Rules of thumb in geotechnical engineering

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ABSTRACT
Ground engineers routinely use simple relationships - rules of thumb – to obtain soil parameters and to design ground works. Some of these have a sound theoretical basis and can be applied generally while some are purely empirical and so should be applied only within the limits of the observations used to derive them in the first place. A classification for rules of thumb was suggested by Wroth (1984) and this has been used to examine the theoretical basis – or lack of it – for some of the more common empirical rules in geotechnical engineering.

RÉSUMÉ
Techniciens au sol utilisent couramment des relations simples - des règles de pouce - pour obtenir des paramètres des sols et de concevoir des travaux de terrassement. Certaines d'entre elles ont une base théorique solide et peut être d'application générale alors que certains sont purement empirique et devrait donc être appliquée que dans les limites des observations utilisées pour les calculer en premier lieu. Une classification des règles de pouce a été suggéré par Wroth (1984), ce qui a été utilisée pour examiner le fondement théorique - ou leur absence - pour certaines des règles les plus communes empirique en génie géotechnique.

1 INTRODUCTION

Geotechnical engineering is essentially a mechanical science which has a strong theoretical basis. Nevertheless many geotechnical engineers use simple relationships – rules of thumb – in routine design. Some of these rules of thumb are based on sound theory and so should be generally applicable; others are purely empirical and so are applicable only within the range of the data from which they were derived. In this paper I will explore the theory, or lack of it, behind some of the commonly used rules of thumb – what are the rules for a reliable rule of thumb?

2 RULES OF THUMB

The origins of the term "rule of thumb" are obscure. Apparently Roman bricklayers used the tip of the thumb from the knuckle as a unit of measure. Brewers used their thumb to test the temperature of fermenting ale. In the Middle Ages a man was permitted to beat his wife with a cane no thicker than his thumb. Nowadays, rule of thumb implies a rough estimate based on experience rather than formal calculation.

An informal survey of colleagues and friends was conducted and it is clear that most geotechnical engineers have their own favourite rules of thumb. Some are trivial – never trust the driller – while some – the bearing capacity of a shallow foundation is twice the undrained strength of the soil - are of fundamental importance.

Clearly it is not possible to cover all the rules of thumb in common use. Instead the rules for rules of thumb can be examined and an understanding of when these are applicable and when not developed.

3 THE WROTH RULES.

A relationship that links two observations and which is purely empirical can really only be used with confidence within the limits of the observations upon which it was obtained in the first place. For example, if a group of engineers find that the drillers they work with give unreliable information does that mean that all drillers everywhere are unreliable? Is there something basic in the human condition that links drilling with reliability?

In his Rankine Lecture, Wroth (1984) gave a set of conditions which should be met for a successful relationship that can be used with confidence outside the immediate context in which it was established. Wroth's rules for a successful relationship are that it should ideally be: (a) based on physical appreciation of why the properties can be expected to be related; (b) set against a background of theory, however idealised this may be and (c) expressed in terms of dimensionless variables so that advantage can be taken of scaling laws of continuum mechanics.

Wroth (1984) was writing specifically in the context of interpretation of in situ soil tests but his rules hold generally.

4 CLASSES OF RULES OF THUMB.

We now have a framework for classifying rules of thumb.

Class 1. These obey the Wroth rules. They have a sound theoretical basis and are generally applicable everywhere. They can be derived from theory alone without need for empirical observations. An example is

$$q_a = 2s_u \quad [1]$$

where q_a is the allowable bearing capacity of a shallow foundation and s_u is the undrained strength of the soil. (I will consider this and other rules of thumb later.)

Class 2. These obey the Wroth rules. They have a sound theoretical basis and are generally applicable everywhere but they require empirical correlations. An example is;

$$q_a = 10N \quad [2]$$

where q_a in kPa is the allowable bearing capacity of a shallow foundation and N is the SPT blow count corrected for depth or effective stress. The relationship in Equation 2 is not in dimensionless variables and it is necessary to state the units of q_a which, in this case are kPa. Equation 2 can easily be recast in dimensionless form by dividing q_a by a reference pressure such as p_r = 1kPa in which case the units of q_a are the same as those of the reference pressure.

Class 3. These violate the letter and the spirit of the Wroth rules. There is no physical appreciation, there is no background theory and the relationship is not expressed in dimensionless variables.

