A new fractal index R_d for rock roughness

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ABSTRACT
Based on the fractal theory, a new index R_d for quantitative description of rock surface roughness is proposed in the paper. The new index can better reflect the influence of large-scale roughness on mechanical properties compared to the traditional fractal dimension *D*. Therefore, it is applicable for the case when the mechanical properties of the interface need to be considered. In this study, a number of natural rough rock surfaces are scanned with laser scanner. R_d and *D* values are calculated and compared according to the measurement data. The advantage of R_d is verified.

RÉSUMÉ
Sur la base de la théorie fractale, un nouvel indice pour la R_d description quantitative de la rugosité de surface de la roche est proposé dans le document. Le nouvel indice peut mieux refléter l'influence de la rugosité de grande envergure sur les propriétés mécaniques par rapport à la dimension traditionnelle fractale *D*. Par conséquent, il est applicable pour le cas où les propriétés mécaniques de l'interface doivent être prise en considération. Dans cette étude, un certain nombre de surfaces naturelles rock bruts sont numérisés avec un scanner laser. valeurs R_d et *D* sont calculés et comparés selon les données de mesure. L'avantage de R_d est vérifiée.

1 INTRODUCTION

In the research of interaction between engineering structures and foundation or geo-masses, the interface roughness is an important parameter which affects the mechanical behavior of the system [1,2]. The stress and displacement are transferred by the interface during the interaction between two bodies, especially dynamic interaction. Thus, how to describe the roughness quantitatively and scientifically is a problem much concerned by the engineering field [3]. Although many researches have been carried out on this issue [4-7], the description indexes presented in the previous studies have some limitations. In this study, the advantages and limitations of common description methods are analyzed and a new description index is proposed to better satisfy the needs in the study of mechanical properties of the interface.

2 ADVANTAGES AND LIMITATIONS OF COMMON DESCRIPTION INDEXES

The parameter for description of surface roughness varies with measurement method and its application. So far, tens of indexes have been proposed to describe the characteristics of rough surface [8]. The traditional indexes can be generally divided to two types: (1) Statistical parameters describing the altitudinal variation and distribution, (2) Statistical parameters describing the relative position and correlation between points on the interface[2].

2.1 Parameters for Altitudinal Features

In the study of surface roughness, the altitude of the surface is usually taken as a random variable. Some indexes have been proposed to describe the altitudinal features, including the average height of center line z_0, the mean square root of height z_1, the skewness coefficient S, the kurtosis coefficient *K* and etc [9].

The above indexes take the altitude of the surface as a random variable and describe the relative difference in

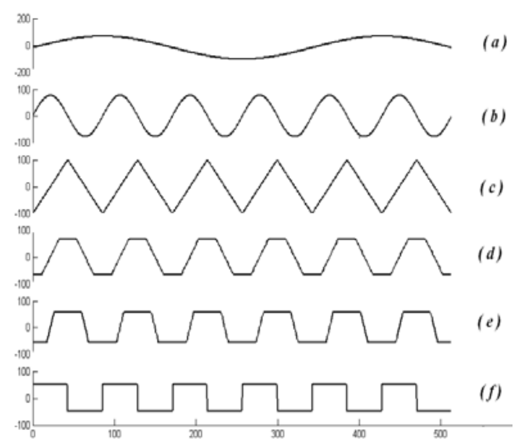


Figure 1.some classic curves with same value of Z_0 and Z_1

the altitude of the surface from the point of view of statistics. However, the information on the gradient, profile and appearance frequency of the peaks and the correlation between points on the surface is absent. Thus, they are unable to reflect the variation laws of the altitude on the surface and the proportion occupied by different range of altitude. The surfaces with same the average height of center line z_0 or the mean square root of height z_1 may have very different profiles (see Fig. 1). Therefore, parameters describing texture features are necessary for describing the surface roughness.

