Considering the influence of neighbouring wells when interpreting a pumping test in a confined aquifer

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ABSTRACT
Pumping tests are generally interpreted without taking into account the activity of neighbouring pumping wells, which can lead to erroneous interpretations. This paper presents a simple method derived from the Cooper-Jacob method to interpret the results of a pumping test influenced by other wells and carried out in a confined aquifer under Theis' conditions. An application using numerical data is also presented. This method requires knowing the time the interfering well started to extract water and its pumping rate, but contrary to previous methods, it does not require knowing the equilibrium water level.

RÉSUMÉ
Les essais de pompage sont généralement interprétés sans tenir compte de l'activité des puits avoisinants, ce qui peut conduire à des erreurs d'interprétation. Cet article présente une méthode simple, dérivée de la méthode de Cooper-Jacob, qui permet d'interpréter les résultats d'un essai de pompage influencé par d'autres puits dans un aquifère à nappe captive, sous les conditions de Theis. Un exemple d'application avec des données numériques est également présenté. Cette méthode nécessite de connaître le temps de départ du pompage du puits interférent et son débit, mais contrairement aux méthodes existantes, elle ne nécessite pas de connaître le niveau de la nappe à l'équilibre.

1 INTRODUCTION

The phenomenon of interferences between wells during a pumping test is an important issue mentioned by Theis as early as 1935 in his well-known article.

However, when interpreting a pumping test in a confined aquifer, neighbouring wells are rarely taken into account, either because they are considered to be too far from the tested well, or because they are pumping at a constant rate (Rushton, 1985).

These reasons are not sufficient to neglect the influence of neighbouring wells. According to Rushton (1985), the distance between the pumping well and the interfering well is in fact of small importance in comparison with the pumping time of the interfering well and with its pumping rate.

The solution of the direct problem is a simple application of the superposition principle thanks to the linearity of the equations in a confined aquifer, and it has been used in many studies (e.g., Engelund, 1957).

The inverse problem, however, has not been much documented. Wenzel and Greenlee (1943) and Carapcioglu (1977) have proposed two graphical methods based on the Theis equation, but these methods suppose that the equilibrium water level is known, while it is generally not the case if the interfering well starts pumping before the beginning of the pumping test.

Cooper and Jacob (1946) developed their equations for many wells. These generalized equations were used with success by Bentley (1977) for an aquifer in Florida, but again, the knowledge of the equilibrium water level is necessary to use this method.

The present article presents a new method to interpret a pumping test influenced by another pumping well, when the aquifer equilibrium level is unknown, that is to say when the interfering well starts pumping before the pumping test begins. The knowledge of the pumping rate and of the pumping time of the interfering well is yet required.

2 THEORETICAL ASPECTS

2.1 During the pumping phase

The new method is based on the Cooper-Jacob equation. As a consequence it applies under the conditions listed by Theis (1935): the aquifer must be confined, horizontal, homogenous, isotropic, and of infinite extension; and the well must be fully penetrating and of infinitesimal diameter. Moreover, the approximation of Cooper-Jacob, $u_k < 0.02$ (Cooper and Jacob, 1946) must be valid for each pumping well (index *k* is used for the well No. k).

$$u_k = \frac{r_k^2 S}{4 T t_k} \qquad [1]$$

The parameter r_k (m) is the distance between the well *k* and the observation well, *S* is the aquifer storativity, *T* (m^2/s) its transmissivity and t_k (s) the time since the well *k* started pumping.

Cooper and Jacob (1946) gave the equation connecting the drawdown s_k (m) induced by the pumping well k at the observation well to the previous parameters and to the pumping rate Q_k (m^3/s).

$$s_k = \frac{2.3Q_k}{4\pi T}\log\frac{2.25Tt_k}{r_k^2 S} \quad [2]$$

From this point, the subscript p and the expression "pumping well" will be used for the tested well and the subscript i and the expression "interfering well" will be used for the pumping well that interfere with the previous one. The theory is presented below for only one interfering well. The origin of time is the beginning of the pumping test and the interfering well started pumping at a time t_0 before the beginning of the pumping test.

