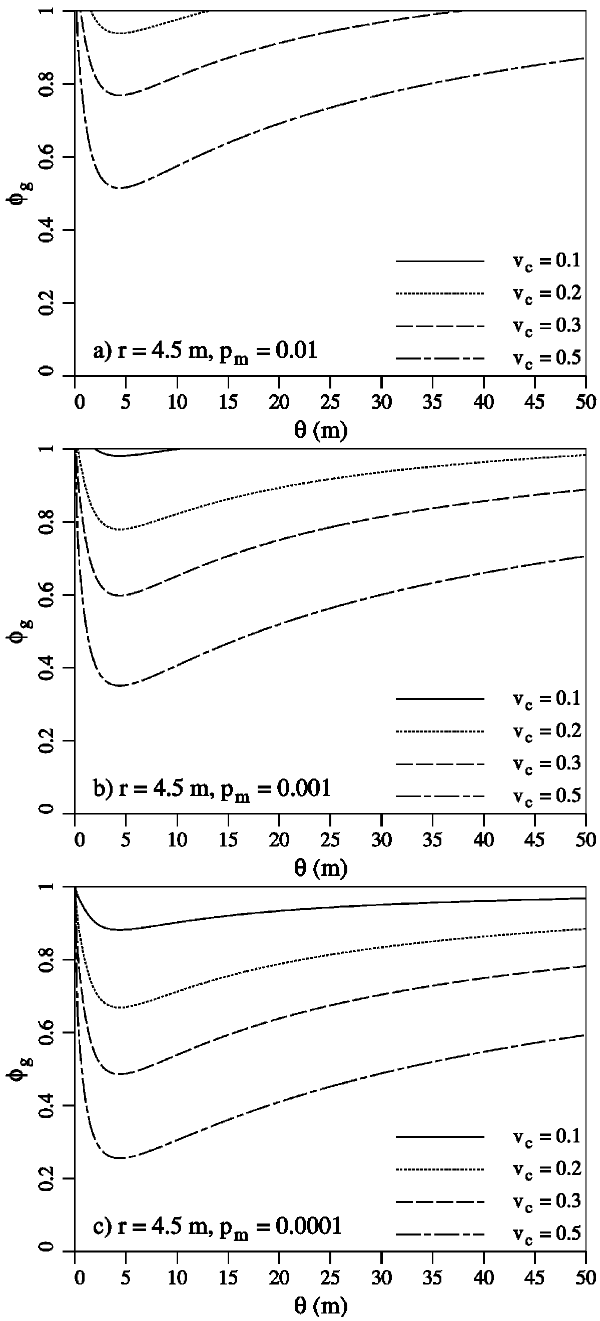

Figure 5. Resistance factors when the soil has been sampled 4.5 m from the pile centerline.

