Evaluating the Liquefaction Potential of Lower San Fernando Dam Sand in a Centrifuge

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ABSTRACT
A series of centrifuge tests was conducted on saturated models of Lower San Fernando Dam (LSFD) sand containing different silt contents to study the influence of non-plastic silt content on the liquefaction potential of LSFD sand. Relationship between energy per unit volume versus silt content was introduced at different effective confining pressures considering the effect of intergranular, interfine and global void ratios. Results indicated that with increasing silt content to a certain threshold value decreases the liquefaction resistance of LSFD sand.

RÉSUMÉ
Des essais en centrifugeuse ont été effectués sur des maquettes saturées de sable fin du LSFD, en variant le contenu de limon pour étudier l'influence du limon non-plastique sur le potentiel de liquéfaction du LSFD sable limoneux. La relation entre l'énergie par volume d'unité et la proportion de limon a été présentée pour différentes valeurs de pression de confinement efficace de façon à tenir compte des variations de vides intergranulaires, entre les particules fines ainsi que le taux de vide global. Les résultats obtenus indiquent qu'une augmentation de la proportion de limon, jusqu'à l'atteinte d'une valeur de seuil, réduit la résistance du LSFD à la liquéfaction.

1 INTRODUCTION

The Northridge earthquake in Los Angeles, 1994, which generated extensive damage to buildings, roadways, bridges, and other civil engineering structures, with evidence of liquefaction occurring in several locations of fairly uniform sand that contains up to 28% of fines, shows the great demand for evaluating the liquefaction potentials of silty-sands. During Imperial Valley earthquake of 1979 liquefaction occurred in silts with as little as 7 percent sand (Holzer et al. 1989). It is noticed that Lower San Fernando Dam (LSFD) silty sand has a lower liquefaction resistance than that of other sands at the same initial confining pressure (Dief 2000). The higher susceptibility to liquefaction of the LSFD silty sand can be attributed to the grain size difference and in particular to the substantially higher silt content found in this sand (up to 28%). The liquefaction potential of LSFD silty sand remains approximately constant for all relative densities, indicating that the influence of the relative density on the energy per unit volume is practically eliminated with increased silt content, regardless the value of the effective confining pressure as concluded by Liang (1995), and Figueroa et al. (1995). This indicated that the influence of relative density on the liquefaction potential of LSFD sand may be neglected with increased silt content, regardless of the effective confining pressure and that the presence of silt may have changed the kinematical behavior of the granular soil.

Previous studies showed that the liquefaction resistance of sands containing non-plastic fines initially decreases as fine content increases until some minimum resistance is reached, and then increases as fine content increases (Law and Ling 1992; Koester 1994). Recent studies (Guo and Parakash 1999; Amini and Qi 2000; Thavanayagam et al. 2000; Dief and Zeng 2009) focused on finding a definite criterion for evaluating the liquefaction potential of sands with non-plastic mixtures. The purpose of this study is to examine the influence of non-plastic silt content, voids ratio and effective confining pressure on the liquefaction potential of the silty LSFD sand.

2 EXPERIMENTAL CONFIGURATION

A dynamic centrifuge test series was conducted at a scale of 50gs on saturated models of LSFD sand containing different silt contents to study the influence of non-plastic silt content on the liquefaction potential of silty LSFD sand. In this series a total of 11 liquefaction tests prepared in a laminar box were conducted at a scale of 50gs using LSFD silty sand (Gs of clean sand (SP group) =2.65, and Gs of fines (ML group) =2.75) with different silt contents of 0.0, 5, 10, 20, 28 and 40 % at different nominal void ratios as shown in Tables 1 and 2. The input horizontal base acceleration time history for LSFD sand tests is simulated the base excitation of the centrifuge test model No 1 used in the VELACS Project (Arulanandan and Scott 1993) as shown in Figure 2. In this series the resistance in terms of energy per unit volume, required for the onset of liquefaction were determined. The non-plastic silt content, voids ratio and the initial effective vertical stress σ'_v as major factors to affect liquefaction of granular soils are considered in this study. The selected voids ratios for LSFD sand were extended to low values since Liang (1995) observed that this sand tends to liquefy at high densities. Lower San Fernando Dam (LSFD) sand, a fairly uniform sand that contains up to 28% of fines at natural conditions. This silty sand was collected from sand boils generated in the Lower San Fernando Dam, near Los Angeles, California during the Northridge Earthquake of January 17, 1994. To achieve the objectives of this research the selected sand is derived by washing the original soil through the No. 200 sieve. In this study, only void ratio is used as an index since there is no ASTM procedure for determining relative density of soils with fine content more than 15%.

