Advances in probabilistic slope stability analysis using the finite element method

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ABSTRACT
New techniques are presented that enable the shear strength reduction (SSR) method to be used in a probabilistic fashion to obtain reliable estimates of probability of slope failure. The combination of Point Estimate methods for generating random samples, along with parallel processing techniques and easy to use interfaces for model set-up and interpretation, allows quick and easy statistical analysis using the SSR approach. Examples are presented and compared with Monte-Carlo limit equilibrium analyses.

RÉSUMÉ
Des nouvelles techniques sont presentées qui permettent la méthode de la réduction de la résistance au cisaillement (RRC) d'être utilisée de façon probabiliste pour obtenir des estimations fiables la probabilité de la rupture de la pente. La combinaison des méthodes d'estimation ponctuelles pour générer des échantillons aléatoires, accompagnées des techniques de traitement parallèles ainsi que des interfaces qui sont faciles à utiliser pour mettre en place le modèle et faire de claires interpretations , permet un analyse statistique qui est rapide et facile en utilisant l'approche de RRC. Des examples sont presentés et sont comparés avec les analyses de la limite de l'équilibre de Monte-Carlo.

1 INTRODUCTION

Slope stability is strongly dependent on the strength of the soil. However, accurate estimates of soil material properties (cohesion and friction angle) are often difficult to obtain. For this reason, probabilistic slope stability analysis is a popular method in engineering design. With limit equilibrium methods, Monte-Carlo simulations with variable strength parameters can be performed quickly, and probability of slope failure can be easily determined. Many software programs have probabilistic capabilities built in to easily enable these types of analyses.

Finite element methods (FE), using a shear strength reduction (SSR) approach can also be used to calculate factors of safety in slope stability analyses (e.g. Duncan, 1996, Dawson et al., 1999). These methods offer many advantages over limit equilibrium methods (e.g. ability to calculate stresses and displacements, model interactions between soil and support systems, etc.), however the disadvantage is that each finite element analysis is significantly more costly to set up and execute than the corresponding limit equilibrium analysis. This drawback has generally prevented the FEM from being used in probabilistic slope stability analyses. The time required to set up and run a Monte-Carlo simulation is usually too long for most practitioners.

This paper will show how three different technologies now enable quick and easy finite element slope stability analyses. Firstly, multi-core computers are now common and parallel processing programming techniques can be used to drastically speed up the finite element analyses. Secondly, the Point Estimate Method can be used instead of Monte Carlo to get good results from fewer simulations. Finally, built-in probabilistic modelling capabilities in finite element software make it significantly easier for engineers to set up simulations and interpret results.

Several examples of probabilistic slope stability analyses are shown and results are compared between the limit equilibrium methods and the finite element method. In particular, the calculated probability of failure and the time required to perform the analyses will be evaluated.

2 METHODOLOGY

2.1 Shear Strength Reduction

The Shear Strength Reduction (SSR) method is now well established as a technique for analysing slope stability. The SSR method calculates the factor of safety for a slope by solving several different finite element models with different material strengths. The following steps are executed:

- The strength parameters of a slope are reduced by a certain factor (**SRF**), and the finite element stress analysis is computed.
- This process is repeated for different values of strength reduction factor (**SRF**), until the model becomes unstable (the analysis results do not converge).
- This determines the critical strength reduction factor (**critical SRF**), or safety factor, of the slope.

This technique has been used with different software programs with much success (see Dawson and Roth, 1999, Hammah et al., 2005 and Griffiths and Lane, 1999 for examples).

The advantage of the SSR method over limit equilibrium analyses is that the shape of the critical failure surface does not need to be pre-defined. The failure surface simply 'falls out' of the analysis. In addition, a finite element analysis enables the observation of stresses, displacements and soil-support interaction. Finite element analyses can also include elements such as joints that are not easily incorporated in limit equilibrium programs. The main disadvantages of the FE-SSR method are the time and expertise required setting up and interpreting a model, and the computer time required to perform the analysis – especially when probabilistic studies are performed.