5 SOME RULES OF THUMB FOR SOIL PROPERTIES.

There are a number of rules of thumb relating simple test results to soil properties. Here is it necessary to distinguish between a material property which depends only on the grains and a state dependent property which depends also on the current water content and effective stress.

5.1 Material Properties.

Soil grains are described by their grading and by their shape, texture and mineralogy. There are a number of fundamental soil properties which depend only on these. The main problem in relating soil properties to the grains is to quantify descriptions of soil grains. The Atterberg limits, liquid limit and plastic limit, describe clay mineralogy and d_{10} (the size of the 10% fraction) and the coefficient of uniformity quantify grading. However the Atterberg limits are measured on only part of a well graded soil and then the fraction tested contains silt sized grains: activity is a better descriptor of clay mineralogy.

If soil is continuously distorted it must reach a state in which stress and water content no longer change; this is the critical state and the parameters ϕ'_c C_c and e_Γ (as defined by Atkinson, 2007) which define the critical state are material parameters. Wood (1991) showed a relationship between ϕ'_c and plasticity index PI that may be approximated to

$$\phi'_c = 50^0 - 15\log PI \quad [3]$$

for PI in the range 10 to 100.

For coarse grained soils the value of ϕ'_c can be taken as about 30^0 for smooth rounded grains to about 40^0 for carbonate sand. These relationships fall into the Wroth class 2. They are entirely empirical and there is some scatter but there are sound reasons for relating friction to plasticity and both variables are dimensionless.

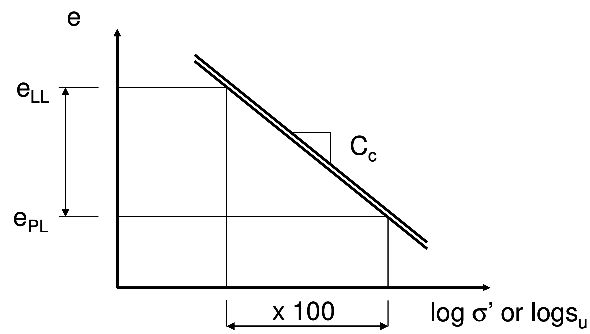


Figure 1 Relationship between Atterberg limits and strength.

When a soil is at its liquid limit or its plastic limit it is at its critical state and the Atterberg limits measure the water contents at which the soil has particular strengths. These are about 1.7kPa and 170kPa respectively and they conveniently differ by 100 times. Both states lie on the critical state line and so there is a strong relationship between C_c and plasticity index as shown in Figure 1. Noting that $e = wG_s$, water content is expressed as a percentage and s_u/σ' at the critical state is constant we have

$$C_c = \frac{PI \times G_s}{200} \quad [4]$$

This falls into Wroth class 1 because the relationship does not depend on an empirical correlation and it arises purely from the definitions of the Atterberg limits.

Schofield and Wroth (1968), showed that critical states lines generally pass close to a single point denoted Ω with the values v_Ω = 1.2 and p'_Ω = 1500lb/in^2 = 10,000kPa. From $e_\Gamma = e_\Omega + C_c \log p'_\Omega$ we have

$$e_\Gamma = 0.2 + C_c \log 10{,}000 = 0.2 + 4C_c \quad [5]$$

Hence the critical state parameters ϕ'_c C_c and e_Γ are all related directly to the plasticity index. These relationships can be considered to be rules of thumb but they are based on sound theory and the only empiricism involved is the 100 times difference between the strengths at the liquid and plastic limits and the existence of the Ω point.

5.2 Undrained strength.

Many soil properties such as undrained strength and stiffness depend both on the grains and on the current state described by both the water content and the effective stress; these are state dependent properties.

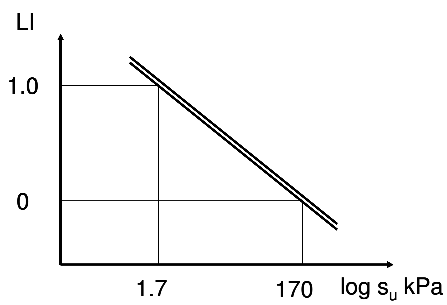


Figure 2 Relationship between undrained strength and liquidity index.

Figure 2, which is similar to Figure 1, shows the variation of undrained strength with liquidity index taking s_u = 1.7kPa at the liquid limit and s_u = 170kPa at the plastic limit. This is a convenient rule of thumb for estimating the undrained strength from the Atterberg limits and the natural water content. It is in Wroth class 1 because it does not require empirical observations and it arises from the coincidence that the undrained strengths at the liquid and plastic limits differ by a factor of 100.