2.2 Parameters for Texture Features [2,10]

The parameters commonly used for describing the texture features of surface roughness include the root-mean-square z_2, the curvature root-mean-square z_3, the self-correlation coefficient and the arithmetical mean deviation of the profile R_a. Among them, R_a is also called the roughness index and most widely used. If l is the sampling length, R_a can be expressed as[5]:

$$R_a = \frac{1}{l}\int_0^l |Z(x)| dx \quad (1)$$

The advantages of the traditional parameters for description of surface roughness are simple and convenient for calculation. They have been widely recognized. However, they can only be adopted to describe two-dimensional profiles. Two dimensional evaluation has local dependency, which can not reflect the microscopic features of the surface as a whole and is not sufficient to describe the three-dimensional profiles[11]. Although R_a has been extended to describe three-dimensional surfaces, the effect is not satisfactory. Therefore, an index which can reflect three-dimensional roughness and describe the surface roughness of rock and other materials, such as fractal dimension, is expected[12].

2.3 Fractal Description of Surface Roughness

Different from Euclidean geometry, the fractal theory considers the geometric dimension of an object is not only an integer. For one-dimensional, two-dimensional or three-dimensional irregular figures (such as fluctuation curve, rough surface and fragmented mass), the fractal dimension is a fraction. Generally, if the topological dimension of a figure is D_t, $D_t<D<D_t+1$ is satisfied. Therefore, a rough surface with fractal features endlessly show new coarse margin on the edge when the observation or measurement scale is small enough. Such surface can be represented by a dimensionless parameter, i.e., fractal dimension D [12-14]. It is commonly recognized that the fractal dimension D is related to the magnitude of altitude variation of the surface roughness. The larger the value of D, the more the high frequency components or more features on the surface. On the other hand, the smaller the value of D, the simpler the profile is.

Description of surface roughness by fractal dimension D is a scientific approach. It is great progress in human thought to represent a dimension of a figure by a fraction instead of an integer. However, the correlation between the fractal dimension D and the roughness R_a is not a monotonic increase or decrease relationship [16]. If the fractal dimension is directly applied to describe the roughness of rock surface, there exist some problems as below:

(1) The basis of the fractal theory is self-similarity of a figure. For a figure which is mathematically self-similar, when the ruler size r is small enough, the measured fractal dimension D is independent of r and is a unique and stable value [12]. However, the natural rock surface is statistically self-similar instead of mathematically self-similar. The fractal dimension of a jointed surface varies with the ruler size r. When the ruler size r is small enough so that the fractal dimension is basically stable, the measured features are too small, which have trivial effects on the mechanical properties of the interface. Hence, this stable fractal dimension D is insignificant for describing the mechanical behaviors of the interface. In another word, description of surface roughness directly by fractal dimension may be applicable to the cases such as mechanical friction. In this case, the surface is relatively smooth and the mechanical behaviors of the interface are highly dependent on the small-scale features.

(2) The domain of fractal dimension is limited. For a rough surface, the fractal dimension D should satisfy $2 \le D < 3$ according to the fractal theory. However, two rough surfaces with very different surface profile may have approximately the same D values.

In view of the above problems, the fractal dimension D for the roughness of rock surface can hardly correspond to the mechanical properties of the interface. Therefore, the fractal dimension for describing surface roughness has been more applied to the cases such as CD-drive abrasion and metal friction in recent years [17,18]. The rock surface is described mostly by statistical parameters [2] and spectral analysis [19,20]. Is it possible to have a roughness index which can correlate with the mechanical properties of rough rock surface based on the fractal theory?

3 MEASUREMENT CONDITIONS AND RESULTS

Two types of fractal figures exist in the nature. One is the figure with regular border and mathematically self-similar, the other is the figure with stochastic border and statistically self-similar.

When a figure satisfies strict mathematical self-similarity and has fractal characteristics, its fractal dimension can be evaluated by various methods and equations [21, 22]. Among them, the equation for the area-covering dimension method is:

$$D = \frac{\log(S/S_0)}{\log(r/r_0)} \quad (2)$$

The equation for the box-counting dimension method is:

$$D = \frac{\log(N/N_0)}{\log(r/r_0)} \quad (3)$$

Where r is the ruler size which is gradually shortened, r_0 is the initial ruler size, S is the area of the rough surface, S_0 is the area of the rough surface corresponding to r_0, N is the number of cubes with side length of r needed to cover the rough surface, N_0 is the number of cubes corresponding to r_0. When the ruler size r is small enough, the value of D calculated by Eq. (2) or (3) becomes stable .