The initial drawdown s_0 (m) induced by the interfering well in the observation well at the time $t = 0$ can be estimated using Equation 2.

$$s_0 = \frac{2.3Q_i}{4\pi T}\log\frac{2.25Tt_0}{r_i^2 S} \quad [3]$$

The apparent drawdown s_a (m) is measured from the water level at the beginning of the pumping test. It is equal to the difference between the real drawdown s (m), measured from the equilibrium water level, and the initial drawdown s_0 (Figure 1).

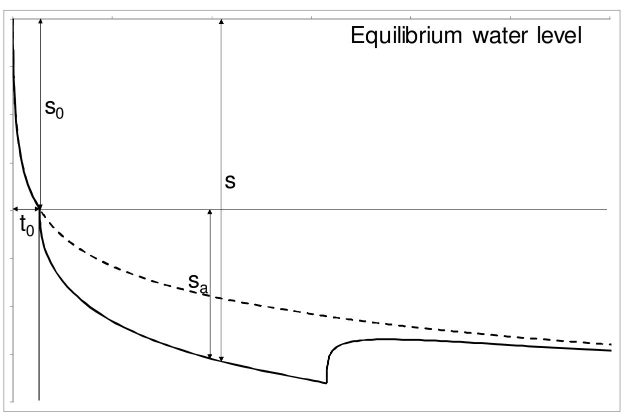

Figure 1. Real drawdown and apparent drawdown (adapted from Rushton, 1985)

On Figure 1, the dashed line represents s vs. t if the interfering well had pumped alone and the solid line is the combination of the drawdown induced by the interfering well and by the pumping test. The latter includes a pumping phase followed by a recovery phase.

The superposition principle can be applied to Equation 2 and the real drawdown is thus given by Equation 4.

$$s = \frac{2.3Q_i}{4\pi T}\log\frac{2.25T(t+t_0)}{r_i^2 S} + \frac{2.3Q_p}{4\pi T}\log\frac{2.25Tt}{r_p^2 S} \quad [4]$$

As a consequence the apparent drawdown is given by Equation 5.

$$s_a = \frac{2.3Q_i}{4\pi T}\log\frac{t+t_0}{t_0} + \frac{2.3Q_p}{4\pi T}\log\frac{2.25Tt}{r_p^2 S} \quad [5]$$

As for the Cooper-Jacob equation, Equation 5 can be written in three different ways according to the available data.

It is important to notice that the distance r_i between the interfering well and the observation point does not appear in Equation 5, confirming Rushton's conclusions.

2.1.1 For one observation well

For one observation well, r_p is constant and Equation 5 can thus be written as follows.

$$s_a = \frac{2.3}{4\pi T}\log\frac{t^{Q_p}(t+t_0)^{Q_i}}{t_0^{Q_i}} + \frac{2.3Q_p}{4\pi T}\log\frac{2.25T}{r_p^2 S} \quad [6]$$

Let A be the value under the first logarithm. The plot of s_a vs. A on semi-logarithm paper is a straight line whose slope and y-intercept are related to T and S according to Equations 7 and 8.

$$T = \frac{2.3}{4\pi\Delta s_a / cycle} \quad [7]$$

$$S = \frac{2.25T}{r_p^2}\exp-\frac{4\pi Ts_{(A=1)}}{Q_p} \quad [8]$$

The difference of drawdown on a logarithm cycle Δs_a/cycle (m) is the slope of the straight line and $s_{(A=1)}$ (m) its y-intercept.

