Tensile Strength of Frozen Soils Using Four-Point Bending Test

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ABSTRACT
The four-point bending test (FPBT) is one possible test method to measure tensile strength of unfrozen soils/rocks. FPBT was conducted on frozen Devon silt at temperatures between 0ºC and -10ºC, and at different loading rates (0.8 to 8 mm/min). Images taken during testing were used to determine strains thus allowing to follow the stress-strain curve. A clear dependency of the tensile strength on the temperature and on the loading rate could be identified. Frozen Devon silt develops significant tensile strength at temperatures close to 0ºC. Furthermore, the elastic modulus increases as the temperature and the loading decrease.

RÉSUMÉ
L'essai de pliage à quatre points (EPQP) est une méthode possible pour mesurer la résistance en traction des sols/roches dégelés. EPQP a été conduit sur le limon Devon congelée aux températures entre 0ºC et -10ºC, et à des taux de chargement (0.8 à 8 mm/min) différent. Des images prises pendant l'essai ont été utilisées pour déterminer des contraintes et suivre la courbe de contrainte-tension. Une dépendance claire de la résistance à la traction sur la température et sur le taux de chargement a pu être identifiée. Le limon Devon congelée développe la résistance à la traction significative aux températures près de 0ºC. En outre, le module élastique augmente que la température et taux de chargement diminue.

1 INTRODUCTION

The tensile strength of both unfrozen and frozen soils plays an important role in geotechnical problems involving tensile failure. For tensile failure to occur, the tensile stress in a soil must exceed the tensile strength of the soil. However, not much research has been done on the tensile strength of soils as compared to the compressive strength. This is mainly because tensile strength is considered insignificant and very small as compared to the compressive strength.

Tensile strength becomes more important once the soil is frozen. The limited studies available on frozen soils show that frozen soils have considerable tensile strength (e.g. Zhu and Carbee, 1987). This is due to their phase composition. Frozen soils are composed of four phases: soil solids, unfrozen water, frozen water (pore ice), and air (if not saturated) (e.g. Andersland and Ladanyi, 1994). The tensile strength of frozen soils depends on the relative proportion of each component.

The tensile strength is an important parameter in the study of ground patterns and ice wedge polygons in permafrost areas (Lachenbruch, 1962). It is also believed to play an important role in the frost heave process during the formation of ice lenses since the frozen fringe has to crack before the ice lenses are formed (Miller, 1978; Arenson et al., 2008; Azmatch et al., 2008). Hence, more research is required in the study of tensile strength of frozen soils especially near 0^{o}C.

The test methods that can be used to determine the tensile strength of soils can be broadly divided into two groups: (i) direct methods such as the direct tension test; and (ii) indirect methods such as split cylinder test, four point bending test, and Brazilian test. Tests proven to provide reliable results for tensile strength tests of unfrozen soils have been applied to test frozen soils: Zhu and Carbee (1985) and Haynes (1978) used direct tension tests; Bragg and Andersland (1980) used split cylinder tests.

Four-point bending test (FPBT) was used successfully to determine the tensile strength of unfrozen soils (Thusyanthan et al., 2007) and also to determine the tensile strength of soft rocks (Coviello et al., 2005). In this study, four-point bending test will be used to study the tensile strength of Devon silt.

The tensile strength of frozen soils depends on temperature, loading rate and unfrozen water content. The effect of each has been investigated by different authors (Zhu and Carbee, 1985; Zhu and Carbee, 1987; Bragg and Andersland, 1980; Haynes et al., 1975; Haynes, 1978).

Bragg and Andersland (1980) used split-cylinder tests to investigate the effect of strain rate on the tensile strength of frozen silica sand at a temperature of -6.0^{0}C. They found the tensile strength to be nearly independent of the deformation rate for values above 1.3 mm/min at -6.0^{0}C. Haynes (1978) conducted direct tension tests to investigate the effect of temperature, loading rate and unfrozen water content on the tensile strength of Fairbanks silt. He conducted the tests over a range of temperature values (-0.1^{0}C to -57.0^{0}C) and over a range of strain rates (1.6x10^{-4} s^{-1} to 2.9 s^{-1}). He stated that the tensile strength doubled over the strain rate range and increased about one order of magnitude over the temperature range. Zhu and Carbee (1987) investigated the effect of temperature, strain rate and density of the tensile strength of Fairbanks silt by using direct tension test method. Their investigation was over a temperature range from -1.0^{0}C to -10.0^{0}C and over a loading rate range of 5.9x10^{-4} mm/min to 5. 9x10^3 mm/min. The peak tensile strength of frozen silt was found to be very sensitive to strain rate. They concluded that for brittle failure, the peak tensile strength slightly decreases with increasing strain rate; and for ductile failure, it significantly decreases with decreasing strain rate. They determined that the peak tensile strength increases with decreasing temperature and that it increases more rapidly when the temperature is lower that -5.0^{0}C. They also concluded that the initial tangent modulus is independent on strain rate. Christ and Kim (2009) used direct tensile test to investigate the effect of moisture content and temperature on the tensile strength of frozen silt over a temperature ranging from -2.0^{o}C to -20.0^{o}C. They observed a strong dependence of the stress-strain behaviour of frozen silt on the moisture content and temperature.

