Modelling coupled sedimentation and consolidation of fine slurries

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ABSTRACT
A model for one-dimensional simultaneous sedimentation and consolidation of a solid-liquid suspension has been derived, using permeability as the unifying concept. The numerical solution is based on an explicit finite difference procedure in material coordinates, with Euler forward-marching scheme in time. The numerical solution solves the problem of internal discontinuities through a combination of convenient theoretical assumptions and the use of Lagrangian coordinates. The model is implemented in Java-based SECO software, with a user friendly graphical interface allowing visualization and animation of the solution process. The model is validated with a fine-grained suspension data set.

RÉSUMÉ
Un modèle de sédimentation et consolidation simultanée unidimensionnelle a été dérivé comme une théorie de mélange, en utilisant la perméabilité comme le concept unifiant. La solution numérique présentée est basée sur une procédure de différence finis explicites en coordonnées matérielles, avec système d'avant-marching Euler dans le temps. La solution permet d'éviter le problème des discontinuités numériques internes grâce à une combinaison de pratiques des hypothèses théoriques et l'utilisation des coordonnées lagrangienne. Le modèle est implémenté dans le logiciel SECO basé sur Java, avec une interface graphique conviviale permettant la visualisation et l'animation du processus de solution. Le modèle est validé avec un ensemble cohérent de données matériel.

1 INTRODUCTION

Sedimentation is a process of the settling of solid particles dispersed in a liquid. The process is driven by gravity and is caused by the inability of the fluid to sustain long-term shear stresses. During sedimentation solid particles gradually form sediment, a saturated soil layer, at the bottom of the suspension column. Consolidation is a time-dependent compaction of the soil skeleton in sediment as a result of a load: self-weight as well as external loads exerted on it. Both processes are of theoretical and practical interest in diverse scientific and technological fields, from chemistry and material engineering to geology and geotechnical engineering.

Historically, two major theories describing these processes have emerged independently: sedimentation has been a concern of chemists, while consolidation has traditionally remained a geotechnical subject. It is not surprising, therefore, that until recently there have been no attempts to integrate the separately developed models into one unifying theory.

Sedimentation became a geotechnical concern in recent times when modern mining industries began to produce enormous quantities of waste material in the form of slurry. These slurries, or tailings, have a high proportion of fine particles of colloidal size and exhibit exceptionally small rates of sedimentation; therefore, the sedimentation from the suspension and the consolidation of sedimented material occur simultaneously.

In geotechnical engineering, sedimentation has often been treated as the consolidation of very loose soils, making use of non-linear finite strain consolidation theories. However, significant differences are encountered when the results of such analyses are compared with observed or experimental responses. Need has thus arisen for a method that will be able to simulate and successfully predict the essential elements of the sedimentation and consolidation as a coupled process.

2 PHENOMENOLOGY

Through extensive experimental work, Been and Sills (1981) have clarified the basics of a continuous process of sedimentation and consolidation of solid particles from an initially suspended state.

Three modes, or regimes, of settling behaviour were identified: free settling in dilute suspensions, hindered settling in concentrated suspensions, and self-weight consolidation. Among these three modes, the hindered settling phenomenon is relatively unfamiliar to the geotechnical engineers. It is characterized by the sedimentation of a suspension as a whole, as if the particles were in a spatial network, but without stable direct contacts and measurable effective stresses.

During settling, zoning occurs in an initially uniform suspension. Three principal zones are distinguished: almost clear water at the top, a suspension in the middle and sediment at the bottom (Figure 1). The settling behaviour of these zones in general corresponds to the above three settling regimes.

Settling velocity of particles in suspension is found to be dependent on their concentration as the main variable (Kynch, 1952). Settling velocity of solids in sediment depends on both permeability and compressibility of the solid structure (skeleton), which are functions of the porosity *n* (i.e. concentration $c = 1 - n$) of the solid phase.