2.2 Point Estimate Method

The most popular method for probabilistic slope stability analysis is the Monte Carlo method. With this method, input variables (e.g. soil strength) are assigned random values from some statistical distribution (e.g. normal distribution) and an analysis is performed. If this is repeated enough times, then an accurate assessment of the probability of failure can be obtained. The method is robust, flexible and easy to implement. The problem is that many simulations may be required to obtain reasonable accuracy. With the FE-SSR approach, each simulation represents multiple solutions of a finite element model, and the Monte-Carlo method quickly becomes practically infeasible.

The Point Estimate Method (PEM) overcomes these problems by requiring much fewer simulations (if the number of random variables is not large). Instead of trying to sample from the all possible inputs, the PEM (Rosenbleuth, 1975 and Rosenbleuth, 1981) uses point estimates at selected values (known as weighting points) of the input random variables. In its simplest form, the PEM uses two weighting values – typically one standard deviation to each side of the mean – for each input random variable. Each point is assigned a weight of $1/n$ where n is the total number of points. For all the different possible permutations of the input, full FE analyses are carried out. The calculation of statistical moments for outputs is based on the results of the computed FE models.

The disadvantage of the PEM is that when many variables are being tested, then many point evaluations are required (the number of simulations required increases exponentially with the number of variable inputs). However for relatively simple slope stability problems, the number of random variables is usually small. Also, the PEM is necessarily not as accurate as a full Monte-Carlo simulation. By only evaluating a few samples, it is possible to miss some failure mechanisms. However this paper will show that the PEM is a quick and easy way to obtain a good estimate of probability of failure. See Hammah et al (2009) for more details on the use of the Point Estimate Method in SSR analyses.

2.3 Parallel Processing

Most desktop computers are now sold with multiple cores. This allows several applications to be run in parallel without a loss in performance. Multi-core architecture can be exploited to drastically speed up finite element calculations. For this paper, the finite element program Phase2 has been used (Rocscience Inc, 2009) with the following parallelizations:

1. The inversion of the stiffness matrix is parallelized through the use of Intel's Math Kernel Libraries (Intel, 2008)
2. The calculation of internal forces is parallelized by splitting up the elements so that elements independently calculate and store their own internal forces on different cores and then all of the internal forces are brought together at the end of each non-linear iteration.

These two techniques drastically speed up the finite element simulations, depending on how many cores are in the computer.

The relative importance of each technique depends on the nature of the finite element calculations. For an 'initial stiffness' approach, the stiffness matrix is inverted once and many iterations are performed in which internal forces are calculated. In this case, technique #2 is dominant. When using the 'Newton-Raphson' method, the stiffness matrix is recalculated and inverted every iteration and much fewer iterations are required. With this method, technique #1 is most important.

For an explicit finite element or finite difference calculation scheme (e.g. Itasca Consulting Group's FLAC), there is no stiffness matrix, so only technique #2 would apply.

2.4 Interface Improvements

One of the problems with performing probabilistic analyses with the finite element method is the time and effort required to set up the models. While most limit equilibrium programs include probabilistic analysis capabilities, few finite element programs used in geotechnical engineering allow for easy set up and interpretation of probabilistic analyses.

To speed up finite element probabilistic analyses, enhancements were made to the Phase2 interface. These enhancements allow the user to specify a mean and standard deviation for any material property, joint property or for in-situ stresses. Point estimate models are then automatically set up and executed on multiple cores. In addition, the interpretation of results is simplified by collating all of the results from the different simulations to enable easy viewing of probability of failure, as well as statistical information on all other results (e.g. mean and standard deviation displacement). These enhancements and parallelizations are currently being tested and are not yet part of the commercially available product.

3 EXAMPLES

3.1 Example 1: Simple slope with one material

The first example is from a set of problems proposed by Giam and Donald (1989) for testing slope stability software. A finite element model of the slope is shown in Figure 1.