The silty sand specimens were prepared by the dry pluviation method that shown to create a grain structure that duplicates closely the anisotropy observed in naturally alleviated marine-deposited sands (Miura and Toki 1984, Lade and Yamamuro et al. 1997) as well as dry pluviation provides very low depositional energy into the specimen which is necessary to imitate alluvial deposits in creating a loose compressible specimen (Yamamuro et al. 1999). Ishihara (1993) found that the steady state line was nearly the same for silty sand samples prepared at the same density by dry pluviation and water sedimentation methods. The accelerometers and the pore pressure transducers were placed at the proper orientations and locations during dry pluviation. The sample was de-aired by the application of vacuum to the prepared soil model and Carbon dioxide is used to displace the less soluble air that followed with the saturation process.

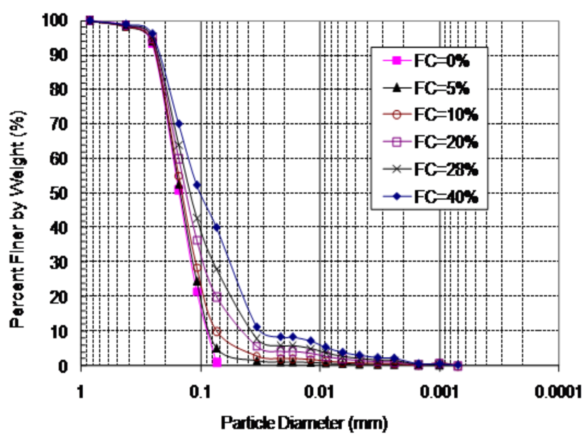

Figure 1. Grain-Size Distribution of LSFD Sand

3 TEST RESULTS AND ANALYSIS

The characteristic behaviour of saturated LSFD sand during an earthquake base excitation was evident in all experiments. A total of 11 tests including six groups of silt contents varying from 0.0% to 40% were conducted. A sketch of the laminar box and instrumentations used for the soil model is presented in Figure 3. A typical liquefaction test of LSFD sand with the silt content of 28% at natural conditions is selected to explain representative results. The specimen was prepared at an equivalent inter-granular void ratio of 1.52 (test # 9 in Table 2) representing a prototype thickness of 7.6 m. The corresponding total saturated and dry unit weights of the sand are 18.37 kN/m^3 and 13.7 kN/m^3 respectively.

Figure 4 shows the excess pore pressure ratios ($r_u = p/\bar{\sigma}_v$) time histories at various depths of the soil deposit. The excess pore pressure ratio of location P1 increased rapidly in the first 2.6 seconds of the base excitation. At this stage, the soil structure loses its integrity indicating initial liquefaction. From this point on, several decreases and increases in the excess pore pressure happen until the end of base excitation. All layers reached final liquefaction after 6 seconds of the start of shaking. As shown in the figure, the excess pore pressures at P1, P2, and P3 continued without any dissipation after stopping the base excitation. As shown in Figure 4, it is noticed that, the excess pore pressure recorded at sensor P4, which is located in the surface layer, began to dissipate at the 12 seconds point where the application of excitation was stopped.

The time histories of the recorded horizontal accelerations in the soil are given in Figure 5. Results show that the soil followed the base excitation only up to approximately 6 seconds from the start of shaking followed by a decrease of the acceleration signals until they are barely noticeable. Similar observations were made by Dewoolkar et al. (1999). The substantial decrease in the acceleration signals within the upper layers indicates excellent consistency among the results. By comparing the acceleration records with the time series of excess pore pressure ratios, an agreement between the acceleration spikes and the instantaneous drops in pore pressure is noticed.