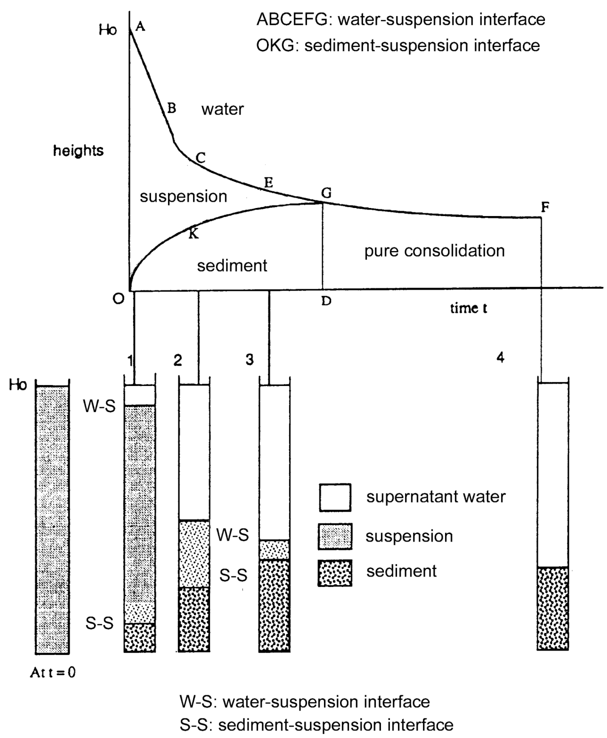

Figure 1. Settling behaviour of a suspension (Been and Sills, 1981)

3 MODELLING

A model for simultaneous sedimentation and consolidation in 1-D was developed by Masala (1998). The sedimentation model does not consider the free settling regime of dilute suspensions, where segregation may occur; it covers only the stage of hindered settling. A full description of the modelling process is described elsewhere (Masala and Chan, 2001), while here we present only a brief description, stating the assumptions made and the final governing equations derived.

It should be noted that the foregoing theories neglect any time-dependent phenomena in the system; specifically, thixotropy in the suspension and creep in the sediment are not considered.

The models for both phenomena were derived beginning from the same theoretical basis – two laws of continuum mechanics: the law of conservation of mass and the principle of linear momentum.

3.1 The conservation of mass

The mass balance (or continuity) equation is derived from considering the suspension as a mixture composed of solid and fluid phases (Figure 2). Both are assumed incompressible so that the change in the total density of the mixture is completely convective—a plain result of the change in the proportion of the components within a control volume due to different fluxes over its boundaries. The same continuity equation is valid over both suspension and sediment regions.

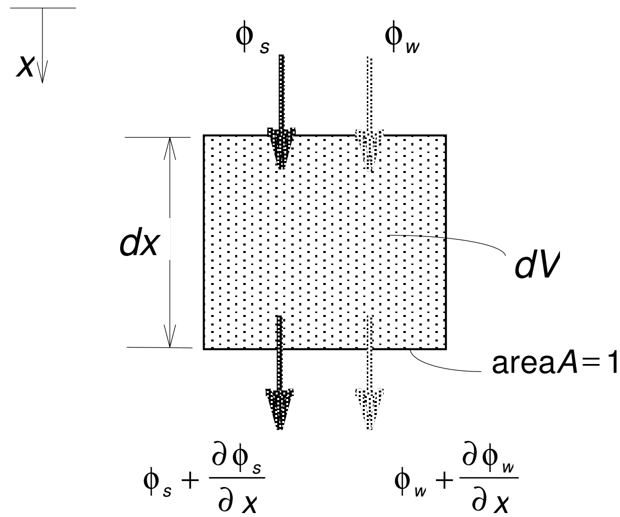

Figure 2. Mass balance for a solid-liquid mixture

3.2 Equilibrium equation

It is assumed that the acceleration terms in the equations of motion vanish ('quasi-static' conditions). As a consequence, the application of the method is restricted to 'slowly settling' suspensions.

Derivation of the equilibrium equation in a one-dimensional case is elementary; see for example Gibson et al. (1967). Considering the settling system as a mixture, the total axial stress is expressed as the sum of the stresses acting in the solid and the liquid phase (Figure 3). The total density is also expressed as the sum of partial densities of the mixture components.

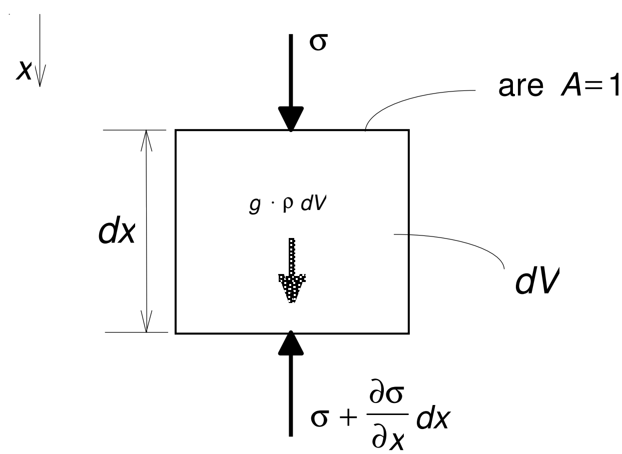

Figure 3. Equilibrium of an elemental volume

Two separate equilibrium equations must be written in this case. In the region of settling suspension, the effective stress in solids vanishes and the total stress is equal to the fluid pressure: $\sigma = u$ (i.e. a suspension behaves as a "heavy" liquid). In the region of consolidating sediment, the total stress is the sum of fluid pressure and a finite effective stress: $\sigma = \sigma' + u$.