In this study, four-point bending test was used to investigate the influence of temperature, strain rate and unfrozen water content on the tensile strength of frozen Devon silt.

2 EXPERIMENTAL PROGRAM AND MATERIAL STUDIED

2.1 Soil Properties and Sample Preparation

The soil tested is Devon silt with a specific gravity of 2.65, a clay fraction of 25% and a silt fraction of 75%. It has a liquid limit of 32% and plastic limit of 20%. Slurry of the soil sample is prepared at a moisture content of 55% and then consolidated at 100 kPa in a consolidation cell. Soil samples of dimension 304.8 mm x 76.2 mm x 76.2 mm are then cut out for the four-point bending test. The dimensions in the test set-up are as shown in Figure 1.

2.2 Soil Freezing

The sample to be used for the FPBT is placed in a freezing cell. The temperature of the freezing cell is controlled by flowing cold fluid through brass coils (placed inside the freezing cell) from a cooling bath. The temperature of the freezing cell is monitored by two RTDs placed at two corners within the cell. The sample is let to freeze isotropically to the desired temperature by placing it in the cell for a minimum of 24 hours.

2.3 Tensile Strength Test using Four-Point Bending Test

After the sample is frozen to the desired temperature, the flexural testing (FPBT) is carried out and digital images are taken at regular time interval during the test. A 15.1 megapixel digital camera (Canon EOS 50D) is used to take the images. The test set-up is as shown in Figure 2. The digital images, together with the marks engraved on the sample, are used to determine the strains. The stresses are determined using beam flexure theory. It is assumed that the frozen soil is elastic and elastic analysis is carried out.

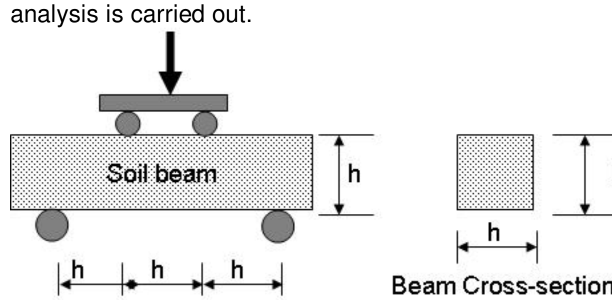

Figure 1. Sample dimensions, h = 76.2 mm.

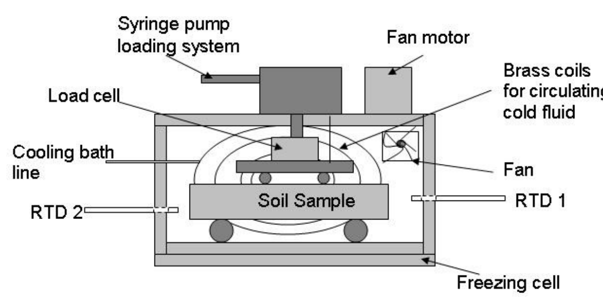

Figure 2. Schematics of (FPBT) tension test set-up.

2.3 Unfrozen Water Content

In a frozen soil, a certain amount of water remains unfrozen at subzero temperatures because of a decrease in the free energy of soil water due to surface forces of soil particles and the pore geometry among soil particles (Dash et al., 1995). There are a number of methods to determine the unfrozen water content (Anderson and Morgenstern, 1973). Some of the methods used are time domain reflectometry (TDR) method, calorimeters method, and nuclear magnetic resonance (NMR) method. For this study, unfrozen water content is measured by TDR. The TDR method measures the dielectric property which is then converted to volumetric water content by using the empirical equation provided by Topp et al. (1980). The TDR was first used for unfrozen soils. Its use was then extended to frozen soils (e.g. Patterson and Smith, 1980).