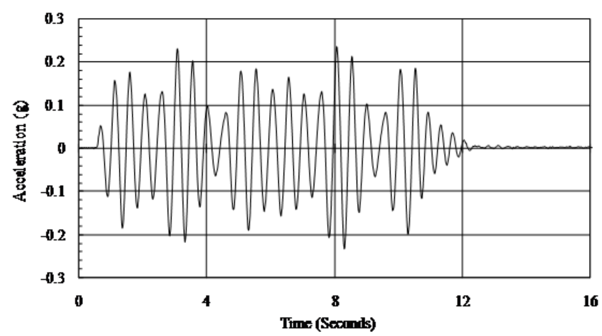

Figure 2. Prototype horizontal base acceleration time series

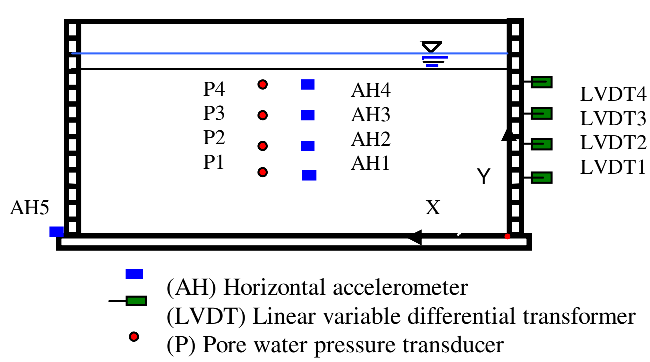

Figure 3. Schematic section of the centrifuge model and instrumentation

As shown in Figure 4, the pore pressure generation is not high enough during these cycles (r_u = 0.86) to produce liquefaction in the soil. At the end of the third cycle, the excess pore pressure exceeds the effective overburden pressure, which is initiated by the largest peak of acceleration of 0.21 g at this layer, as shown in Figure 5.

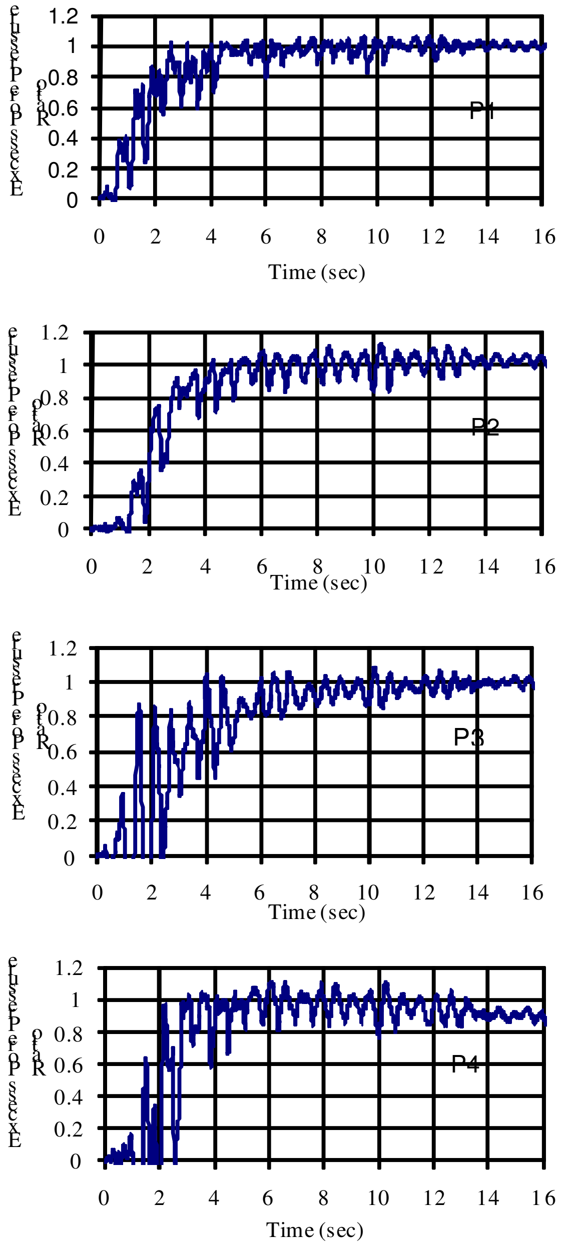

Figure 4. Excess Pore Pressure Ratio Time Histories, LSFD sand with 28% silt (Test #9)