3.3 Connecting continuity and equilibrium

The missing equation to complete the problem is a definition of the mutual interaction force between the solids and the liquid. In this derivation we have introduced the permeability as a measure of the interaction in a solid-liquid mixture, using Darcy-Gersevanov's law:

$$v_w - v_s = -\frac{k}{n}\frac{1}{\gamma_w}\frac{\partial \bar{u}}{\partial x}$$

[1]

where: v_w and v_s are true velocities of water and solids, k is permeability, n is volumetric concentration of water (or porosity in geotechnical terms), γ_w is the unit weight of water, $\bar{u}$ is excess pore water pressure (over hydrostatic) and x is the spatial coordinate.

It is worth noting here that the concept of permeability has already been extended to the sedimentation stage and a suspended state of solids by Pane and Schiffmann (1997). In other words, it was assumed that the expression for the coefficient of permeability applies smoothly over a range of void ratios which encompasses both soil and suspension, regardless of whether effective stresses are present or absent. Permeability thus appears as a measure of hydrodynamic interaction between solid and liquid in a suspension. In a very general sense, it may be understood as a 'constitutive law' for the solid-fluid interaction in suspensions.

3.4 Governing equations

The following governing equations were derived for sedimentation and consolidation, respectively:

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}\left\{c^2 k\left(G_s - 1\right)\right\} = 0$$

[2]

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}\left\{c^2 k\left(G_s - 1\right) - c\frac{k}{\gamma_w}\frac{\partial \sigma'}{\partial c}\frac{\partial c}{\partial x}\right\} = 0 \qquad [3]$$

where: c = volumetric concentration of solids; G_s = specific weight of solids, γ_w = unit weight of liquid, $k(c)$ and $\sigma'(c)$ are the permeability and compressibility relationships, both being functions of porosity n (i.e. solids concentration c).

Both equations are expressed in terms of solids concentration c as the independent variable. The use of solids concentration c is preferred over void ratio e, which is typical in soil mechanics, because of its mathematical convenience. In the sedimentation stage e becomes infinite when all particles have settled and concentration decreases to zero.

It is easily seen that the governing equation for sedimentation may be obtained from the governing equation for consolidation by simply setting $\sigma' = 0$. This shows that the concept of permeability offers a base for the unification of the theories of sedimentation and consolidation.

Sensitivity analysis of the model shows that the principal parameters for settlement behaviour are permeability and initial concentration, while compressibility and critical concentration (the transitional concentration between suspended and sedimented state) are of dominant importance for the solids content distribution in the sediment.

4 NUMERICAL SOLUTION

The governing equation for sedimentation (2) is a hyperbolic partial differential equation (PDE) while the consolidation equation (3) is of the parabolic type. Both equations are derived in the same form – the conservation law form, which again gives evidence for the fact that both processes have the same underlying physics.

4.1 Difficulties with numerical solution of coupled system

The 'weak link' in the governing system is the sedimentation equation, as a hyperbolic PDE. The solution of such equations may have discontinuities which originate from the discontinuities in the initial and boundary data or, in the case of a non-linear PDE - equation (2) - discontinuities may develop at some later time even from smooth initial data.

In our problem, discontinuities such as the water-suspension and the suspension-sediment interface normally appear in a solution, and they are an essential part of the solution. The discontinuities propagate along characteristic lines, and in any particular sedimentation problem, the upper boundary of the suspension layer is a characteristic line. The sediment-suspension interface is not a discontinuity in its original meaning in PDE theory, but rather a consequence of the 'redundant' boundary conditions at the bottom boundary. Nevertheless, it behaves as a discontinuity in a numerical scheme.

It is important to mention that two phenomena, numerical dispersion (artificial oscillations) and numerical diffusion (artificial dissipation - change of shape of the solution profile), are responsible for the deviations of a particular numerical solution from the actual one.