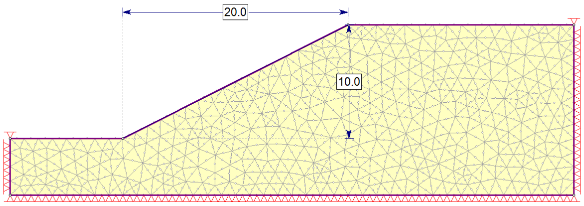

Figure 1. Finite element model of first example.

The soil properties for the model are:

Unit Weight (γ)	=	20 kN/m^3
Tensile strength	=	3 kPa
Cohesion (c)	Mean = 3 kPa, Std. Dev. = 1 kPa	
Friction angle (ϕ)	Mean = 19.6°, Std. Dev = 5°	

The cohesion and friction angle were assigned statistical (normal) distributions. Since there are two changing variables, the point estimate method requires 2^2 (=4) models to be run. In this case, the following models are executed:

$c = 2 \quad \phi = 14.6$
$c = 4 \quad \phi = 14.6$
$c = 2 \quad \phi = 24.6$
$c = 4 \quad \phi = 24.6$

With a good interface, this model can be set up in minutes. By simply entering the mean and standard deviation for the cohesion and friction (or any other material parameter) the necessary models are set up automatically. The user can then specify the number of cores to be used when processing. When computation is started, all models are automatically calculated in an efficient manner according to the number of cores specified. For this model, the calculations were finished in less than 4 minutes on a dual core machine.

A traditional Monte-Carlo simulation with the limit equilibrium (LE) program Slide (Rocscience, 2007) was also executed to check the equivalency of the finite element results. Several different LE methods were tested as shown in Table 1. A good description of the different methods can be found in Abramson et al. (2001). For the limit equilibrium Monte-Carlo simulation, 1000 random simulations were executed. To ensure accuracy, a full search was performed for each simulation (rather than simply altering the cohesion and friction on the deterministic failure surface). The calculation took approximately 13 minutes on the same dual core PC (note that there is no parallelization in Slide). All of the results are shown in Table 1.

It is clear that the FE-SSR model using the Point Estimate method agrees well with the Monte-Carlo limit equilibrium analyses. Also – the calculation time for the FE-SSR approach was significantly less than the limit equilibrium calculation times. The time required to set up the models and do the interpretation was about the same for both methods.

Table 1. Calculation results for the first example

Model	Mean FS[1]	Probability of Failure (%)
FE-SSR	0.990	51.5
LE (Bishop)	0.989	51
LE (Janbu)	0.939	60.6
LE (Spencer)	0.985	51.5
LE (GLE[2])	0.985	51.5

[1]Factor of Safety
[2]General Limit Equilibrium method. Result is the same as the Morgenstern-Price method.

Obviously, the FE-SSR was faster because much fewer models were run, and because of the parallel processing capabilities of the FE-SSR method. However, most limit equilibrium programs are not set up to use the point estimate method and are not parallelized, so the time and effort required to set up, run and interpret a point estimate simulation with a limit equilibrium program would probably still be longer than solving the problem with a finite element program in which the process is automated and parallelized.

3.2 Example 2: Slope with three materials

This example is a slope composed of three different materials and is also from Giam and Donald (1989). A finite element model of the slope is shown in Figure 2.

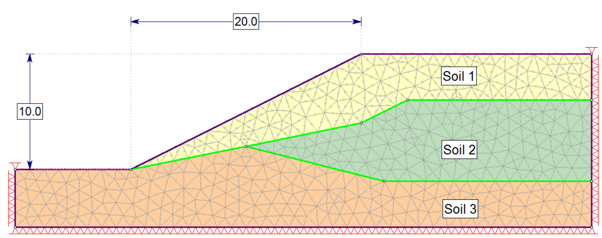

Figure 2. Finite element model of the second example.

The cohesions and friction angles were assigned normal distributions as shown in Table 2. All of the soils were assigned a unit weight of 19.5 kN/m^3.

Table 2. Soil properties used in the second example.