4 APPLICATION OF ENERGY CONCEPT TO LIQUEFACTION

Nemat-Nasser and Shokooh (1979) introduced the energy concept for the analysis of densification and liquefaction of cohesionless soils and they related the dissipated energy to the number of cycles and the area of the hysteretic loop, which can be estimated from the number of stress cycles and the amplitude of shear stress or shear strain depending on the test condition. This dissipated energy can be estimated by calculating the area within the hysteric shear stress-strain loop and can be determined experimentally. Hysteresis loops can be approximated to ellipses to facilitate the determination of the unit energy absorbed by the specimen during a complete load cycle (Figueroa 1990). Davis and Berrill (1982), Law et al. (1990) and Kanamori et al. (1993) showed that it is possible to estimate the energy released by an earthquake.

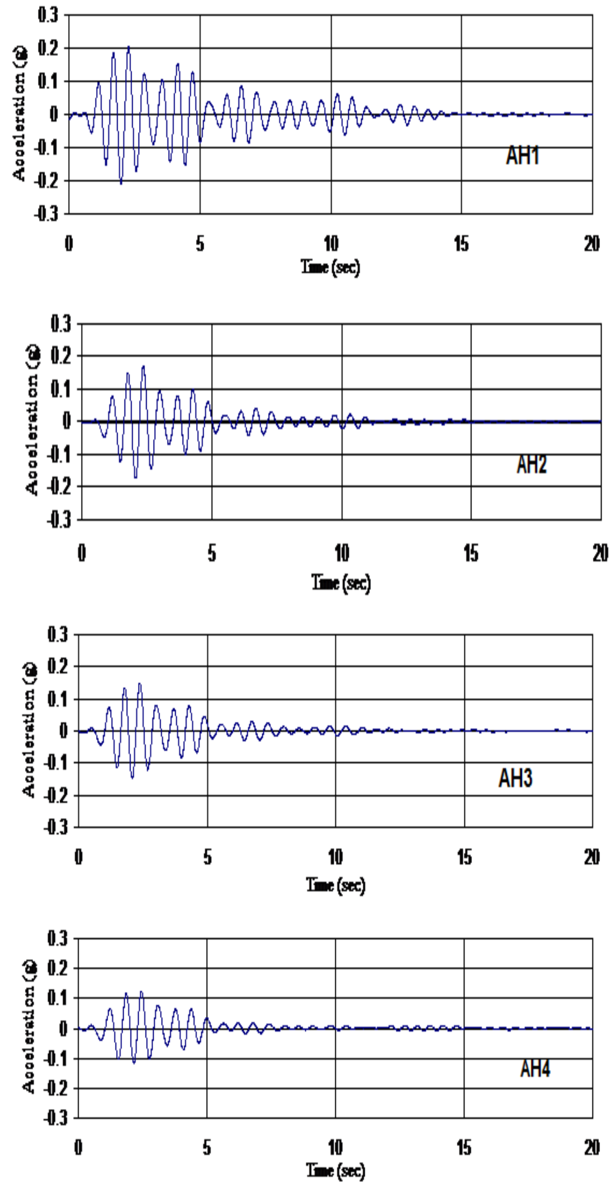

Figure 5. Horizontal Acceleration Time Histories, LSFD sand with 28% silt (Test #9)

Davis and Berrill (1982) assumed a linear relationship between the pore pressure build up and the dissipated energy. The nonlinear relationship proved later by Law et al. (1990); Figueroa and Dahisaria (1991); Dahisaria (1991); Liang (1995), Dief (2000) and Dief and Figueroa (2007). Extensive research using torsional shear-controlled strain liquefaction tests has been conducted at Case Western Reserve University to introduce and evaluate the energy concept in defining the liquefaction potential of soils when subjected to dynamic loads.

Geotechnical centrifuge modeling is considered the most appropriate method to simulate the boundary conditions, to predict actual prototype behavior and to represent a full soil deposit. Dief (2000) and Dief and Figueroa (2007) conducted a series of liquefaction centrifuge tests using the same soils previously examined at CWRU to compare the relationships developed through torsional series tests conducted by Liang (1995) and Rokoff (1999) to validate the energy concept in defining the liquefaction potential of soils when subjected to dynamic loads. The influence of relative density and effective confining pressure as well as the effect of different grain size distribution on the energy per unit volume required for liquefaction were studied, Dief (2000); Dief and Figueroa (2007).