4.2 Solution method

Although the governing equations were derived in spatial coordinates, the successful numerical procedure in fact makes use of a Lagrangian finite difference grid, which follows the motion of the material. A similar idea was applied in the moving mesh method, which uses an initially uniform grid that deforms with time and the nodal points which move relative to each other, adapting their spatial distribution to the requirements of an optimal description of the deformation process. When each nodal point remains connected to the same material point to which it had been connected in the initial state, then one says that the grid is a Lagrangian one.

A simple Eulerian time-forward marching scheme was chosen for advancing the solution in time. Although not very sophisticated by itself, it behaved well in the examples calculated. Sporadic problems with numerical dispersion (oscillations) could be removed by reducing the time step. This approach works well for reasonably smooth initial conditions (approximately uniform suspension in the initial state).

In the solution presented, the water-suspension discontinuity is dealt with explicitly, while the suspension-sediment interface is actually 'smeared off' between the two consecutive grid nodes that bound a temporary position of this discontinuity.

Spatial discretization of the settling column is shown in Figure 4. The sedimenting system is divided into an array of equal cells with central nodal points. The origin of the spatial coordinate x is placed at the surface of the suspension.

The state variable is the volumetric concentration c(x,t), i.e. in the nodal points c_i^n, where the subscript i denotes the spatial position (node), while the superscript n denotes the time instant (Figure 4). It is assumed that the initial state is known, i.e. that $c(x,0) = c_0(x)$.

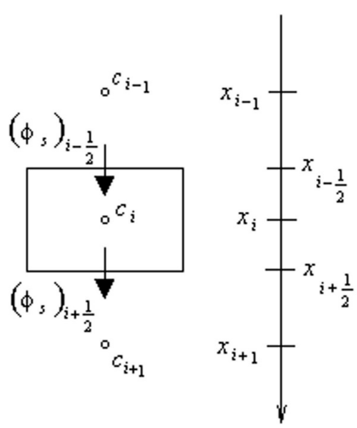

Figure 4. Numerical scheme: discretization

The solid fluxes over the cell boundaries are denoted by a fraction in the subscript, see Figure 4.

The system of governing equations is expressed in the form of conservation law for both regions of sedimentation (the suspension) and consolidation (the sediment):

$$\frac{\partial c}{\partial t}+\frac{\partial(\phi_s)_{sed}}{\partial x}=0 \quad ; \quad \frac{\partial c}{\partial t}+\frac{\partial(\phi_s)_{cons}}{\partial x}=0 \qquad [4]$$

$$(\phi_s)_{sed}=c\cdot(v_s)_{sed} \quad ; \quad (\phi_s)_{cons}=c\cdot(v_s)_{cons}$$
[5]

where: v_s = settling velocity of solid particles. Its value is calculated according to two different expressions, depending on the region:

$$(v_s)_{sed}=(G_s-1)\cdot c\cdot k$$
[6]

$$(v_s)_{cons}=(G_s-1)\cdot c\cdot k-\frac{k}{\gamma_w}\frac{d\sigma'}{dc}\frac{\partial c}{\partial x}$$
[7]

The calculation cycle consists of the following steps:

(1) calculate concentrations at cell boundaries as the averages of concentrations in adjacent nodes:

$$c^n_{i+\frac{1}{2}}=\frac{1}{2}\left(c^n_{i+1}+c^n_i\right) \; ; \qquad c^n_{i-\frac{1}{2}}=\frac{1}{2}\left(c^n_i+c^n_{i-1}\right) \qquad [8]$$

(2) calculate boundary velocities and solid fluxes

$$(v_s)^n_{i+\frac{1}{2}}=v_s\left(c^n_{i+\frac{1}{2}}\right) \quad ; \quad (\phi_s)^n_{i+\frac{1}{2}}=c^n_{i+\frac{1}{2}}\cdot v_s\left(c^n_{i+\frac{1}{2}}\right) \qquad [9]$$

(3) update geometry:

$$x^{n+1}_{i+\frac{1}{2}}=x^n_{i+\frac{1}{2}}+(v_s)^n_{i+\frac{1}{2}}\cdot\Delta t \quad ; \quad \Delta x^{n+1}_i=x^{n+1}_{i+\frac{1}{2}}-x^{n+1}_{i+\frac{1}{2}}$$
[10]

(4) update concentrations:

$$c_i^{n+1} = \frac{(V_s)_i^0}{\Delta x_i^{n+1}}$$
[11]

$$(V_s)_i^0 = c_i^0 \cdot \Delta x_i^0$$
[12]

$(V_s)_i^0$ being the total volume of solids in cell *i* at time t = 0.