The TDR test for this study was carried on samples prepared similar to the samples used in the tensile strength testing. The temperature of the samples is also measured identically by using RTDs placed within the samples. The results were then used to create the soil freezing characteristic curve (which is the unfrozen water content versus temperature).

3 EXPERIMENTAL RESULTS AND DISCUSSION

Frozen Devon silt showed a significant increase in tensile strength compared to its unfrozen state. The experimental investigations show that frozen Devon silt exhibits considerable tensile strength even at subzero temperature values close to 0°C.

Figure 3 shows a sample loaded to failure. It is seen that the sample cracked just at the middle span. All the samples tested cracked at the middle-third span. The marks engraved on the sample are used to measure the strain development during the test.

Figure 3. Soil sample after loading.

A summary of the tensile test results discussed in this paper is presented in Table 1.

Table 1: Tensile strength test results.

Test Number	Temperature, θ (°C)	Loading Rate (mm/min)	Peak Tensile Strength, σ_T (kPa)
15	+2.25	0.8	7
10	-0.65	0.8	827
4	-0.95	0.8	965
9	-0.95	0.8	982
8	-1.40	0.8	1223
19	-3.9	0.8	1536
20	-5.45	0.8	2413
16	-5.45	3	2855
18	-5.45	8	3175
21	-9.0	0.8	3256

3.1 Effect of Subzero Temperatures on Tensile Strength

To investigate the effect of subzero temperatures on tensile strength of frozen soils, tests were conducted at different temperatures ranging from -0.65°C to -9.0°C. These tests were carried out under a loading rate of 0.8 mm/min. The results of these tests are as shown in Table 1. The results show that the peak tensile strength is significantly influenced by the temperature; the tensile strength increases with a decrease in temperature.

The tensile strength of the unfrozen soil was also determined at a temperature of +2.25°C. Devon silt in the unfrozen state has a peak tensile strength of 7.0 kPa under the test conditions in this study. An increase of two orders of magnitude (from 7.0 kPa to 827 kPa) is observed as the soil changed from an unfrozen state to a frozen state at a temperature of -0.65°C.

The tensile strength tests carried out over the temperature range of the frozen fringe (zone between 0°C isotherm and the base of the warmest ice lens during frost heave) in this study (-0.65°C and -0.95°C) indicate that the frozen fringe has considerable tensile strength (982 kPa at -0.95°C and 827 kPa at -0.65°C).

Zhu and Carbee (1987) suggested a relationship for the peak tensile strength of frozen soils as a function of temperature as:

$$\sigma_T = A(\theta/\theta_o)^m \qquad [1]$$

.

Where θ is the negative temperature in °C, θ_o is a reference temperature taken as -1.0 °C, and A (in kPa) and m are empirical parameters.

Figure 4 shows the variation of the peak tensile strength (σ_T) with temperature expressed as θ/θ_o. It was determined that for Devon Silt under the conditions of investigation A = 997.4 kPa and m = 0.49, in Eq. 1.

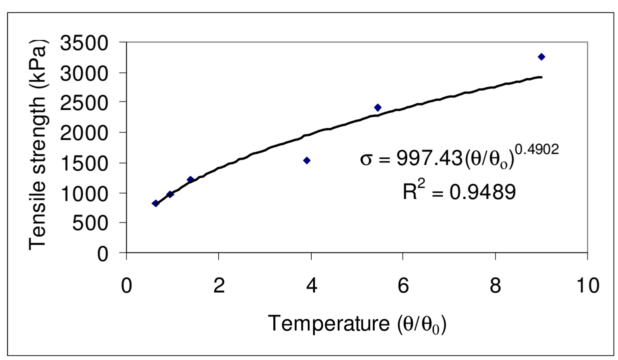

Figure 4.Tensile strength as a function of temperature.

3.2 Effect of Loading Rate on Tensile Strength

To investigate the effect of loading rate on tensile strength, tests at different loading rates were conducted on samples frozen at -5.45°C. The loading rates used were 0.8 mm/min, 3.0 mm/min and 8.0 mm/min. The results from these tests are also presented in Table 1. The results are plotted as shown in Figure 5. Only three data points are available, but the results show that the peak tensile strength is influenced by the loading rate. As the loading rate increases, the tensile strength increases.