In dynamic centrifuge testing, the seismic response of horizontal soil layers can be monitored to give shear stress and shear strain histories that can be determined for each layer. From the formed hysteretic loops, the amount of dissipated energy per unit volume can be determined for each layer up to the end of the earthquake (Dief 2000, Dief and Figueroa 2000, Dief et. al. 2001 and Dief and Figueroa 2007).

Figure 6 shows the variations of the total accumulated energy per unit volume versus time for each layer of this test. It is observed that the major contribution to the energy per unit volume occurs at the time of high pore pressure build up. After reaching the point of complete liquefaction, the specimen is not able to absorb any more energy because of the lack of shearing resistance, however a continuous increase in energy after reaching the point of complete liquefaction is observed, because of the inherent residual friction in the laminar plates. Referring back to Figures 4 and 6 it is observed that the accumulated energy per unit volume increases as the pore pressure increases and as the shear modulus decreases. As shown in Figure 6, the energy per unit volume needed for liquefaction increases in conjunction with a rise in the initial effective overburden pressure.

5 THE CORRELATION OF CONTACT INDICES WITH EXPERIMENTAL RESULTS

Global, intergranular and interfine void ratios are selected as constant parameters for all test series, Figure 7. Thevanayagam (2000) and Thevanayagam et al. (2002) proposed the equivalent intergranular void ratio $(e_c)_{eq}$ that considers the secondary influence of fines contributing to the active intergrain contacts and this relationship is represented by the factor b and $(e_c)_{eq}$ is considered as a modification of intergranular void ratio e_c (Mitchell 1993 and Vaid 1994) into a contact density index at $FC<FC_{th}$:

$$(e_c)_{eq} = \frac{e+(1-b)f_c}{1-(1-b)f_c}; \quad 0<b<1 \qquad [1]$$

$$e_c = \frac{e+f_c}{1-f_c} \qquad [2]$$

Where $f_c = f_c/100$; FC = fine grains content; f_{cth} = threshold fine grains content; b = portion of the fine grains that contribute to the active intergrain contacts; and e=global void ratio, Thevanayagam (2000). The $(e_c)_{eq}$ values for the two tested sand series are listed in Table 2.

Figure 8 shows that there is no unique relationship between the global void ratio and the energy per unit volume required for liquefaction of LSFD sand silt mix results however $(e_c)_{eq}$ showed a consistent relationship with these data at factor b=0.12 as shown in Figure 9 with $FC<FC_{th}$, Table 2. In this test series only one test was conducted with $FC>FC_{th}$ which made it difficult to obtain parameter *m* required for the equivalent interfine void ratio $(e_f)_{eq}$ at $FC>FC_{th}$ which defined by Thevanayagam et al. (2002):

$$(e_f)_{eq} = \frac{e}{f_c + \dfrac{1-f_c}{(R_d)^m}} \qquad [3]$$

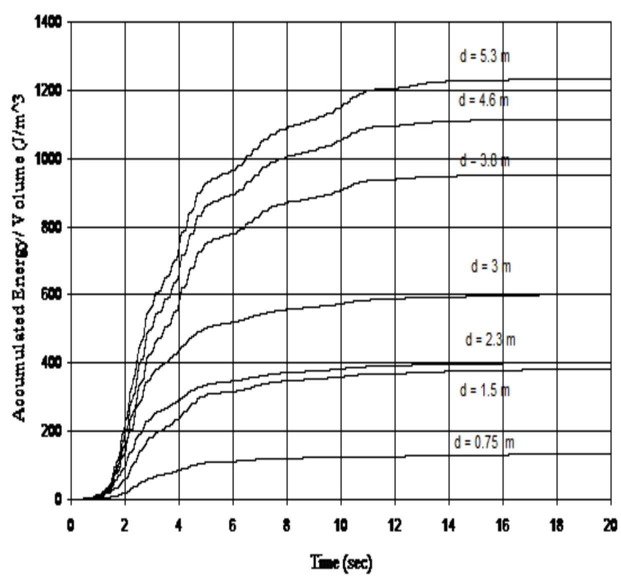

Figure 6. Accumulated Energy per Unit Volume Time History at Different Depths for LSFD Sand (Test #9)