The boundary conditions consist of prescribed solid and fluid fluxes at the outer boundaries of the first and the last cell. These are transformed into corresponding concentrations in two imaginary cells before the first and after the last cell, and two additional imaginary nodes in these cells, denoted as nodes (0) and (N+1); for example, for zero solid fluxes at both ends:

$$c_0^n = -c_1^n \Rightarrow (\phi_s)_{\frac{1}{2}}^n \equiv 0$$
[13]

$$c_N^n = -c_{N+1}^n \Rightarrow (\phi_s)_{N+\frac{1}{2}}^n \equiv 0$$
[14]

Since the x-coordinate origin is set at the suspension surface, the settlement of the water-suspension interface is merely the displacement of the left boundary of cell 1. The internal boundary between suspension and sediment is determined in each step by comparing the nodal concentrations with the critical concentration c_{cr}.

5 SECO SOFTWARE

The original software for numerical solution of the coupled sedimentation – consolidation problem (Masala, 1998) was programmed in Fortran 90, with a textual user interface for data input, while the output was directed to a custom structured textual file that was further plotted in a spreadsheet/charting commercial program. This additional step (data files) used to make the whole analysis a little cumbersome, particularly when a quick parametric analysis had to be conducted, or specific observed or measured data had to be accurately fitted. Input files were then introduced to save time by avoiding re-typing of data, but a disadvantage was the need to use text editors to inspect and modify file content. Although successful applications of the original SECO software were recorded at a graduate research level (Azam, 2003; Bartholomeeusen et al, 2002) it was assessed that the program's user (un)friendliness was a main obstacle for its wider industrial application. In an attempt to overcome this problem, a modern graphical user interface (GUI) has been developed as a part of a new version of the SECO software. It should be noted that this software is under development and all options have not been implemented yet.

5.1 Functionality and GUI

SECO functional flowchart is shown in Figure 6. A problem can be defined from scratch or using an input file. Input data are checked for completeness and correctness; they can be modified at any time before the solution process is started. The results of analysis can be visualized during solution or after it has been completed. Solution can be paused, then restarted or stopped. Problem definition and solution results may be stored in a file.

This scheme is realized in SECO GUI, designed as a Multiple Document Interface (MDI) software, using Swing Application Programming Interface (API) (Sun, 2003). The MDI technique uses a single application primary window, called a parent or main window (Figure 7) to visually contain a set of related documents or child windows. Each child window is constrained to appear only within the parent window instead of on the desktop. The examples of child windows are input panels and output charts.

The menu bar in the main window contains five menus: File, Input, Analysis, Output and Help. Some of the menu bar functionality is replicated by the button bar.

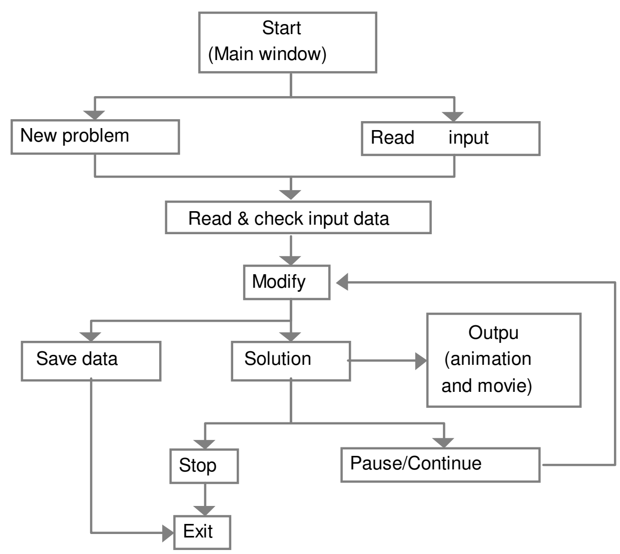

Figure 6. SECO functional flowchart

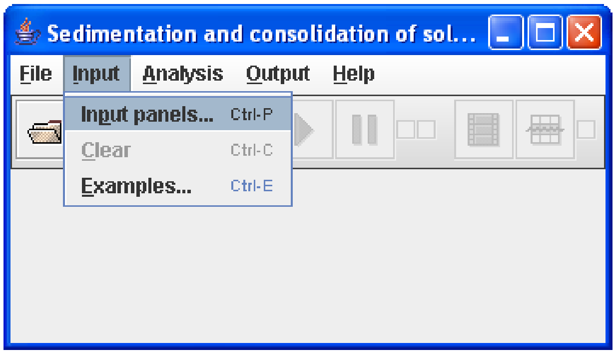

Figure 7. SECO Main window

All operations relating to problem creation from a file or saving to a file are grouped under the File menu. The files are intended for both input and output, if there is any. The files have a predetermined structure that is checked by the software on input.

