Recent advances in debris flow modelling

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ABSTRACT
This paper presents the basic concept of an energy approach in debris flow runout analysis. The framework is based on mass and energy conservation in the global and local sense and takes into account external and internal energy dissipation. The formulation of the governing equations is unique in its treatment of energy calculation adopting the Rankine active and passive state of failure. Although the energy approach degenerates to the existing momentum approach for special cases, it is potentially more robust in accounting for various internal mechanisms that may occur in debris flow.

RÉSUMÉ
Cet article présente le concept d'une approche énergétique à l'analyse des coulées de débris. La démarche est dérivée de la conservation de la masse et de l'énergie tant au niveau global que local. La formulation des équations de base est unique dans son traitement du calcul de l'énergie en utilisant l'approche active et passive de l'état de rupture de Rankine. Bien que pour certains cas spéciaux, l'approche énergétique dégénère vers l'approche moment existante, elle est potentiellement plus robuste en tenant compte des diverses mechanismes internes qui peuvent apparaître en coulées de débris.

1 INTRODUCTION

Debris flows incorporate a broad range of sediment-fluid flows intermediate between dry rock avalanches and hyperconcentrated flow (Hungr et al. 2001). Debris flows and related phenomena are encountered in a variety of geological and geomorphological settings. Because of high flow velocities, large impact forces, long runout distances, and poor temporal predictability, debris flows are among the most dangerous and destructive natural hazards. Debris flows often have severe social, economic, and environmental consequences in mountainous environments, particularly on potential debris flow areas where settlements and infrastructure have been built.

To mitigate and manage hazards induced by debris flows, it is necessary to understand the mechanisms of debris flow initiation, transport, and deposition and develop reliable analytical models to predict its behaviour. Numerical simulation of debris flows often adopt an equivalent fluid concept and momentum approach (Savage and Hutter 1989; Hungr 1995) in calculating the flow velocities, runout distance, and depositional profiles. Formulation of governing equations is based on the application of mass and momentum conservation equations on the flowing mass with simplifications using depth averaging techniques (Savage and Hutter 1989, 1991).

In this paper, an analytical model based on energy conservation is formulated on a macroscopic scale. Compared with the existing depth-averaged continuum models, deformation work and internal energy dissipation are introduced into the governing equations of the new model. Model verifications are undertaken by comparison of numerical predictions with analytical solutions. Results of back-analysis of debris flow cases are presented following the model verifications.

2 AN ANALYTICAL MODEL BASED ON ENERGY CONSERVATION LAWS

Mathematical modeling of granular flows was originally introduced by Savage and Hutter (1989, 1991). Starting from the mass and momentum conservation equations for flow on a rough inclined plane and using the depth averaging process, Savage and Hutter (1989) derived one-dimensional, depth-averaged equations for the shallow free surface flow of dry granular materials. The Savage-Hutter model simplifies a moving granular mass as a cohesionless Coulomb frictional material. The relationship between shear and normal stresses on internal and rough bounding surfaces obeys the Coulomb friction law. Multi-dimensional extensions of the Savage-Hutter model have been formulated for analyzing dry granular flows over complex topography (Denlinger and Iverson 2004; Iverson and Denlinger 2001; Pudasaini and Hutter 2007; Wang et al. 2004).

Hungr (1995) developed a dynamic model (DAN) for the runout analysis of rapid landslide from a geotechnical perspective. DAN was based on an explicit solution of the Saint Venant equations with the integration of a variety of constitutive relationships. The sliding mass in DAN was simplified as an equivalent fluid and represented a number of boundary and mass elements. Formulation of the governing equations was based on the principles of momentum conservation for the boundary elements. Mass conservation was applied to the mass elements to calculate changes in element depths. Extension of the DAN model to simulate fast moving landslides over three dimensional terrains was formulated and presented by McDougal and Hungr (2004).