Figure 3(a) shows the relationship for normally consolidated soils between s_u/σ'_v and PI: it is the well known Skempton (1957) relationship

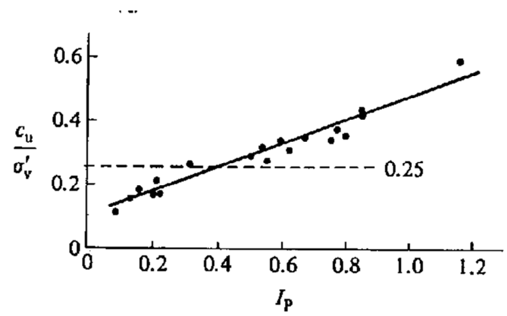
$$\frac{s_u}{\sigma'_v} = 0.11 + 0.0037\text{PI} \quad [6]$$

The parameter s_u/σ'_v is the ratio of a strength to an effective stress and so it is related to a friction angle and to the pore pressure developed during undrained shearing. However Equation 3 shows that friction angle *decreases* with plasticity so Equation 5 implies that undrained pore pressures increase rapidly with increasing plasticity.

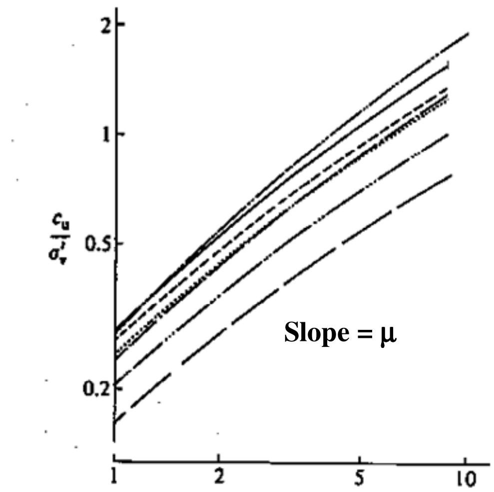
Figure 3(b) shows the relationship between undrained strength and overconsolidation ratio. Combining Figures 5(a) and (b) we have

$$\frac{s_u}{\sigma'_v} = B \times R_0^{\mu} \quad [7]$$

where B is the expression for PI in Equation 1 and μ has a unique value = 0.8. Equation 7 is a rule of thumb to estimate undrained strength from the in situ stress and the overconsolidation ratio. It is dimensionally correct and relies on empirical observations to determine values for the parameters B and μ.



(a) Normally consolidated soils



(b) Overconsolidated soils

Figure 3 Relationships between undrained strength, state and plasticity index (Wood 1991)

5.3 State parameter

While not strictly a rule of thumb, the concept of state and state parameter is fundamental to interpreting soil behaviour.

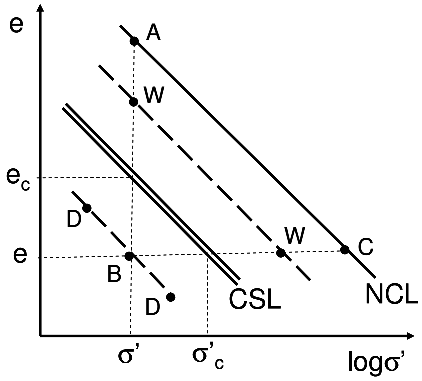


Figure 4 State parameters.

Figure 4 shows a normal compression line (NCL) and a critical state line (CSL). Soil at A is loose or soft and the same soil at B is dense or stiff. Soil at C is loose or soft because, due to the very high stress, it will compress when sheared. When sheared, soils with states at W (on the wet side of critical) will all compress by the same amount and soils with states at D (on the dry side of critical) will all dilate by the same amount.

The behaviour of soil is governed by its initial state and this is represented by a state parameter which is the distance from some reference line. All soils on the broken line through W behave in the same way and so do all soils on the broken line through D. The concept of state parameter was introduced by Schofield and Wroth (1968) and has been used by Been and Jefferies (1985). For the state at B the state parameter is either $(e - e_c)$ or σ'/σ'_c (Atkinson, 2007).