Interactive input is provided in the Input menu. Each data group (material properties, geometry, initial and boundary conditions, etc.) has its own input panel (see for example Figure 8). An elementary check of entered data is automatically performed and, if they are in error, a message will be displayed indicating the source of error. One menu item is provided here specifically to facilitate the use of the program by novice users. The Examples submenu lists a number of typical situations (stagnant pond, filling pond, etc.) with predefined input data that the user can modify to his needs and save under a different name.

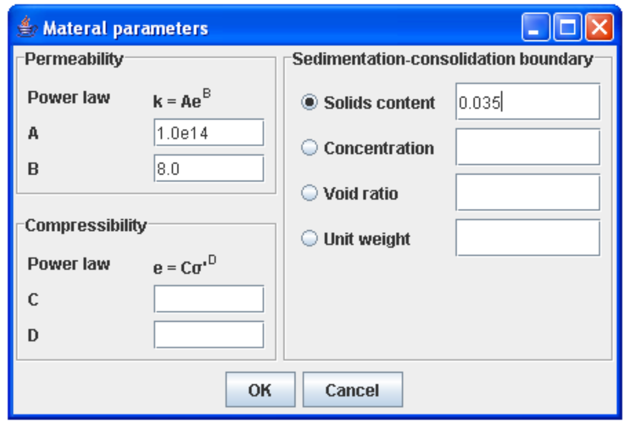

Figure 8. SECO Material properties input panel

The Analysis menu contains all tools needed to start and manage the solution procedure: pause it, then continue or abort it. As analysis results are typically shown ‘live’ on-screen, this allows the user to interrupt a solution that produces undesirable output.

The Output menu provides a selection of visualization options for an ongoing or a completed analysis. It is possible to plot a settlement-time chart and a number of profiles (concentration, pore pressure, effective stress, etc.). All charts are created using a free Java class library JFreeChart (JFree, 2005); see Figure 9 for an example of a graphical output window. All graphical output can be saved as a still picture in the JPEG format. In addition, it is possible to create a digital video in the QuickTime movie format, containing the animation of the output charts. The video is created at the end of analysis from a series of still pictures taken at regular times during the computation.

The Help menu provides only essential program information at the moment, but a full cross-linked description of the program options is planned for the final version.