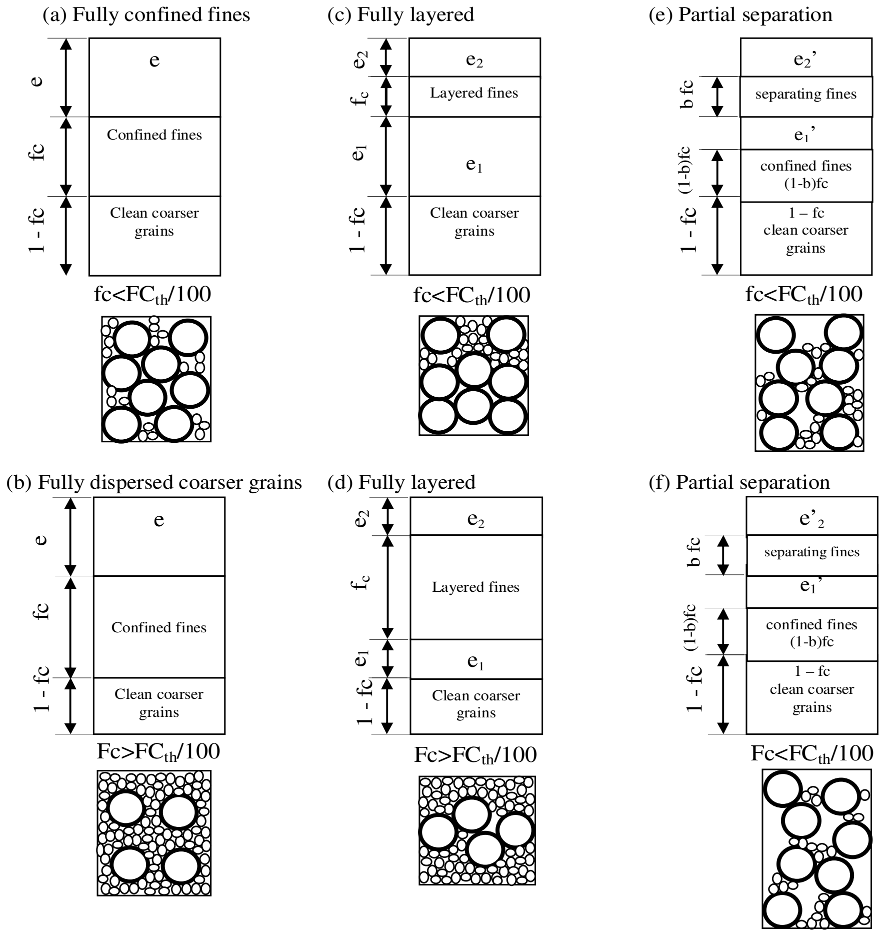

Figure 7. Phase diagram of microstructure and intergranular soil mix classification (after Thevanayagam 2000)

Table 1. Index properties for tested sand mixes

Sand	FC%	D_{50} (mm)	d_{50} (mm)	e_{maxHC}	e_{maxHf}	C_{uc}	C_{uf}	R_d	χ	b	m
LSFD 0.5*	0 to 40	0.151	0.043		0.95	1.526	1.95	6.94	3.5	2.0	0.12

*Based on data obtained from Tao (2003)

Table 2. Properties and microstructure classifications of LSFD sand slit mixes

Test #	% Fine	e	$(e_c)_{eq}$	e_f	$e_f > e_{max,HF}$	e_c	$e_c > e_{max,HC}$	Case	$(e_c)_{eq}$	$(e_f)_{eq}$
1	0	0.88	0.88	0	--	0.88	No	--	--	--
2	0	0.84	0.84	0	--	0.84	No	--	--	--
3	5	0.95	1.04	19	Yes	1.05	Yes	III	Yes	No
4	5	0.86	0.94	17.2	Yes	0.95	Approaching	II	Yes	No
5	10	0.95	1.12	9.5	Yes	1.16	Yes	III	Yes	No
6	10	0.85	1.02	8.5	Yes	1.06	Yes	III	Yes	No
7	20	0.95	1.34	4.75	Yes	1.44	Yes	III	Yes	No
8	20	0.86	1.23	4.3	Yes	1.32	Yes	III	Yes	No
9	28	0.94	1.52	3.36	Yes	1.69	Yes	III	Yes	No
10	28	0.85	1.41	3.0	Yes	1.57	Yes	III	Yes	No
11	40	0.76	1.93	1.52	No	2.52	Yes	IV	No	Yes