However, when a figure can only satisfy statistical self-similarity instead of strict mathematical self-similarity, a series of S or N values are obtained with shortening ruler size r. The fractal dimension can be calculated by regression analysis. In engineering practices, fractal figures with statistical self-similarity are usually encountered, including the natural rock surface in this study. In order to quantitatively explain the limitations of the existing indexes and explore a new and more rational description method, the rough surfaces of a number of rock specimens were measured and studies. Rock samples were fresh limestone taken from *Mentougou* Quarry, Beijing, China. The samples were trimmed into hexahedrons with a natural tough surface and five orthogonally cutting surfaces. The dimension of the natural rough surface was 100×100mm.

In this study, a large-scale surface topography laser scanner developed by Tianjin University was employed to scan specimens with different surface roughness. The scanner works in a line scan mode with minimum scanning interval of 0.1mm. The scanning speed is about 600 points/s. Six natural rock surfaces with different surface roughness were scanned with intervals of 0.1mm and 0.2mm, respectively. The scanned surface images are shown in Fig. 2.

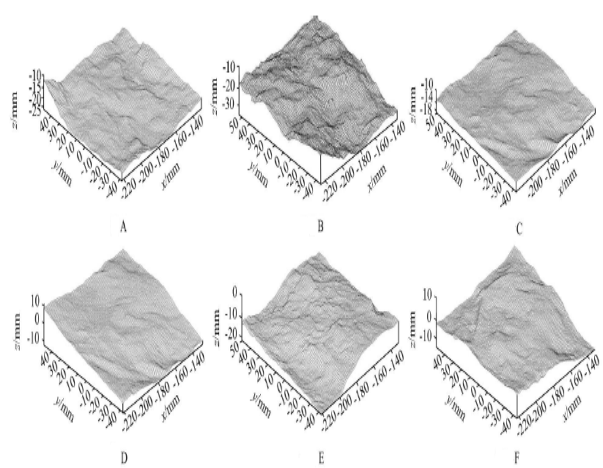


Figure 2 Rough surface images taken by laser scanner

In order to investigate the effect of ruler size r on the fractal dimension D, the fractal dimension of the rough surfaces were calculated for intervals of 0.1mm, 02mm, 0.4mm, 0.8mm and 1.6mm, respectively. The data for calculation with the latter three intervals were taken from the scanning results with interval of 0.1mmm. The calculation results are shown in Table 1.

4 RESULTS ANALYSIS

4.1 Proposal of A New Description Index

Fig. 3 shows the variation of fractal dimension D for Specimen C with the measurement interval r. The variation trends of fractal dimensions for other specimens are similar and not discussed one by one here.

As the profile of rock surface is statistically self-similar, it can be seen from earlier sections and Fig. 3 that its fractal dimension D is affected by the ruler size (scanning interval) r. The fractal dimension tends to be stable with the decrease of the measurement interval r. However, the measurement interval is too tiny and the measured features have trivial effects on the mechanical properties of the surface.

Under this circumstance, the idea of fractal may be applied to combine the fractal dimension D_n and the corresponding rule size r_n. Even if D_n is not stable, a new index can be formed to describe the roughness of a curve or surface.

Let $A=[a_1\ a_2 \dots a_i \dots a_m]$ represent an array composed of the measurement intervals for calculation of fractal dimensions. The element $a_i=r_i\ /R$, where R is the measurement scope and is the overall dimension of the cross section for a specimen, r_i is the rule size for the i th step. The element in A is in a descending order .If D_T is the topological dimension of a figure, $D_T=2$ for a curved surface and $D_T=1$ for a curve. Let D_{ni} denote the fractal dimension calculated by Eq. (2) or (3) corresponding to $r=r_i$. Let b_i denote the difference between D_{ni} and D_T, i.e., $b_i=\Delta D_{ni}=D_{ni}-D_T$. Let B represent a matrix composed of b_i, i.e., $B=[\ b_1\ \ b_2 \dots b_i \dots b_m\ ]^T$.