5.4 Peak strength

The peak strength of soil depends on its state. It can be expressed by either of

$$\tau_p = c'_p + \sigma' \tan\phi'_p \qquad [8]$$

$$\tau_p = A'\sigma'^b \qquad [9]$$

where c'_p, ϕ'_p, A and b are parameters which depend on the material and on the current state. There are no reliable rules of thumb for these and none would be expected.

The peak strength of uncemented soil arises from friction and dilation and is given by

$$\frac{\tau_p}{\sigma'_p} = \tan(\phi'_c + \psi_m) \qquad [10]$$

where ϕ'_c is the critical state friction angle and ψ_m is the maximum angle of dilation. The value of ψ_m depends on state parameter. This is the basis of the rule of thumb given by Bolton (1986) for plane strain as

$$\phi'_m = \phi'_c + 3I_R \qquad [11]$$

where I_r is a function of relative density and stress (i.e. a function of state). Equation 11 falls into the Wroth class 2; it has a sound theoretical basis but requires empirical correlations.

5.5 Stiffness

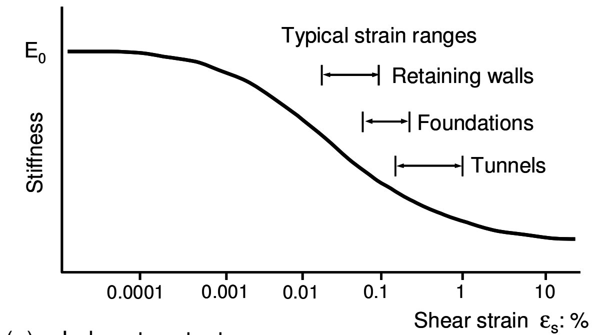
The stiffness of a soil, either Young's modulus E, shear modulus G or one-dimensional modulus M, depends on the grains, the state, on whether the soil is drained or undrained and on the strain. This means that rules of thumb for soil stiffness will at best be very approximate.

The drained one-dimensional constrained modulus $M' = 1/m_v$ is related to C_c by

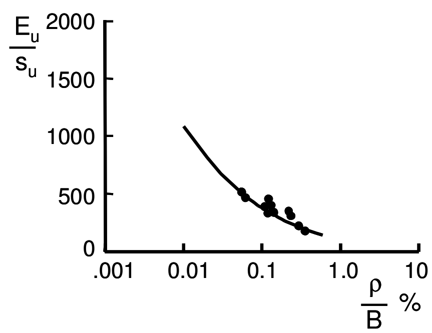
$$M' = \frac{\sigma'(1+e)}{C_c} \qquad [12]$$

The coefficient of compressibility C_c is a material parameter related to the grains by Equation 4 and so Equation 12 demonstrates how stiffness varies with stress and voids ratio. Young's modulus and shear modulus vary with state in a similar way.

The variations of undrained Young's modulus E_u and undrained strength s_u with state follow similar patterns and so the ratio E_u/s_u should be reasonably constant for many soils. However soil stiffness is markedly non-linear and the variation of Young's modulus with strain is now well documented (Atkinson, 2000). A typical stiffness decay curve is shown in Figure 5(a) together with typical ranges of strains in the ground near structures. A rule of thumb is the stiffness should correspond to strains of the order of 0.1% (Atkinson, 2000). Figure 5(b) shows data from foundations in central London. The values of E_u were found from the observed settlements ρ using simple elastic analyses and the values of s_u were found from site investigation data.



(a) Laboratory tests



(b) Field observations

Figure 5 Degradation of stiffness with strain.

A commonly used rule of thumb is to take

$$\frac{E_u}{s_u} = 300 \qquad [13]$$

From Figure 5(b) this corresponds to strains ρ/B of the order of 0.1%. Thus the rule of thumb given by Equation 13 is in Wroth class 2; it has a sound theoretical basis but includes an empirical stiffness decay curve.

6 SOME RULES OF THUMB FOR INTERPRETATION OF IN SITU TESTS

There are several common in situ tests involving penetrometers, expanding cavities, shearing and so on. For each there are analyses and interpretations most of which require rules of thumb. To illustrate the general principles I will consider the common probing tests. These are the standard penetration test (SPT) and the cone penetration test (CPT) usually with measurement of sleeve friction and sometimes with measurement of pore pressure. There are several rules of thumb for estimating soil properties from SPT and CPT test results.