$e_{max,HC}$=0.949, $e_{max,HF}$=1.526, e: global void ratio, e_c: intergranular void ratio, e_f: interfine void ratio

However parameter *m* is estimated for LSFD sand-silt mix from 18 cyclic torsional shear tests, Table 1, that were conducted by Tao, 2003 on thin hollow cylindrical specimens of LSFD sand with silt contents >FC_{th} of 35% and 45% at different effective confining pressures and different void ratios. The $(e_f)_{eq}$ values for the LSFD sand series are listed in Tables 2. Case I, Figure 7, shows that finer grains just act as fillers and not active in the transfer of interparticles forces (Thevanayagam et al. 2000) and to fulfill this condition the coarser grain size has to be at least 6.5 times larger than the finer grain size (i.e. Rd=D50/d50 ≥ 6.5) (Thevanayagam et al. 2000). Table 1 shows that R_d= 3.5 for LSFD sand-silt mix therefore it was not possible to satisfy the conditions for case I.

The microstructure of these soil mixes showed an increase in both intergranular void ratio (e_c> $e_{max,HC}$) and interfine void ratio (e_f>$e_{max,HF}$) for LSFD sand-silt mix at FC 10%-28% while it was predictable that finer grains would actively contribute to the stress-strain response and match case III (Figure 7) at 0.0<FC<FC_{th}. The data for these sand-fines mixes fall in the vicinity of the data of the respective host clean sands (Thevanayagam et al. 2002) while some fines act in load transfer between some coarse grain particles and the reminder act as filler of voids (Thevanayagam et al. 2000) with limited threshold values of FC_{th}=28%. The existence of these non-plastic fines between the coarser particles reduces the interlocking shear strength mechanisms as well. Most of the LSFD sand-silt mixes (test # 3, 5 to 10, Table2) show an increase in the intergranular void ratio to indicate the existence of partial layering and partial separation of coarser grains of LSFD sand by the finer silt grains (case III) however FC= 5% and e_c=0.95 (test # 3, Table2) shows that most of the fine particles are confined and some of them support the coarser grain skeleton and passively participating in the internal force chain (case II), Figure 7. For LSFD silt mixes the expected FC_{th} values based on equation (3) (Thevanayagam et al. 2000) are ≤ 40%.

$$FC_{th} \leq \frac{100\, e_c}{1 + e_c + e_{\max,\, Hf}} \% \qquad [4]$$

Results for LSFD sand-silt mix (test # 11, Table2) shows that fines carry the contact and shear forces while the coarse grains may act as reinforcing elements embedded within the fine grain matrix (case IV).

LSFD sand-silt mixes with e~0.95 show an increase in the intergranular void ratio to indicate the existence of partial layering and partial separation of coarser grains of LSFD sand by the finer silt grains (case III). However, for e~0.86 it does not exclude the intergranular matrix from partial layering and partial separation (case III). Comparing LSFD test series of e~ 0.95 with e~0.86, the separation fines for e~0.95 is more than for e~0.86 resulting an increase of unstability for the intergranular matrix for e~0.95. Therefore, it is more susceptible for liquefaction than the second case and requiring less energy/unit volume to liquefaction compared with e~0.86, Figure 10.