In this study, a new index for roughness R_d is defined as $R_d=10^k \times A \times B$, where 10^k is an amplification factor so as to avoid too small R_d. k is an positive integer and

Table 1 Fractal dimensions D for various ruler size r for images in Fig. 2

Specimen No.		A	B	C	D	E	F
Area-covering method (Eq.2) D	r_1=0.1mm	2.1172	2.1966	2.0767	2.1000	2.1234	2.2170
	r_2=0.2mm	2.0389	2.0865	2.0238	2.0370	2.0488	2.0784
	r_3=0.4mm	2.0159	2.0402	2.0089	2.0130	2.0189	2.0282
	r_4=0.8mm	2.0115	2.0315	2.0069	2.0091	2.0171	2.0132
	r_5=1.6mm	2.0101	2.0301	2.0091	2.0107	2.0160	2.0070
Box- counting method (Eq.3) D	r_1=0.1mm	2.1795	2.2640	2.0916	2.1765	2.1869	2.3434
	r_2=0.2mm	2.0379	2.1253	2.0114	2.0469	2.0504	2.1411
	r_3=0.4mm	2.0102	2.0647	2.0026	2.0124	2.0192	2.0393
	r_4=0.8mm	2.0066	2.0746	2.0052	2.0100	2.0184	2.0107
	r_5=1.6mm	2.0040	2.0494	2.0000	2.0069	2.0028	2.0000

determined by r_i /R. In this paper, k=2+lg(R/r_1). R_d can then be expressed as:

$$R_d = \frac{10^k}{R}\left(r_1 \cdot \Delta D_{n1} + r_2 \cdot \Delta D_{n2} + \cdots + r_i \cdot \Delta D_{ni} + \cdots + r_m \cdot \Delta D_{nm}\right) = \frac{10^k}{R}\sum_{i=1}^{m} r_i \cdot \Delta_{ni} \tag{4}$$

The index R_d can describe the roughness of both surfaces and curves. In fact, R_d is a parameter reflecting scale-dependent fractal dimension. It can better reflect the scale characteristics of a rough surface than a simple fractal dimension. It can be seen from Eq. (4) that the index R_d not only considers the rough features under different scale, but also assign higher weight to larger-scale features.

Fig. 4 shows an amplified curve satisfying statistical self-similarity. Let D_{n1} be the fractal dimension corresponding to r=r_1. If the shadow regions are ignored, the fractal dimension corresponding to r< r_1 remains as D_{n1} as the fractal dimension is stable in this case. If the shadow regions are considered, the fractal dimension corresponding to r_2 (r_2< r_1) is D_{n2}. Obviously, D_{n2}> D_{n1} as the fractal dimension is not stable yet.

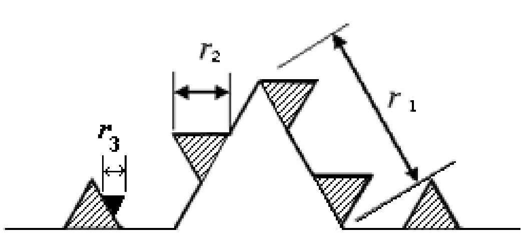


Figure 4. Measuring rough curve with different r

For the convenience of discussion, m=2 is assumed in Eq. (4). If the shadow regions are ignored,

$$R_{d1} = \left(r_1 D_{n1} + r_2 \cdot D_{n1}\right)/R$$

If the shadow regions are considered,

$$R_{d2} = \left(r_1 D_{n1} + r_2 \cdot D_{n2}\right)/R$$

Obviously, R_{d2} > R_{d1}. Thus, it is shown that R_d increases with the rough features represented by the shadow regions. The more the features, the higher the value of R_d.

The waveforms shown in Fig. 1 mainly reflect the difference in texture and topography. From Section 2, we know that indexes reflecting altitudinal characteristics cannot describe roughness well. The values of R_d are calculated for the typical curves in Fig. 1 and the results are listed in Table 2. It can be seen that the fractal index R_d proposed in this paper can distinguish the difference in texture which can hardly be described by the index designed for altitudinal characteristics

Table 2 Calculation result of R_d index from Fig.1

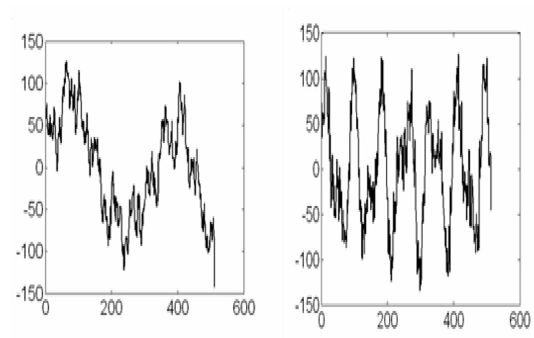
Curve No.	A	B	C	D	E	F
R_d	2.91	83.78	118.14	35.61	65.22	61.23