	Mean	Std. Dev.
Soil 1, *c (kPa)*	0	-
Soil 1, ϕ (degrees)	38	6
Soil 2, *c (kPa)*	5.3	1.5
Soil 2, ϕ (degrees)	23	5
Soil 3, *c (kPa)*	7.2	2
Soil 3, ϕ (degrees)	20	4

For this problem, 5 variables are being tested so 2^5 (=32) models are required. It is clear how the addition of more variables quickly increases the number of required simulations. However, the number required is still significantly less than would be required to obtain accurate results with a Monte-Carlo approach.

Since the process for performing the point estimate method is mostly automated, the time required to set up the model(s) was about 10 minutes. The computation time to solve all 32 models on a dual core PC was about 25 minutes. On a 6-core PC, the time required was ~10 minutes. All computation times are summarized in Table 3.

Table 3. Calculation times for the second example

Model	No. Of samples	Computer	Time to solve
FE-SSR (PEM)	32	2-core[1]	25 min.
		6-core[2]	10 min.
FE-SSR (M-C)	1,000	2-core	13.5 hrs.
		6-core	5.1 hrs
LE (Monte-Carlo)	10,000	2-core	2.5 hrs.

[1]Intel Core 2 Duo, 2.33 GHz
[2]Intel Xeon 2.67 GHz

As with the previous example, a Monte-Carlo limit equilibrium analysis was performed for comparison purposes. The probability of failure for this example was quite low, so to ensure sufficient sampling, 10,000 random simulations were run for this example (instead of 1,000 as in Example 1). As in Example 1, a full search for critical failure surface was performed for each random sample. This required about 2.5 hours of computer time.

The calculation results are shown in Table 4. It shows that the mean factor of safety and the probability of failure for the FE-SSR model are slightly less than the limit equilibrium model. To test if the differences were because of the sampling method (Point Estimate) or calculation method (FE-SSR), a Monte Carlo analysis was performed using the finite element method. It was not feasible to run 10,000 simulations so 1,000 models were run. This required approximately 13.5 hours on a dual core PC (about 5 hours on a PC with 6-cores). The results from this test are also shown in Table 4. It can be seen that the factor of safety from the Monte-Carlo FE-SSR test is the same as from the Point Estimate method but that the probability of failure has increased and is now closer to the values obtained from the Limit Equilibrium tests. The number of samples used (1000) is probably still too small to draw any firm conclusions but it appears that in this particular example, the Point Estimate approach may slightly underestimate the probability of failure.

Table 4. Calculation results for the second example

Model	Mean FS	Prob. of Failure (%)
FE-SSR (Point Estimate)	1.30	3.34
FE-SSR (Monte Carlo)	1.30	4.95
LE (Bishop)	1.40	4.19
LE (Janbu)	1.26	8.67
LE (Spencer)	1.37	4.55
LE (GLE)	1.37	4.51

This example is interesting because different failure surfaces emerge for different sets of parameters. Often probabilistic analyses are performed by first locating the critical failure surface deterministically, and then randomly altering the material properties and calculating the factor of safety for this surface. For the scenario shown in Figure 2, this approach would result in a whole suite of failures being missed.

Figure 3 shows the 10,000 critical failure surfaces found with the limit equilibrium probabilistic analysis. It is clear that there are two main failure surface clusters; one that cuts through Soil 1 to create a failure surface that approximately intersects the crest. A second failure surface is also common in which the surface follows along the boundary between Soil 1 and Soil 2 until eventually cutting up the surface. This mechanism occurs when Soil 2 is relatively weak.

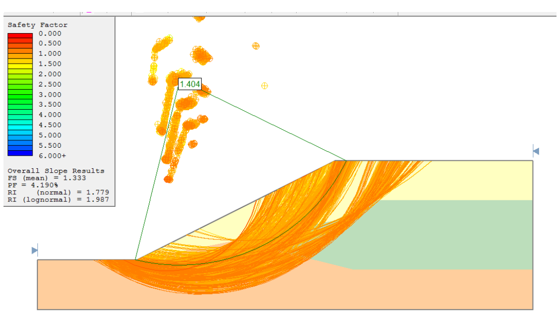

Figure 3. Critical failure surfaces found in the limit equilibrium Monte-Carlo simulation.