2.1.2 For several observation wells at a given time

At a given time t, Equation 5 can be written as follows.

$$s_a = -\frac{2.3Q_p}{2\pi T}\log r_p + \frac{2.3Q_i}{4\pi T}\log\frac{(t+t_0)}{t_0} + \frac{2.3Q_p}{4\pi T}\log\frac{2.25Tt}{S} \quad [9]$$

The plot of s_a vs. r_p is a straight line on semi-logarithm paper. We can deduce the transmissivity and the storativity of the aquifer through Equations 10 and 11.

$$T = -\frac{2.3Q_p}{2\pi\Delta s_a / cycle} \quad [10]$$

$$S = 2.25Tt\exp-\frac{4\pi Ts_{(rp=1)} - Q_i\ln\left(\frac{t+t_0}{t_0}\right)}{Q_p} \quad [11]$$

Where $s_{(rp=1)}$ is the y-intercept of the straight line.

Equation 10 shows that the T value can be estimated through the Cooper-Jacob s vs. log(r) plot even if other wells are interfering with the pumping well.

2.1.3 For several observation wells at several times

If neither t nor r_p are constant, Equation 5 can be written as follows.

$$s_a = \frac{2.3}{4\pi T}\log\frac{t^{Q_p}(t+t_0)^{Q_i}}{r_p^{2Q_p}t_0^{Q_i}} + \frac{2.3Q_p}{4\pi T}\log\frac{2.25T}{S} \quad [12]$$

Let B be the value under the first logarithm. The plot of s_a vs. B on semi-logarithm paper is a straight line. The parameter $s_{(B=1)}$ is the y-intercept of this straight line. T and S are deduced from the following equations.

$$T = \frac{2.3}{4\pi\Delta s_a / cycle} \quad [13]$$

$$S = 2.25T\exp-\frac{4\pi Ts_{(B=1)}}{Q_p} \quad [14]$$

2.2 During the recovery

2.2.1 Residual drawdown method

The recovery can be represented by a new well located at the same place as the pumping well and starting injecting water when the pump is stopped, at the same rate as the pumping test was conducted (Theis, 1935).

As previous equations can easily be transformed for as many interfering wells as wanted through the superposition principle, recovery data can be interpreted in case of interferences with a similar reasoning.

If we suppose that the storativity remains the same during the pumping and recovery phases, the residual apparent drawdown s'_a (m) is given by Equation 15.

$$s'_a = \frac{2.3}{4\pi T}\log\frac{(t+t_0)^{Q_i}t^{Q_p}}{t_0^{Q_i}t'^{Q_p}} \quad [15]$$

Where t' (s) is the time elapsed since the pumped has been stopped.

The plot of s'_a vs. C, where C is the value under the logarithm, is a straight line, whose slope $\Delta s'_a$/cycle depends on T through Equation 16.

$$T = \frac{2.3}{4\pi\Delta s'_a / cycle} \quad [16]$$

As without interferences, the storativity value cannot be found by this method.

2.2.2 Extended drawdown method

After application of the superposition principle and the simplification of the obtained expression, the same equation as the one developed by Johnson (1966) is found (Eq.17).

$$s_p - s'_a = \frac{2.3Q_p}{4\pi T}\log\frac{2.25Tt'}{r_p^2 S} \quad [17]$$

The difference with the Johnson equation is that the extended drawdown s_p (m) is found by extending the straight line of the plot of s_a vs. A and not the straight line in a Cooper-Jacob plot of s vs. t.

The interpretation is yet the same with or without interferences.

$$T = \frac{2.3Q_p}{4\pi\Delta(s_p - s'_a)/cycle} \quad [18]$$

$$S = \frac{2.25T}{r_p^2}\exp-\frac{4\pi Ts_{(t'=1)}}{Q_p} \quad [19]$$

Where $\Delta(s_p\text{-}s'_a)$/cycle (m) is the slope of the straight line and $s_{(t'=1)}$ (m) its *y*-intercept

3 APPLICATION

3.1 The model

The application of the method is illustrated using the finite element code SEEP/W (Geo-Slope International Ltd., 2010). The numerical results obtained were also used to compare the method with the previous methods that neglect interfering wells.