After a comprehensive review of existing models for debris flow simulation, a new computational model based on mass and energy conservation laws was formulated by Wang (2008). As shown in Figure 1, in the slice-based model the flowing mass is represented by an ensemble of contiguous slices which are subjected to gravitational forces, basal resistance, and internal forces. Forces acting on a typical slice of width b and height h are shown in Figure 2. The governing equation for an individual slice can be formulated based on mass and energy conservation laws:

$$\frac{d}{dt}\left(\frac{1}{2}m\bar{u}^2\right) = mg\bar{u}\sin\theta + \frac{1}{2}mgh\bar{e}_{zz} + P_L\bar{u}_L\cos\theta_L - P_R\bar{u}_R\cos\theta_R - T\bar{u} - \int_V \tau_{ij}e_{ij}dV \quad [1]$$

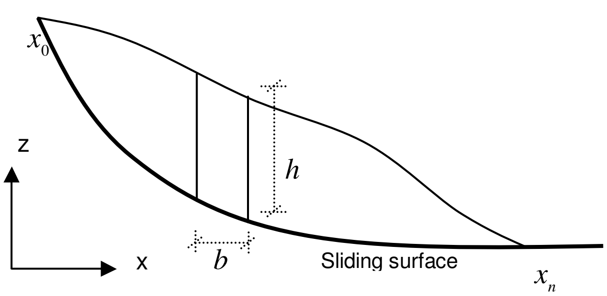

Figure 1. Representative slice in the slice-based model

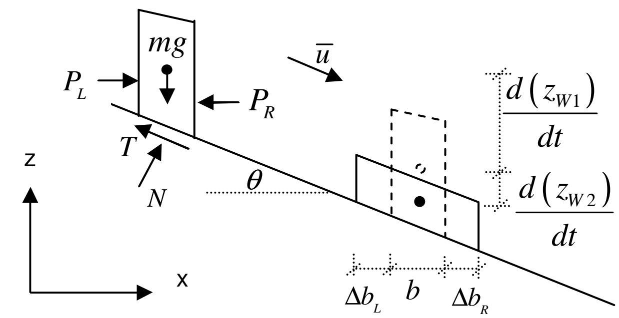

Figure 2. Forces acting on a typical slice

Following the procedures of Savage and Hutter (1989), a Lagrangian finite difference scheme was developed for solving the equations of the slice-based model. Solution of the governing equations of rapid landslides requires determining the positions of the boundaries of each slice at time t. The numerical scheme assumes that all the variables involved in the calculation at $t+\Delta t$ are known from the previous time t, where Δt is a time step interval. In the Lagrangian finite difference scheme, the governing equation for the slice k can be written as:

$$\frac{E_k^{t+\Delta t} - E_k^t}{\Delta t} = \dot{W}_k^t \quad [2]$$

$$E_k = \frac{1}{2}m_k\bar{u}_k^2 \quad [3]$$

$$\dot{W}_k = m_k g\bar{u}_k\sin\theta_k + \frac{1}{2}m_k g h_k\left(\bar{e}_{zz}\right)_k + \left(P_L\bar{u}_L\cos\theta_L\right)_k - \left(P_R\bar{u}_R\cos\theta_R\right)_k - T_k\bar{u}_k - \int_{V_k}\tau_{ij}e_{ij}dV \quad [4]$$

where E_k is the kinetic energy of slice k, $\dot{W}_k$ is the sum of the rate of the work done by body force, surface force, and energy dissipation due to the deformation of slice k. E_k and $\dot{W}_k$ are determined from equations [3] and [4], respectively.

The constitutive law and assumptions regarding the interslice forces and deformation work are required to calculate the rate of work in equation [4]. In debris flow simulations, a frictional model is generally used as a constitutive law to calculate bases' shear resistance. The lateral pressures can be approximated as a product of hydrostatic pressure and the coefficient of lateral pressure. The lateral stress coefficient can be active, passive, or hydrostatic (the coefficient of lateral stress is equal to 1) based on the local strain rate (velocity gradient) of a slice in the longitudinal direction. The values of lateral stress coefficients for a frictional material are calculated using the Rankine equation.

Deformation of the slice in two dimensions is simplified as a pure shear deformation and the deformation work rate is approximated by:

$$\int_{V_k} \tau_{ij} e_{ij} dV = \left(\overline{\tau}_{xx}\overline{e}_{xx} + \overline{\tau}_{zz}\overline{e}_{zz}\right)_k b_k h_k \quad [5]$$

where

$$\left(\overline{e}_{xx}\right)_k = -\frac{\partial}{\partial x}\left(u\cos\theta\right)_k = -\frac{\left(u_R\cos\theta_R - u_L\cos\theta_L\right)_k}{b_k}$$

is the mean strain rate. Following the conventions of stress and strain representation usually adopted in geotechnical engineering, negative signs have been introduced in order that compressive stresses and compressive strains are positive quantities. For an incompressible fluid:

$$\overline{e}_{zz} = -\overline{e}_{xx} \quad [6]$$

$\overline{\tau}_{xx}$ and $\overline{\tau}_{zz}$ are mean horizontal and vertical stresses calculated using the following equations:

$$\left(\overline{\tau}_{xx}\right)_k = K_k\left(\overline{\tau}_{zz}\right)_k \quad [7] \text{ and}$$

$$\left(\overline{\tau}_{zz}\right)_k = \frac{\gamma h_k}{2} \quad [8]$$

where K_k is calculated by averaging the coefficients of lateral stress on left and right sides of the slice:

$$K_k = \frac{\left(K_L\right)_k + \left(K_R\right)_k}{2} \quad [9]$$

This model based on energy conservation laws has been tested against analytical solutions (Wang, Morgenstern, and Chan 2010). Simulations of simple granular flows have also been undertaken to examine the plausibility of the model and applicability of the numerical scheme (Wang 2008; Wang, Morgenstern, and Chan 2010). Results of the numerical tests indicate that the model based on energy conservation laws is robust and applicable to modelling of flow slides and debris flows.

3 MODEL VERIFICATION - COMPARISON BETWEEN NUMERICAL PREDICTIONS AND ANALYTICAL SOLUTIONS

Mangeney et al. (2000) presented an analytical solution for a one-dimensional granular avalanche over a uniform slope of arbitrary inclination. The analytical solution describes the motion of a flow front of the dam break granular flow over an infinite, uniform slope with a Coulomb-type friction acting at the base of the flow. The performance and computational accuracy of the slice-based model and associated numerical scheme were tested by comparing numerical predictions with analytical solutions of one-dimensional granular flows. It should be noted that the analytical solution of Mangeney et al. (2000) can be applied only to an idealized flow where the lateral earth pressure is assumed to be hydrostatic, the basal friction angle is not greater than the slope angle, and the flow never stops on the slope.

Figures 3 and 4 present comparisons between the analytical solutions and the numerical simulations of dam break scenarios on horizontal and inclined planes, respectively. In both cases, an internal friction angle of zero was applied to provide the hydrostatic lateral pressure distribution in all the simulations. Figure 3 shows the result of the dam break scenarios over a horizontal plane with zero basal friction. Figure 4 presents comparisons between analytical solutions and numerical predictions for dam break scenarios on a 30º slope with a 20º basal frictional angle.

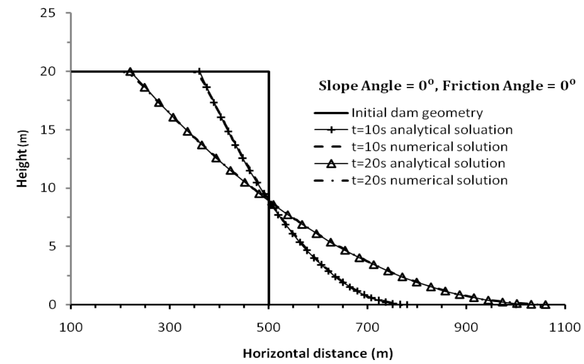

Figure 3. Comparison between a numerical simulation and an analytical solution of a dam break in a horizontal plane

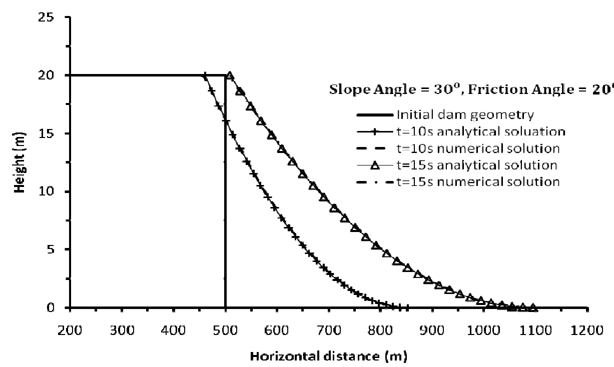

Figure 4. Comparison between a numerical simulation and an analytical solution of a dam break on a 30º slope

As shown in Figures 3 and 4, the numerical predictions accurately reproduce the analytical solutions of the dam break-induced granular flows over both horizontal and inclined planes.

4 MODEL APPLICATION – NUMERICAL ANALYSIS OF DEBRIS FLOWS ON NATURAL SLOPES

The analytical model and numerical method presented in this paper was used to back analyze a well-documented debris flow case – the Tsukidate landslide.