This behaviour is further emphasized when looking at the finite element models. Figures 4-6 show different possible failure mechanisms observed in the different manifestations of the finite element model (Point Estimate analysis). It is clear that different mechanisms are occurring depending on the relative strengths of the different soil layers.

These figures further emphasize the usefulness of the SSR method. With the SSR method, the critical failure surface emerges from the model without having to predefine its shape or location. Figure 5 shows how the failure surface can be non-circular when Soil 2 is weak. Figure 6 shows a slumping mechanism in the Soil 1 layer that is not observed in the limit equilibrium analyses.

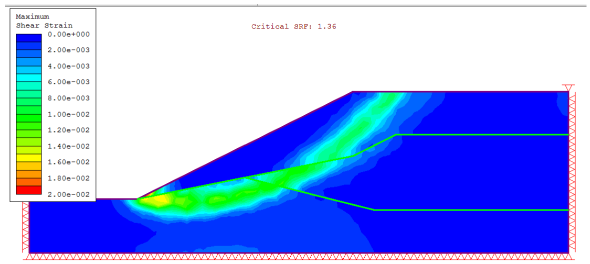

Figure 4. Maximum shear strain in a finite element model with mean material properties.

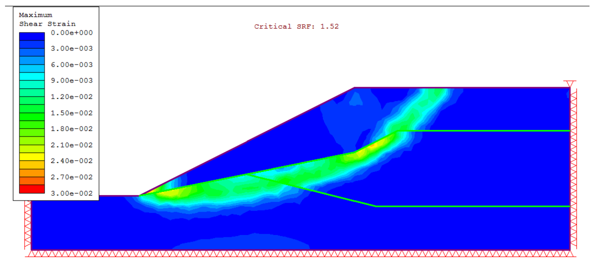

Figure 5. Maximum shear strain in a finite element model when Soil 2 is weak.

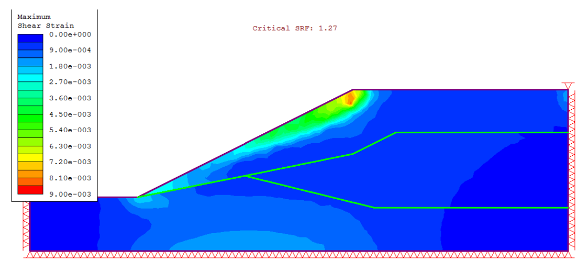

Figure 6. Maximum shear strain in a finite element model when Soil 1 is weak.

3.3 Example 3: Jointed Rock

This example is from Lorig and Varona (2001). The model geometry is shown in Figure 7. The slope is 260 m high and dips at 55°. A regular set of joints with a 10 m spacing dipping at 35° is shown.

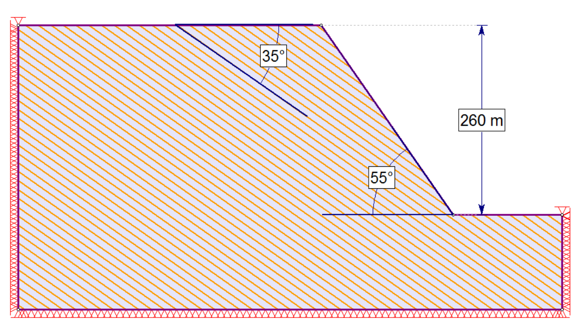

Figure 7. Geometry of jointed model of the third example.

The properties of the rock material are:

Unit Weight (γ)	=	26.1 kN/m^3
Young's modulus (E)	=	9.072 GPa
Poisson's ratio (ν)	=	0.26
Tensile strength	=	0 kPa
Cohesion (c)	=	675 kPa
Friction angle (ϕ)	=	43°

The rock material properties are assumed to be deterministic, and therefore not statistically varied for this example. Instead the joint cohesion and friction are assigned normal distributions with the following parameters:

Tensile Strength =	0 kPa
Cohesion (c_J)	Mean = 100 kPa, Std. Dev. = 25 kPa
Friction angle (ϕ_J)	Mean = 40°, Std. Dev = 5°

Since only 2 parameters were statistically varied, only 4 Finite Element models were executed when using the Point Estimate method. This took about 15 minutes on a dual core PC. As with the other examples, assigning the mean and standard deviation for joint properties is automated in the interface and the relevant models are automatically generated and computed. The model setup took about 15 minutes, for a total solution time of about half an hour. Results are also collated by the program so interpretation was similarly simple and quick.