SEEP/W has many applications in hydrogeology, in saturated as well as unsaturated conditions. Its capacity to solve such problems has been studied in detail by Chapuis and al. (2001) and the code was proven to be accurate.

Concerning saturated problems as studied in this article, SEEP/W has been used successfully, for example, to detect the frontiers from recovery data (Chenaf, 1997), to study leaky aquifer (Gauthier, 2003) or to determine how the water stored in well pipes modifies the pumping test data (Chapuis and Chenaf, 2003).

For commodity, the notation *s* will be used for both the drawdown when the interfering well influence is neglected and the apparent drawdown when it is taken into account. Note that it is the same value, though its interpretation is different in each case. In the first case, the water level at the beginning of the pumping test is in fact supposed to be the equilibrium level, which is not true.

The model consists in a plan view of an aquifer of 20 km long by 20 km wide to avoid boundaries interferences. The aquifer transmissivity and storativity are respectively $8.7.10^{-3}$ m^2/s and $4.2.10^{-5}$. Two wells with a diameter of 40 cm are located near the center of the model, 200 m apart. Recharge boundaries constitute the left and right side of the model whereas impermeable boundaries are located at its top and bottom.

The mesh is refined till 2 cm around the wells and is constituted of squares 100m-side when the distance to the wells is greater than 300 m. The number of elements is thus important (around 44000), but the calculations remain quick thanks to the linearity of the equations in saturated conditions.

Time-steps increase exponentially from one second to about an hour for each part of the pumping sequence.

The pumping sequence is presented in Table 1.

Table 1. Pumping sequence.

Duration	Interfering well	Tested well
1h30	180 m^3/h	X
16h	180 m^3/h	36 m^3/h
16h	180 m^3/h	X

The first sequence has been ignored in the interpretation and the origin for time and drawdown are respectively the start of the tested well and the drawdown observed in each piezometer at that time.

Table 2. Piezometers and their distance to the wells.

Piezometer	r_p (m)	r_i (m)
Pz1	0.51	199.49
Pz2	1.05	199.31
Pz3	2.75	202.74
Pz4	5.25	205.24
Pz5	8.85	208.66
Pz6	92.20	111.80
Pz7	149.71	349.71

The drawdown was observed at the nodes listed in Table 2, representing ideal piezometers.

3.2 Interpretation of the numerical results

For each method, the plots obtained with the new method and with the corresponding method without interference are shown on the same figure. The transmissivity and storativity values calculated using each method are presented and discussed in section 4.

3.2.1 Drawdown vs. *A*

The plot of the drawdown vs. the parameter *A* defined in section 2.1.1 is presented in Figure 2 for Pz4 and Pz6.

If the pumping rates are too small, the straight lines obtained with the new method can be too vertical to distinguish the straight part because of the logarithmic scale. In that case, it is possible to calculate the logarithm of *A* and to use a linear scale.

After a short time, the Cooper-Jacob approximation is respected and the plot of *s* vs. *A* becomes a straight line. The corresponding method is the Cooper-Jacob *s* vs. *t* plot. The plot also seems to present a straight line after a short time, but far from the pumping well (Pz6), this straight line is less obvious to distinguish. The following data are slightly curved and the last data points could be interpreted as a second straight line. The slope of this second straight line is around 4.8 times that of the first straight line in the case presented here. Such a plot would usually be interpreted as two impermeable boundaries forming an angle of 75°.

3.2.2 Drawdown vs. r_p

For this method, the plot is the same whether the interfering well is considered or not. As a consequence, it is not presented here.

The only difference between the new method and the Cooper-Jacob one is the equation used to calculate *S*.

The results of the calculation for t = 16 minutes and t = 7 hours 5 minutes are presented further.

3.2.3 Drawdown vs. *B*

Figure 3 presents the data obtained in all the piezometers, that is to say s vs. B with the new method and s vs. t/r^2 with the Cooper-Jacob method.