Zhu and Carbee (1985) observed that for brittle failure, the peak tensile strength slightly decreases with increasing strain rate; and for ductile failure, it significantly increases with increasing strain rate. For the conditions of investigation in this study, it is observed that the tensile strength increases as the loading rate increases. Hence, it suggests that the soil behaved in a ductile manner under the conditions of investigation.

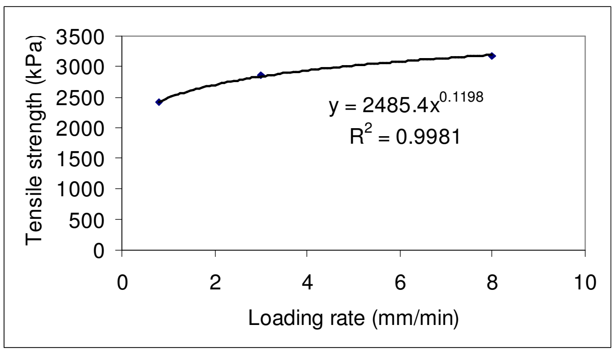

Figure 5. Tensile strength as a function of loading rate.

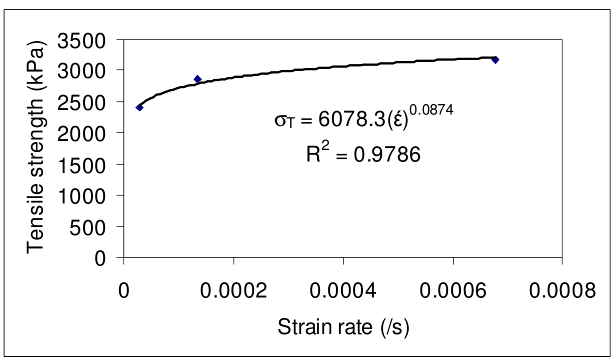

Figure 6.Tensile strength as a function of strain rate.

The variation of peak tensile strength with strain rate is shown in Figure 6. Haynes (1978) expressed the tensile strength as a function of strain rate by:

$$\sigma_T = A\dot{\varepsilon}^b \qquad [2]$$

Where σ_T is the strength in kPa and $\dot{\varepsilon}$ is the strain rate in s^{-1}; A (in kPa) and b are constant for a given temperature. This equation, for Devon Silt, is shown in Figure 6. The values of A and b in Eq. 2 for Devon silt are 6078 kPa and 0.087, respectively.

3.3 Relationship between Unfrozen Water Content and Tensile Strength

Unfrozen water content was measured using TDR to establish the relationship between tensile strength and unfrozen water content. The unfrozen water content variation with temperature for Devon silt is shown in Figure 7. The results from this study compared well with the results reported by Konrad (1990), who measured the unfrozen water content for Devon silt using calorimetery method. The unfrozen water content curve indicates that there is a steep decrease in unfrozen water content in a temperature range from 0°C to -1.0°C. The change in unfrozen water content is small from -1.0°C to -5.0°C. Then the unfrozen water content remains almost constant at 6.5%.

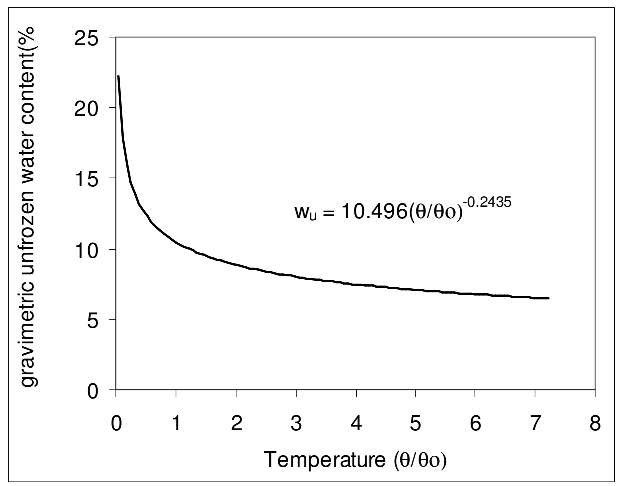

Figure 7.Freezing characteristics (unfrozen water content) curve for Devon Silt.