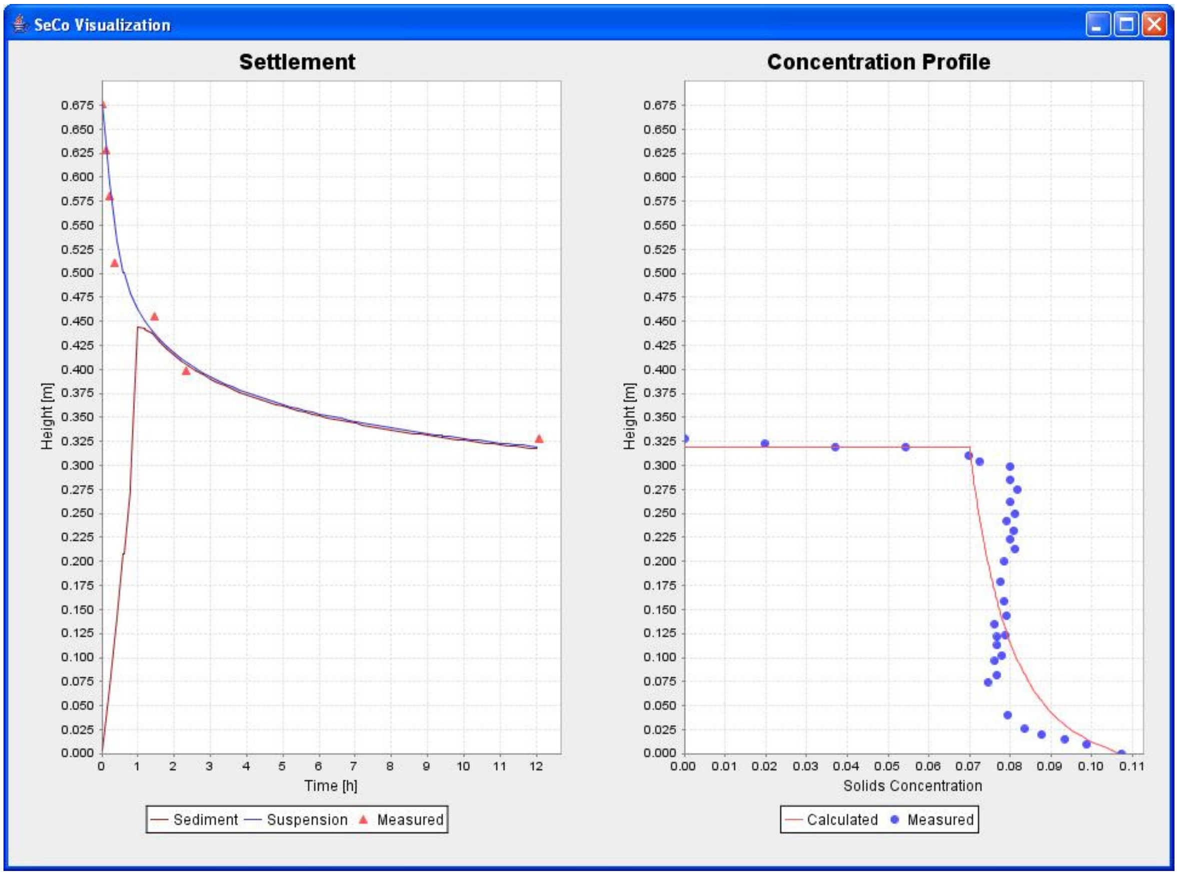

Figure 9. SECO graphical output window

5.2 Programming issues

The software is programmed in Java using Sun JDK version 1.5. Java was chosen particularly because of the requirements for a user friendly GUI. Furthermore, Java is very convenient because of its portability – the same program will work in various hardware/software environments (Windows, Unix) without any adaptation. This choice also had its relative disadvantages. Java is a language under permanent development, fast changing, and requires from developers frequent re-coding to keep pace with current trends and capabilities. With regard to computational efficiency, Java applications were once inferior to software made in other programming languages like C++ or FORTRAN, although recent Sun compilers make this gap almost invisible. Experience with the software has shown that the speed of computation is not a critical issue.

SECO is an object-oriented program (OOP). This means that all important entities, actual or abstract, are defined as independent program units; for example, permeability and compressibility laws are objects, and the computational procedure is also an object. Objects communicate with each other using pre-defined procedures that (in general) do not interfere with object internal structure and functionality. The OOP programming paradigm thus allows for easy software maintenance and development. Adding new options or modifying existing ones requires changing only affected portions of the code (affected objects), keeping the rest of the code unchanged.

This approach is facilitated through the use of Java interfaces which, in very general terms, define object's functionality and behaviour, without specifying the ways how it can be realized. Each object implementing an interface must satisfy these specific 'rules of conduct.' For example, the Permeability interface in SECO requires that each Permeability object must provide, as a minimum:

- the value of the permeability coefficient k for a given value of void ratio e, and vice versa;
- its own input panel (reflecting the specific mathematical definition of the k(e) function) with a specific input data verification procedure; and
- methods for data transfer to and from the object, to set user entered parameters and to provide information about them when required by the main program.

How the value of permeability is calculated, i.e. which actual mathematical expression is used to define the void ratio-permeability relationship, is internal to the permeability object. The solution procedure is not interested in it; it only expects to get a permeability value from the object. Adding a new permeability law is as simple as writing a new permeability object, without any intervention in the rest of the code.

6 EXAMPLE

The model behaviour can be demonstrated by simulation of laboratory sedimentation and self-weight consolidation tests. The model behaves equally well in both coarse-grained material (rigid particles, stiff sediment) and fine-grained material scenarios. The problem presented involves a fine-grained material suspension, the tests carried out by Bowden (1988) on Combwich mud, a natural mud from the Parret River estuary (Toorman 1999).

The original experimental data for the settling rate as a function of the excess density were first converted to the concentration – permeability relationship k = k(c) and then fitted by a power law function for k(c), Figure 10. Therefore, the main input data were obtained independently from the simulated settling test itself.