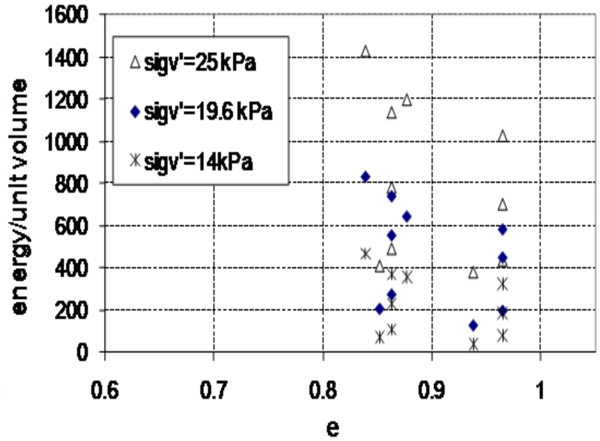

Figure 8: LSFD sand at different silt content with e

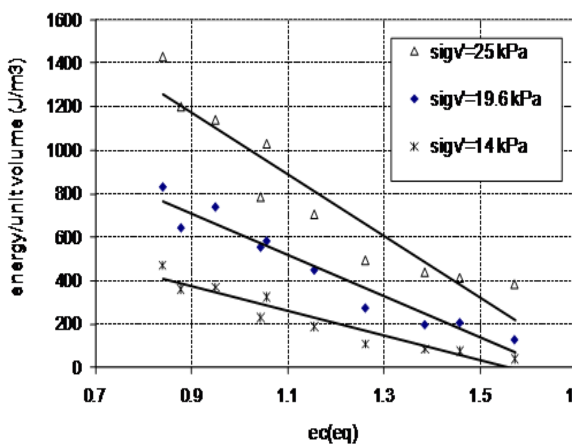

Figure 9: LSFD sand at different silt content with $(e_c)_{eq}$ with b=0.12

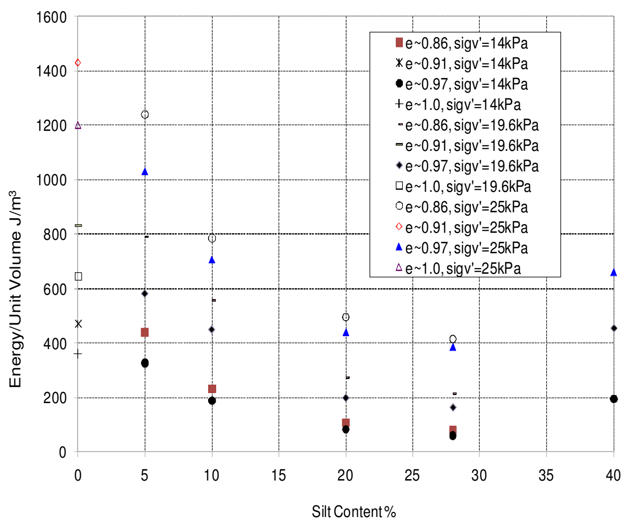

Figure 10: Energy/Unit Volume to liquefy at different silt content

6 CONCLUSIONS

Previous studies showed that the energy per unit volume required for liquefaction of LSFD silty sand remains approximately constant for all relative densities, indicating that the influence of the relative density on the energy per unit volume is practically eliminated with increased silt content. A total of 11 liquefaction tests were conducted on saturated models of silty LSFD sand with different silt contents to study the influence of non-plastic silt content on the liquefaction potential of silty sands. Parameters such as acceleration, displacement, and pore pressure were monitored throughout the tests. In these tests the energy per unit volume required for liquefaction was determined. Test results show that the energy per unit volume required for liquefaction decreases with increasing fines content to a certain threshold value then increases.

Test results showed that there is no unique relationship between the global void ratio and the energy per unit volume required for liquefaction of LSFD sand silt mix results. However, $(e_c)_{eq}$ showed a consistent relationship with these data with $FC<FC_{th}$. The microstructure of the tested soil mixes showed an increase in both intergranular void ratio ($e_c> e_{max,HC}$) and interfine void ratio ($e_f>e_{max,HF}$) at FC 10%-28% while it was predictable that finer grains would actively contribute to the stress-strain response and match case III at $0.0<FC<FC_{th}$.

Results for LSFD sand-silt mix at FC= 5% and e_c=0.95 shows that most of the fine particles are confined and some of them support the coarser grain skeleton and passively participating in the internal force chain (case II).

Results for LSFD sand-silt mix at FC 40% shows that fines carry the contact and shear forces while the coarse grains may act as reinforcing elements embedded within the fine grain matrix (case IV).

Comparing LSFD test series of e~ 0.95 with e~0.86, the separation fines for e~0.95 is more than for e~0.86 resulting an increase of unstability for the intergranular matrix for e~0.95 therefore more susceptible for liquefaction than the second case and requiring less energy/unit volume to liquefaction compared with e~0.86.

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