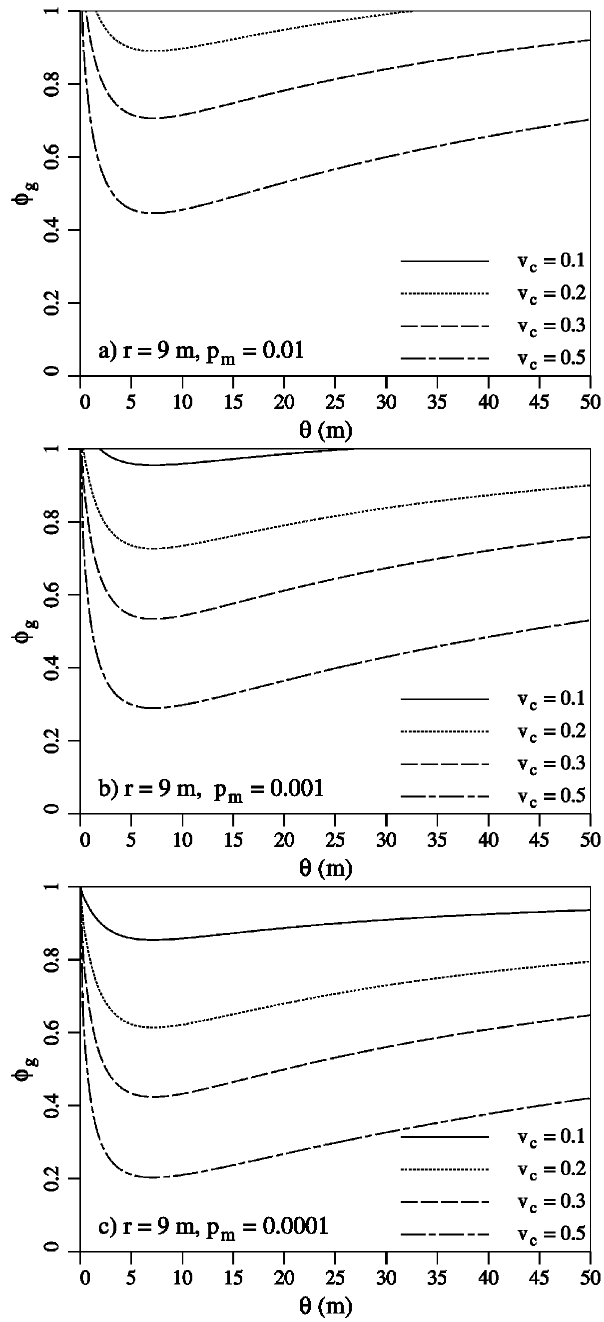

Figure 6. Resistance factors when the soil has been sampled 9 m from the pile centerline.

As shown in Figure 6 the smallest resistance factors correspond the smallest acceptable failure probability shown, $p_m = 0.0001$, when the soil is sampled 9 m away from the pile centerline. When the cohesion coefficient of variation is relatively large, $v_c = 0.5$ the worst case values of φ_g dip down to 0.2 in order to achieve $p_m = 10^{-4}$. In other words, there will be a significant construction cost penalty if a highly reliability pile is to be designed using a site investigation which is insufficient to reduce the residual variability to less than $v_c = 0.5$.

The "worst case" resistance factors required o achieve the indicated maximum acceptable failure probabilities, as seen in Figures 4 through 6, are summarized in Table 2 (which includes the $p_m = 10^{-5}$ case). To compare the resistance factors recommended in Table 2 to resistance factors recommended in the literature and to current geotechnical LRFD codes, changes in the load factors from code to code need to be considered. The resistance factors recommended in this study for $p_m = 0.01$ are greater than 1.0, which may be because the load factors provide too much safety for the larger acceptable failure probabilities when the site is well understood. Also, due to redundancy in pile groups, it is reasonable to use a lower reliability for a single pile. For example, if a single pile in a group has the smallest resistance and begins to fail, the load is transferred to other piles in the group with greater resistance and the overall foundation is less likely to fail. A reasonable value of target reliability index for single driven piles may be in the range of 2.0 to 2.5, corresponding to p_m between 0.01 and 0.001.

Table 2. Worst case resistance factors for various coefficient of variation, distance sampling location and acceptable failure probabilities.

r (m)	v_c	Resistance Factor, φ_g			
		$p_m = 10^{-2}$	$p_m = 10^{-3}$	$p_m = 10^{-4}$	$p_m = 10^{-5}$
0.0	0.1	1.20	1.08	0.99	0.91
0.0	0.2	1.10	1.05	0.96	0.88
0.0	0.3	1.10	1.01	0.91	0.84
0.0	0.5	1.10	0.92	0.82	0.74
4.5	0.1	1.11	0.98	0.88	0.80
4.5	0.2	0.93	0.78	0.68	0.58
4.5	0.3	0.76	0.60	0.49	0.40
4.5	0.5	0.52	0.35	0.27	0.19
9.0	0.1	1.10	0.95	0.85	0.77
9.0	0.2	0.90	0.72	0.62	0.53
9.0	0.3	0.72	0.53	0.42	0.34
9.0	0.5	0.45	0.29	0.20	0.14