6.1 Standard penetration test

The SPT returns a value of N, the number of blows for a given penetration. Often several corrections are applied and there is a general view that the test is far from standard. Given that a typical SPT takes a few minutes to complete only very fine grained soils will be undrained and only very coarse grained soils will be drained and in most cases the soil around the tool will be partly drained. Leaving these issues aside it is not clear exactly what is being measured and to what the N-value can be safely correlated.

The value of N must depend on both strength and stiffness but there is no unique relationship between strength and stiffness, even for soils with the same grains, and the ratio of stiffness to strength – known as rigidity – varies with state (Atkinson, 2000). As the SPT tool penetrates it shears the soil and expands a cavity so both stiffness and strength contribute to penetration resistance. The best that can be said is that N should be related to state.

If SPT test observations are corrected for depth, or effective stress, then N is a measure of liquidity index or relative density. This means that N cannot be uniquely related to ϕ'.

There are several rules of thumb in common use which relate the SPT N value to soil parameters and a common one is

$$\frac{s_u}{p_r} = 5N \qquad [14]$$

where the reference pressure p_r is included to make Equation 14 dimensionless and the factor 5 is purely empirical. From Figure 1 soil which has a liquidity index of 0.5 (i.e. its water content is midway between the liquid and plastic limits) has an undrained strength s_u = 17kPa. From Equation 14 the soil would have N ≈3 to 4.

6.2 Cone penetration test

A standard CPT returns values of cone resistance q_c and sleeve shear stress f_s and from these a friction ratio $F_r = f_s/q_c$ is calculated. In a piezocone pore pressures are measured as well but the values depend on the exact location at which they are measured. Many of the criticisms of the SPT apply also to the CPT. Methods for interpretation of the CPT were given by Lunne et al (1997). Most of these fall into the Wroth class 2. They have a sound theoretical basis but depend on empirical correlations.

There are a number of charts which relate CPT observations to soil type and characteristics. A typical example is given by Douglas and Olsen (1981): others are given by Lunne et al (1997). These rules of thumb are entirely empirical and fall into the Wroth class 3.

7 SOME RULES OF THUMB FOR GEOTECHNICAL DESIGN AND ANALYSIS

As well as rules of thumb for estimating soil properties and parameters, there are rules of thumb for assessing the behaviour of foundations, slopes and walls. Most engineers have their own personal rules of thumb. They use these for preliminary design and, importantly, as a check that a full analysis has given reasonable results. As before, I cannot possibly consider all the rules of thumb routinely used by geotechnical engineers in their day–to–day work. Instead I will consider some common ones for shallow foundations and piles to explore the rules for these rules of thumb.

7.1 Allowable bearing capacity of foundations

Common rules of thumb for design of shallow foundations are in Section 4 and are

$$q_a = 2s_u \qquad [1]$$

$$q_a = 10N \qquad [2]$$

where q_a is the allowable bearing pressure. The ultimate bearing capacity of a shallow foundation is

$$q_a = N_c s_u \qquad [15]$$

With N_c = 6 for a square foundation and a load factor L_f = 3 we get

$$q_a = N_c \frac{s_u}{L_f} = 2s_u \qquad [16]$$

which is the same as Equation 1. (I will consider the rule of thumb for load factor later.) Thus the rule of thumb in Equation 1 is in the Wroth class 1; it has a sound theoretical basis and does not require empirical observations.

Equation 2 is a rule of thumb for estimating the allowable bearing capacity of a shallow foundation from an SPT N value. It is intended for a foundation which can settle 1 inch (25mm) and for smaller settlements the bearing pressure should be reduced proportionally. Together Equations 1 and 2 lead to Equation 14 which is probably the origin of this rule of thumb.

7.2 Ultimate capacity of piles

For piles in coarse grained soils rules of thumb relate shaft friction and end bearing to the SPT N value. Commonly used ones were given by Poulos (1989) for driven piles q_s = 2N and q_b = 400N and for bored piles q_s = N and q_b = 100N (with all stresses in kPa). These rules of thumb are similar to Equation 2. They depend on there being a linear relationship between soil strength or stiffness and N but the relationships are entirely empirical.