Results are shown in Table 5. There is no easy way to perform a limit equilibrium analysis for this example due to the distinct joints. Instead, a Monte-Carlo FE-SSR simulation was performed with 1000 random models. It was estimated that this would take about 3 days to run on a dual core machine, so the model was set up on a 6-core computer and the 1000 models were run in about 24 hours. These results are also shown in Table 5. Table 5 shows a basically good agreement between the Point Estimate results and the Monte-Carlo results. However, as with example 2, there are probably not enough models to accurately sample the 'tails' of the normal distribution. Since the probability of failure is quite low, then it is probably underestimated by the Monte-Carlo method with too few samples.

Also shown in Table 5 are the deterministic factors of safety calculated for the same scenario using a distinct element model (see Lorig and Varona, 2001) and a finite element model (see Hammah et al., 2007).

Table 5. Calculation results for the third example

Model	Mean FS	Prob. of Failure (%)
FEM (Point Estimate)	1.31	4.63
FEM (Monte Carlo)	1.31	2.87
FEM (deterministic)[1]	1.32	-
DEM (deterministic)[2]	1.27	-

[1]Hammah et al. (2007)

[2]Lorig and Varona (2001)

As with example 2, this example is interesting in that different failure mechanisms occur depending on the joint properties. Figure 8 shows the maximum shear strain and joint failure in a model with strong joints. It is clear that the mechanism involves a combination of shear failure through intact material at the toe, slip along a joint near the bottom of the slope and then shear failure through the rock material at the top of the slope. With a weak joint (Figure 9), it appears that there is no band of localized shearing through the rock and that most of the displacement is likely occurring along the joints.

This interpretation is emphasized in Figures 10 and 11 which show the total displacement for strong joints and weak joints respectively. Figure 10 (strong joints) suggests a rotational type failure, whereas Figure 11 (weak joints) suggest more of a sliding wedge type failure mechanism.

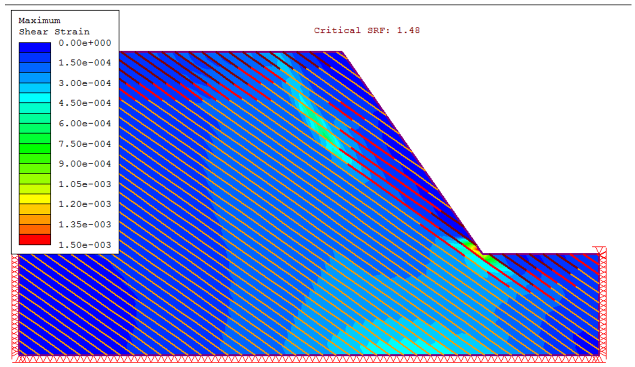

Figure 8. Maximum shear strain and joint failure (coloured red) in model with strong joints.

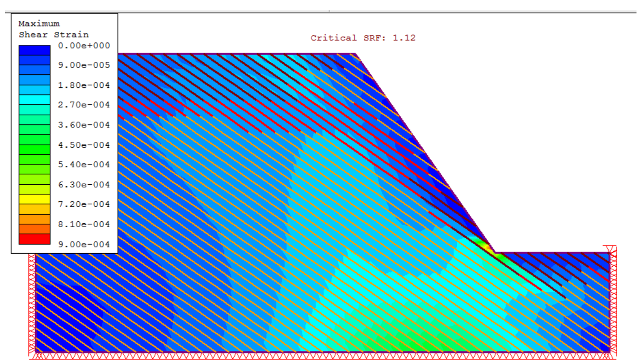

Figure 9. Maximum shear strain and joint failure (coloured red) in model with weak joints.