4.1 Overview of the Tsukidate landslide

On May 26, 2003, an earthquake with a moment magnitude of 7.0 occurred in northern Japan. The earthquake triggered a number of landslides. One of the earthquake-induced large landslides was located in the Tsukidate area. The Tsukidate landslide originated from a failure in a gentle natural slope with an inclination of approximately 13.5°. The source area of the landslide was about 40 m wide and 80 m long, with a maximum depth of about 5 m. It was estimated that the landslide volume was about 8,100 m^3. The deposition area was about 50 m wide and 120 m long. The apparent friction angle of the landslide was about 7.3º (Fukuoka et al. 2004; Uzuoka et al. 2005).

Field investigation indicated that the soils in the source area were composed mainly of pyroclastic deposits. Soil samples were taken from the source area and deposition area of the landslide after the earthquake. The grain size analyses of soil samples taken from the landslide indicated that pyroclastic deposits involved in the landslide consisted of about 20% gravel, 50% sand, 20% silt, and 10% clay. The gravel was mainly composed of pumice.

Undrained cyclic ring-shear tests were conducted on samples from the landslide source area to study the triggering mechanisms of the Tsukidate landslide. The tests revealed that soils in the Tsukidate landslide were highly liquefiable and the apparent friction angle of the liquefied soils was about 7.5º (Fukuoka et al. 2004). It was concluded that the failure of the slope was the result of high pore-water pressures generated by seismic loading during the earthquake. After the original slope failure, persistent high pore water pressure due to widespread shear deformation within the soils resulted in the lower resistance and high mobility of the landslide (Fukuoka et al. 2004).

4.2 Numerical simulation of the Tsukidate landslide

Study of topographic features of the Tsukidate landslide was undertaken based on field measurements and survey data (Fukuoka et al. 2004; Uzuoka et al. 2005). The central longitudinal section of the landslide from the source area to the deposition area was used to obtain the sliding surface and pre-failure geometry in the dynamic analysis.

Simulation of the Tsukidate landslide was carried out using an analytical model based on energy conservation. The frictional model was used as the constitutive law to calculate flow resistance. The same values for internal and basal friction angles were used in the analysis. The post-failure profiles of the landslides from the field observation and dynamic analyses of the Tsukidate landslide are presented in Figure 5.

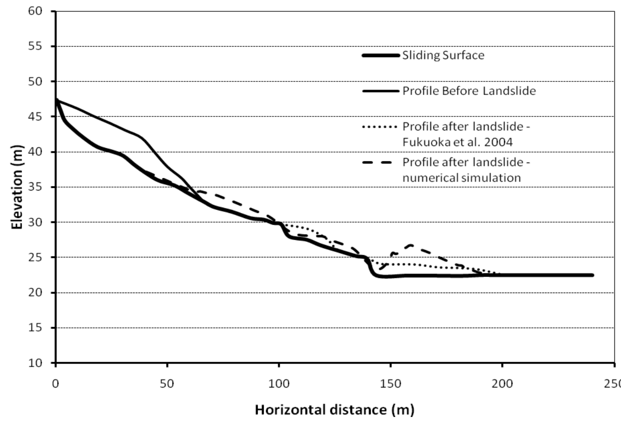

Figure 5. Numerical simulation of the Tsukidate landslide

As shown in Figure 5, the front runout distance calculated from the dynamic analysis is approximately 130 m, which is very close to the runout distance of 135 m measured in the field. The back-calculated friction angle from the model based on energy conservation laws is 8°, which matches very well with the apparent friction angle of 7.5º measured from undrained ring-shear tests.

5 CONCLUSION

A new computational model is introduced for analyzing the motion of granular flows such as flow slides and debris flows. The new model is formulated based on mass and energy conservation laws. Comparison of numerical predictions and analytical solutions demonstrates the predictive power of the new model. The model was also applied to back-analyzing a well-documented debris flow case. The back-calculated friction angle from the proposed model matches well with experimental results from ring-shear tests.

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LIST OF SYMBOLS

P interslice force
T shear force acting on slice base
N normal force acting on slice base
K coefficient of lateral earth pressure
$V = b \times h$ volume of slice
b width of slice
h height of slice
$\overline{u}$ mean velocity of slice
θ inclination of the slice base with respect to the horizontal
L subscript denoting slice property on the left side
R subscript denoting slice property on the left side
k subscript denoting slice number
m mass of slice
g gravitational acceleration
γ unit weight of sliding mass
$\tau_{ij}e_{ij}$ deformation work rate
$\overline{e}_{xx}$ mean horizontal strain rate of slice
$\overline{e}_{zz}$ mean vertical strain rate of slice
$\overline{\tau}_{xx}$ mean horizontal stress
$\overline{\tau}_{zz}$ mean vertical stress