The dependence of unfrozen water content on temperature can be expressed as (Tice et al., 1976)

$$w_u = \alpha(\theta/\theta_o)^{\beta} \qquad [3]$$

Where θ is the negative temperature in ^{0}C; θ_o is a reference temperature taken as -1.0^{0}C; α and β are empirical parameters; and w_u is the gravimetric unfrozen moisture content expressed in percentage. For Devon silt consolidated at 100 kPa, the values of α and β in Eq. 3 are 10.50 and -0.244, respectively.

By using the temperature-tensile strength relationship and temperature-unfrozen water content relationship, the relationship between unfrozen water content and tensile strength can be established. This relationship for Devon silt consolidated at 100 kPa is shown in Figure 8. For a small change in unfrozen water content (e.g. from 5.5 % to 10.5%), the tensile strength changes significantly (from 3200 kPa to 800 kPa).

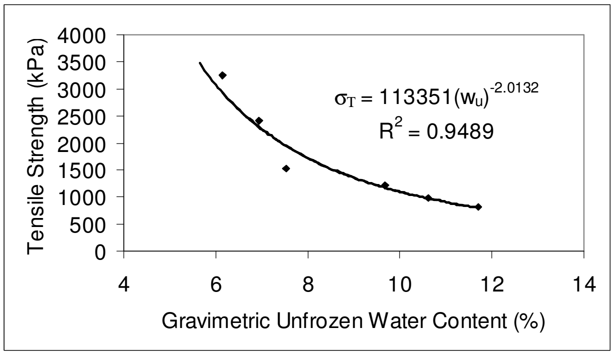

Figure 8. Effect of unfrozen water content on tensile strength.

3.4 Stress-Strain Relationship and Modulus of Elasticity

The stress-strain diagrams for the tests conducted at different temperatures but at a loading rate of 0.8 mm/min are shown in Figure 9. Figure 10 shows the stress-strain relationships for the tests conducted at different loading rates at a temperature of -5.45°C. ImageJ software is used in calculating the strains. The digital images taken at different times during the test together with the linear marks engraved on the soil sample made the strain measurement possible. The change in length of the linear marks was measured using ImageJ software.

3.4.1 Modulus of Elasticity

The modulus of elasticity values, calculated from the initially linear portion of the stress-strain diagram are shown in Table 2. Figure 11 shows the variation of modulus of elasticity with temperature. The modulus of elasticity increases significantly with a decrease in temperature. It is also influenced by loading rate.

Table 2: Modulus of elasticity values.

Test Number	Loading rate (mm/min)	Temperature (°C)	Modulus of Elasticity (MPa)
10	0.8	-0.65	21.50
8	0.8	-1.40	43.50
19	0.8	-3.90	163.47
20	0.8	-5.45	356.42
16	3.0	-5.45	134.30
18	8.0	-5.45	120.24

3.4.2 Strain at Failure

The failure strain as a function of temperature for a given loading rate is shown in Figure 12. The failure strain decreases with a decrease in temperature: It decreased from 14.35 % at -0.65 °C to 5.84 % at -5.45 °C. Similar trend was observed by Zhu and Carbee (1987).

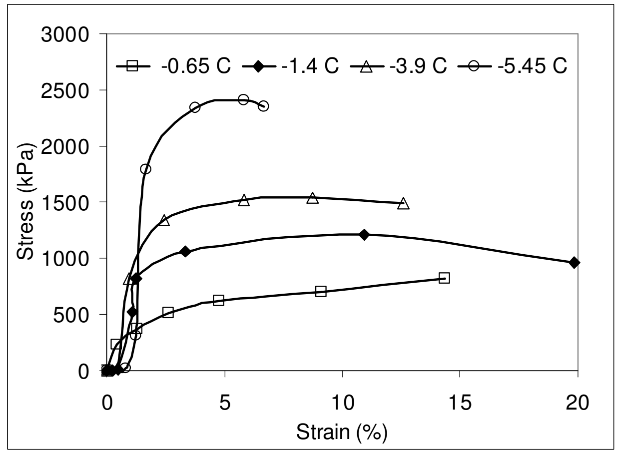

Figure 9.Stress-strain plot for the tension tests at different temperatures for a loading rate of 0.8 mm/min.