In order to verify whether R_d is suitable to describe the roughness of a curve with fractal features, a series of representative curves were generated. The W-M (Weierstrass-Mandelbrot) Function [20] is continuous, non-differentiable and self-affine, which is similar to the natural rock surface profile. The function is expressed in the following form:

$$Z(x) = G^{(D-1)}\sum_{n=n_l}^{\infty}\frac{\cos 2\pi\gamma^n x}{\gamma^{(2-D)n}} \tag{5}$$

Where Z(x) is a stochastic surface profile function, D is the fractal dimension, G is the characteristic scale coefficient, γ is a constant greater than 1, γ^n is the spatial frequency of the stochastic surface profile and n_l is a coefficient corresponding to the lowest cut off frequency of the profile.

The two curves shown in Fig. 5 are obtained by Eq. (5). In Fig. 5(a), γ=1.05, G=0.01. In Fig.5(b), γ=1.22, G=0.01. Both curves have the same fractal dimension D=1.5 and the same R_a=50mm. However, the texture is very different for the two curves and obviously the mechanical



(a) D=1.5 R_a=50 R_d=135.15 (b) D=1.5 R_a=50 R_d=199.93

Figure 5 Comparison of curves with same D and R_a but different R_d

properties of the two curves are different. Neither R_a nor D can reflect the difference. Nevertheless, the fractal roughness index R_d proposed in this paper is 135.15 and 199.93, respectively. R_d can clearly differentiate between the two curves. Hence, the fractal index R_d can be used to describe the rough curves which cannot be described by parameters for texture features or fractal dimension.

4.2 Description of Rough Surface by R_d

In order to verify the capability of R_d for description of surface roughness, R_a and R_d were calculated and analyzed for the six rough surfaces shown in Fig. 2. The results are shown in Table 3. By comparing the results with the scanned images in Fig. 2, it can be seen that R_d can be better correlated with the surface roughness, rather than R_a.

Table 3. R_d and R_a values from rough surfaces shown in Fig. 2

Specimen No.	*A*	*B*	*C*	*D*	*E*	*F*
R_a	2.358	3.131	1.31	3.314	2.903	2.789
R_d	5.114	12.6492	3.601	4.698	6.900	7.051

It should be pointed out that the applicability of R_d depends on the proper selection of the array A for the ruler size. For the array $A=[r_1\ \ r_2 \ldots r_i \ldots r_m]/R$, parameters such as the initial ruler size r_1, the value of $r_{i+1}/\ r_i$ and m affect the value of R_d. If m is very large, the small-scale features occupy a small fraction in R_d and are covered by the large-scale features. In order to unify the processing rule, r_1 is the scanning precision which is 0.1mm, r_i is in a descending order, $r_1\ /\ R = 10^{-3}$, $r_{i+1}\ /\ r_i = 2$, and $m = 5$.

Although R_d can improve the description of the surface roughness, the problem of insufficient information by a single index still exists when one tries to establish one-to-one correlation between R_d and the mechanical properties of the rough surface. For instance, if a rough surface is subjected to forces from different directions, its mechanical behaviors are different. To solve this problem, other indexes are necessary. The authors will discuss the multi-index system in another paper.

5 CONCLUSIONS

1) For an irregular rough surface, large-scale features have more effects on the mechanical behaviors of the surface. When the rule size r is small enough so that the fractal dimension D is stable, the measured rough features have trivial effects on the mechanical behaviors. Hence, roughness description directly by fractal dimension is not suitable for one-to-one correlation between the roughness index and the mechanical behaviors of the surface.
2) A new roughness index R_d is proposed by combining the measured fractal dimension Di and the ruler size r_i. The new index can better reflect the surface topography than the fractal dimension D and the traditional roughness index R_a. R_d can be used to establish the correlation between the roughness and the mechanical properties of a surface.
3) The applicability of R_d depends the proper selection of the array A for the ruler size. The initial ruler size r_1, the value of $r_{i+1}/\ r_i$ and m affect the value of R_d.
4) As a single index contains limited information, one-to-one correlation between the roughness and the mechanical behaviors of the surface can hardly be established only by the roughness index R_d. Other parameters are needed to build a multi-parameter system for rational correlations.

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