Table 3 compares the resistance factors recommended in this study with some other sources. The individual "current study" values correspond to the moderate case where $v_c = 0.3$ and $r = 4.5$ m for acceptable failure probabilities $p_m = 10^{-3}$, 10^{-4}, and 10^{-5}. As can be seen, the Australian Standard, AS5100.3 (2004) approximately spans the range of resistance factors proposed here, although the AS5100 values are somewhat higher. The slightly higher resistance factors may be because their live load factor is also significantly higher. It is also noted that the range given in AS5100.3 corresponds to the degree of site understanding, not the consequence level (e.g. 0.45 would be selected if the site were poorly understood). The resistance factors recommended by NCHRP 343 (Barker et al., 1991) and NCHRP 507 (Paikowsky, 2004) are based on a reliability index 3.0 ($p_m = 0.0013$) which is very close to the first level recommended by the current study in Table 3 ($p_m = 10^{-3}$). The NCHRP resistance factors are somewhat lower than the 0.59 suggested here, which is perhaps slightly surprising considering the significantly higher load factors they are using.

The AASHTO factors (2002, 2004) are based on the reliability index 2.0 ($p_m = 0.0227$) which is very close to the recommended resistance factor corresponding to $p_m = 0.01$ in this study (for v_c = 0.5). The recommended resistance factor by AASHTO (2007) tend to be in the range suggested in this research for $p_m = 10^{-2}$ and $p_m = 10^{-4}$, despite their larger load factors. More research is needed to clearly compare these factors.

Table 3. Comparison of resistance factors recommended in this study (r = 4.5 m)to those recommended by other sources.

Source		Load Factors			Resistance Factor, φ_g
		μ_D / μ_L	α_L	α_D	
Current Study	$p_m = 10^{-2}$	3.00	1.50	1.25	0.76
	$p_m = 10^{-3}$				0.60
	$p_m = 10^{-4}$				0.49
	$p_m = 10^{-5}$				0.40
CFEM (2006)		3.00	1.50	1.25	0.50
AS 5100.3 (2004)		3.00	1.80	1.20	0.45 -0.65
CHBDC (2006)		3.00	1.70	1.20	0.50
NBCC (2005)		3.00	1.50	1.25	0.60
AASHTO (2007)		3.00	1.75	1.25	0.40
AASHTO (2004)		3.70	1.75	1.25	0.54
AASHTO (2002)		3.70	2.17	1.30	0.59
NCHRP 343 (1991)		2.00	2.17	1.30	0.55
NCHRP 507 (2004)		3.00	1.75	1.25	0.50

The load factors and the dead to live load ratio used in CFEM (2006) and NBCC(2006) are exactly as same as the values used in the current study and the recommended resistance factors in these codes are very close to the recommended values in this study for $p_m = 10^{-4}$ and $p_m = 10^{-3}$, respectively.

6 CONCLUSIONS

This paper studied reliability-based design, specifically the load and resistance factor design (LRFD) of deep foundations. The load factors and load combinations are as used in the National Building Code of Canada (NRC, 2005). A mathematical theory was developed to analytically estimate the probability of pile failure. The theoretical model assumes a stationary random soil with lognormally distributed cohesion, c. The effect of the soil's spatial variability and the site investigation intensity on the resistance factor has been investigated via simulation and theory by considering various soil statistics and sampling locations. The simulation involved 2000 realizations for each set of parameters and the results of the Monte Carlo simulation were compared to the proposed theory. Optimal resistance factors were recommended for the design of deep foundations for three target probability of failure.

Both the theory and the simulation demonstrates that a 'worst case' correlation length exists, and resistance factors based on this worst case, shown in Table 2, agree quite well with current literature and LRFD code recommendations, assuming moderate variability and site understanding, suggesting that the theory is in reasonable agreement with past experience.

The overall agreement between the analytically derived resistance factors proposed in this study and those currently used in other codes, as shown in Table 3, is encouraging. The current study now provides a rigorous basis for the determination of resistance factors in pile design and the theory provides a framework to extend code provisions beyond calibration with the past.

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