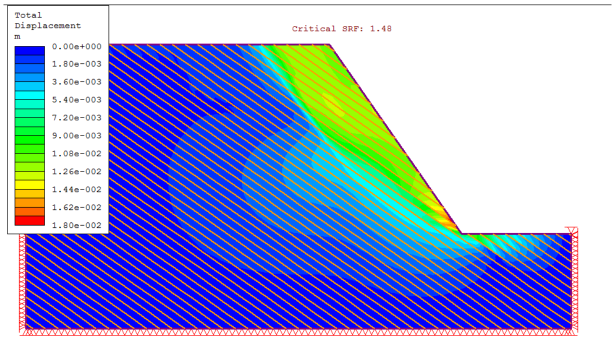

Figure 10. Total displacement in model with strong joints.

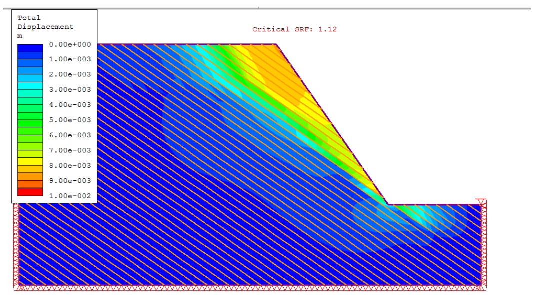

Figure 11. Total displacement in model with weak joints.

4 DISCUSSION

The preceding examples demonstrate how advances in computer modelling make it quick and easy to perform a probabilistic analysis to estimate the probability of failure in slopes when the soil properties (or joint properties) are uncertain. These examples however, all assume that each material layer is homogeneous. When the soil properties are altered in the probabilistic study, the soil properties throughout each layer are assigned the same values.

What the above examples do not show, is the effect of spatial variation within a single model. It is probably more realistic to consider a distribution of soil properties within a single model than a distribution of models with uniform soil properties. A correlation length can then be used to dictate the 'frequency' of variation within each material. Griffiths and Fenton (2004) give a good description of how this could be done with FE-SSR applied to slope stability problems.

Unfortunately, the Point Estimate approach could not be used in these types of models. The spatial variability in these models can lead to different failure mechanisms and factors of safety even when the same parameters are used. This is because different manifestations of randomness may lead to weak seams within the material layers or some other spatial peculiarity that may affect the slope stability. Also, limit equilibrium methods could not easily be used in this type of analysis. To perform these studies, FE-SSR methods with Monte-Carlo randomizations are required.

Similarly, models with different random geometries could not be simulated with the Point Estimate method. Hammah et al. (2009) show how different random manifestations of joint networks can have significant effects on factors of safety. These types of analyses also require the Monte-Carlo method. Determining the probability of failure for uncertain dam heights (Heidari and Roudsari, 2009) or uncertain slope geometries (El-Ramly, et al, 2007) would fall into the same category.

Griffiths and Fenton (2004) perform 1,000 Monte-Carlo simulations to test their spatially correlated randomness. This level of resources (in model set-up, calculation and interpretation) is currently well beyond what most practitioners can afford. However, as multi-core processors become more prevalent and software is written to take advantage of this new architecture and new modelling paradigm, then these types of analyses may become more common in the near future.

5 CONCLUSIONS

This paper shows how advances in finite element modelling methods can make it possible for time-challenged engineering practitioners to perform probabilistic analyses using the finite element method combined with the Shear Strength Reduction technique. By using improved program interfaces, multi-core processing technologies and the Point Estimate method for randomization, FE-SSR analyses can actually be quicker than the equivalent limit equilibrium solutions.

The paper also shows that the FE-SSR approach gives similar mean factors of safety and probabilities of failure as the limit equilibrium method. In addition, the FE-SSR approach offers other advantages over the limit equilibrium method such as the ability to observe displacements and stresses and the ability to include joints and other complex elements. As technologies continue to improve it is envisaged that the probabilistic FE-SSR approach could be used for problems with spatial variability and/or geometric uncertainties such as slope height or joint spacing.

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