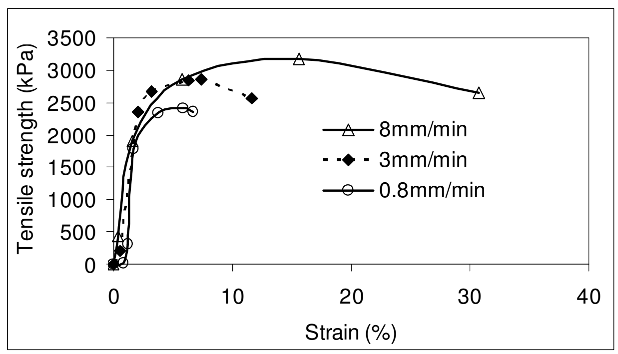

Figure 10.Stress-strain plot for the tension tests at different loading rates at a temperature of -5.45 °C.

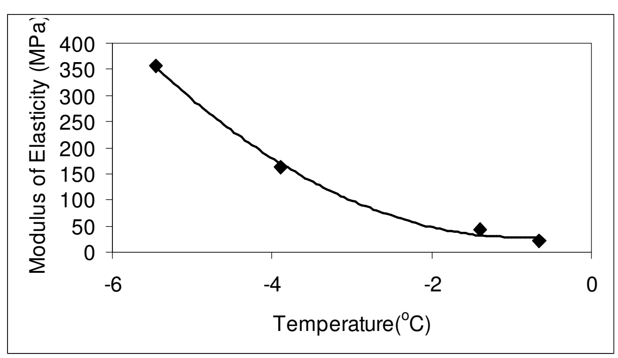

Figure 11. Variation of modulus of elasticity with temperature.

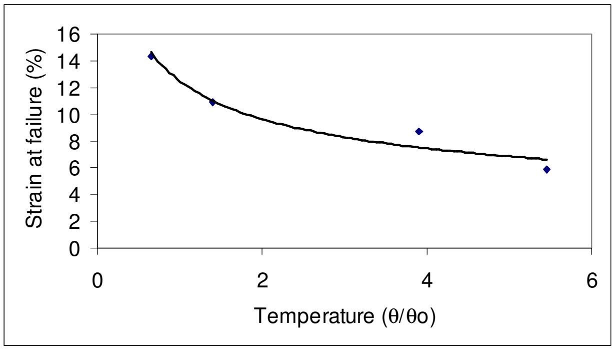

Figure 12. Variation of failure strain with temperature.

4 CONCLUSION

Four-point bending test was used to investigate the tensile strength of Devon silt. The tests were conducted on samples prepared by consolidating slurry of Devon silt at 100 kPa. The influence of subzero temperatures, loading rate/strain rate, and unfrozen water content on tensile strength of Devon silt was investigated.

To investigate the influence of subzero temperatures on tensile strength, tests were conducted at temperatures ranging from -0.65°C to -9.0°C. The peak tensile strength increased as the temperature decreased. It changed from 827 kPa at -0.65 °C to 3256 kPa at -9.0 °C.

To investigate the influence of loading rate on tensile strength, tests were carried out at loading rates of 0.8 mm/min, 3.0 mm/min and 8.0 mm/min on samples frozen at -5.45°C. The results showed that the peak tensile strength increased as the loading rate increased. It increased from 2413 kPa at 0.8 mm/min to 3175 kPa at 8.0 mm/min.

Frozen Devon silt has significant tensile strength even at negative temperatures close to 0°C. Devon silt at unfrozen state possessed a tensile strength of 7 kPa. It showed a peak tensile strength of 827 kPa at a temperature of -0.65°C.

The influence of unfrozen water content on tensile strength was examined by determining the unfrozen water content using TDR. From the relationship established between gravimetric unfrozen water content and tensile strength, it is observed that a small change in unfrozen water content produced a significant change in tensile strength. For example, a change in gravimetric water content from 10.5 % to 5.5 % results in a change in the peak tensile strength from 800 kPa to 3200 kPa.

The tests carried out over the temperature range of the frozen fringe (-0.65°C and -0.95°C) indicate that the frozen fringe possesses considerable tensile strength: 827 kPa at -0.65°C and 982 kPa at -0.95°C. Additional tests are being carried out on the tensile strength under temperatures found in the frozen fringe.

The stress-strain plots showed that the modulus of elasticity is influenced by the freezing temperature and the loading rate: it increased from 21.5 MPa at -0.65°C to 356.4 MPa at -5.45°C; it decreased from 356.4 MPa at 0.8 mm/min to 120.4 MPa at 8.0 mm/min.

The failure strain was influenced by the freezing temperature. It decreased from 14.35 % at -0.65 °C to 5.84 % at -5.45 °C

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