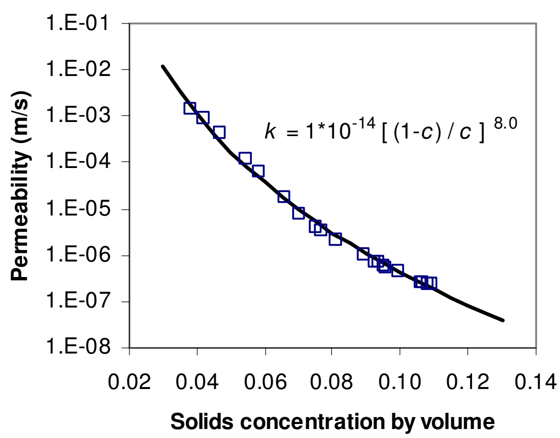

Figure 10. Permeability law for Combwich mud

The settlement of the suspension surface is shown in Figure 11. The simulation covered all 300 hours of the actual testing time; however, only the first 12 hours are shown in the figure. It can be seen that the sedimentation stage is quite short and that consolidation is the prevailing phenomenon. Although a good match with experimental data was obtained, it should be noted that much simpler models can also fit this curve easily.

The evolution of concentration (density) profiles is shown in Figure 12. The model both qualitatively and quantitatively predicts the typical features of the concentration distribution, and it can be shown that the maximum deviation between model and measurement is about twice the experimental error. The capacity of a model to correctly predict the density profile development with time (and the pore pressure distribution as well) is of much greater significance from an engineering point of view than to merely match measured settlement curves.

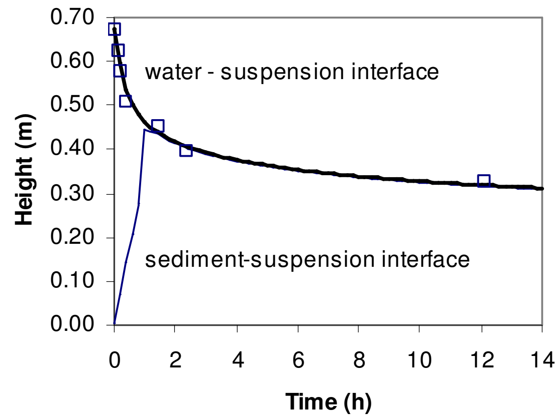

Figure 11. Settlement curves for Combwich mud

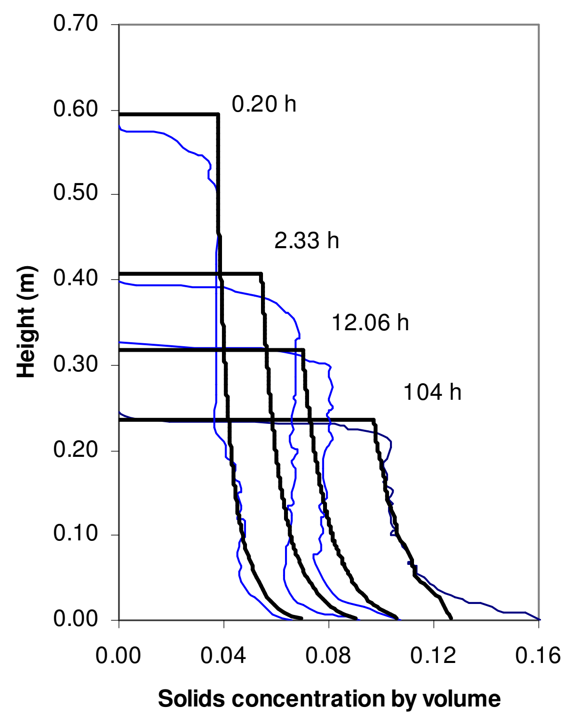

Figure 12. Density profile evolution for Combwich mud

7 CONCLUDING REMARKS

A model for coupled sedimentation and consolidation of a solid-liquid suspension was derived, with permeability as the unifying concept. The solution method for coupled PDE governing equations was developed as a finite difference scheme with Lagrangian grid. It was capable of dealing with the discontinuities – interfaces, which appear as a regular and an essential part of the solution.

The solution deals with the problem of discontinuities through a combination of convenient theoretical assumptions and the use of Lagrangian coordinates. Determination of the position of the water-suspension interface was eliminated by the use of Lagrangian coordinates that are connected to material points (solids) and move with them. Determination of the suspension-sediment interface was solved: (1) by assuming the continuous validity of the permeability law k(e) in both sedimentation and consolidation regions, and (2) by adopting the fluid nature of the suspension zone with effective stress $\sigma' = 0$, and with effective stress in the sediment gradually increasing downward from zero value at the suspension-sediment interface.

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