For piles in fine grained soils rules of thumb relate shaft friction and end bearing to the undrained strength s_u. The ultimate end bearing capacity is given by

$$q_b = 9s_u \qquad [17]$$

This is equivalent to taking a bearing capacity factor N_c = 9 for a deep foundation and so this is in the Wroth class 1, it is theoretically sound and does not require empirical observations. The shaft resistance is given by

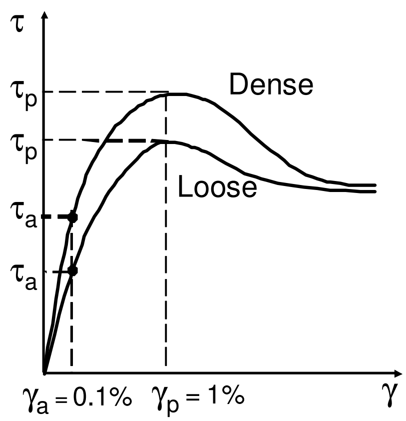
$$q_s = \alpha s_u \qquad [18]$$

where α is usually taken to be about 0.5 for both driven and bored piles. The form of Equation 18 is sound, the shaft resistance for undrained loading should be related to the undrained strength, but the factor α = 0.5 is empirical and it is interesting that the same value should be appropriate for both driven and bored piles. It is probable that the loss of strength for a driven pile is associated with the residual strength while for a bored pile it is associated with softening and it is fortuitous that the reductions are about the same for each case.

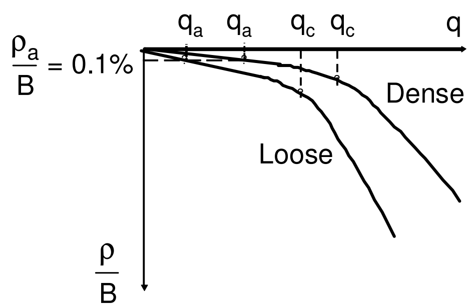
7.3 Factor of safety and load factor

A common method for geotechnical engineering design is to calculate a state of collapse, usually from a limit equilibrium analysis and then to apply a factor. To prevent collapse at the ultimate limit state the factor is a factor of safety and this is often taken to be about 1.25. To prevent excessive movement at the serviceability limit state the factor is a load factor and is often taken to be about 3 as in the analysis leading to Equation 16 above.

A factor of safety to prevent the ultimate limit state of a slope or retaining wall is there to account for uncertainties and to provide a margin of safety. The number is a purely empirical rule of thumb. Presumably the value chosen by the designer reflects his confidence in his analyses and the consequences of failure. This really is an individual rule of thumb.



(a) Laboratory tests



(b) Foundation settlement

Figure 8 Foundation design for dense and loose soils.

A load factor to prevent excessive movement is there to bring the design point to a place on a load displacement curve where the movements will be small. Figure 8(a) illustrates characteristic shear test stress-strain curves for dense and loose samples of the same soil sheared with the same normal effective stress. They have the same critical state strength but different peak strengths but these occur at about the same strain and up to the peak the curves are geometrically similar. This means the peak strength is an indirect measure of stiffness and if the values of peak strength are divided by the same factor the design stresses τ_d occur at the same strain γ_d. Figure 8(b) illustrates characteristic foundation load settlement curves for the same loose and dense samples of the same soil as that in Figure 8(a). There is an ultimate bearing capacity σ_c and an allowable design bearing capacity σ_d where the settlements in both cases are the relatively small allowable design settlements ρ_d. Again, if the load settlement curves are geometrically similar up the point of failure, dividing σ_c by the same load factor will produce allowable bearing pressures which cause the same settlements ρ_d. Inspection of typical stress strain and load settlement curves demonstrates that a load factor $L_f = \sigma_c/\sigma_d = 3$ leads to settlements of the order of ρ_d/B of the order of 0.1%. The rule of thumb using a load factor of about 3 to design foundations to limit settlements has a sound theoretical basis: the value of 3 is empirical although there is some theoretical justification.

8 CONCLUSIONS

A rule of thumb implies a rough estimate based on experience rather than formal calculation. In geotechnical engineering practice there are very many rules of thumb in common use and most experienced engineers have their own personal favourites.

Some rules of thumb have a sound theoretical basis while others are purely empirical and seem to have no theoretical basis. Rules of thumb can be classified by their theoretical standing following criteria set by Wroth (1984).

Rules of thumb in class 1 have a sound theoretical basis and do not require empirical observations. They can be used with confidence in many different circumstances. Rules of thumb in class 2 have a sound theoretical basis but require empirical correlations. Before these are employed in design the basis of the empiricism should be shown to be applicable. Rules of thumb in class 3 have no apparent theoretical basis. They are entirely empirical and so should be used with caution and only in the context within which they were developed.

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