The plot is a straight line when B is the x-axis and it looks as if there were a boundary for t/r^2: there are as many parallel straight lines as piezometers for high values on the x-axis, and the data are all joining an asymptotic line for shorter times.

The interpretation was thus done using the asymptotic line drawn on the plot.

We can notice that the data from the two piezometers that are located far from the pumping well do not join this asymptotic line. Piezometers near the pumping well are thus necessary to draw an accurate asymptotic straight line.

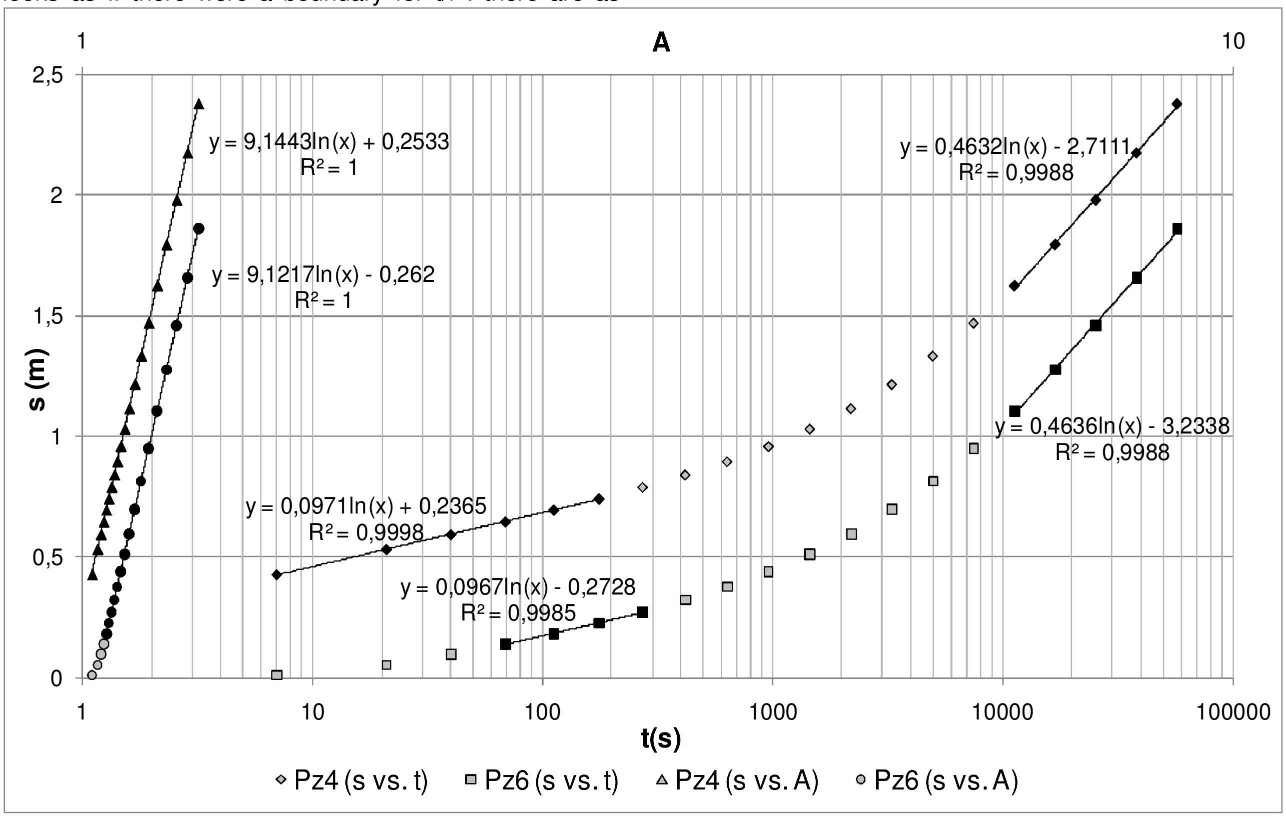

Figure 2. Comparison of the new method for one piezometer with the Cooper-Jacob s vs. t method

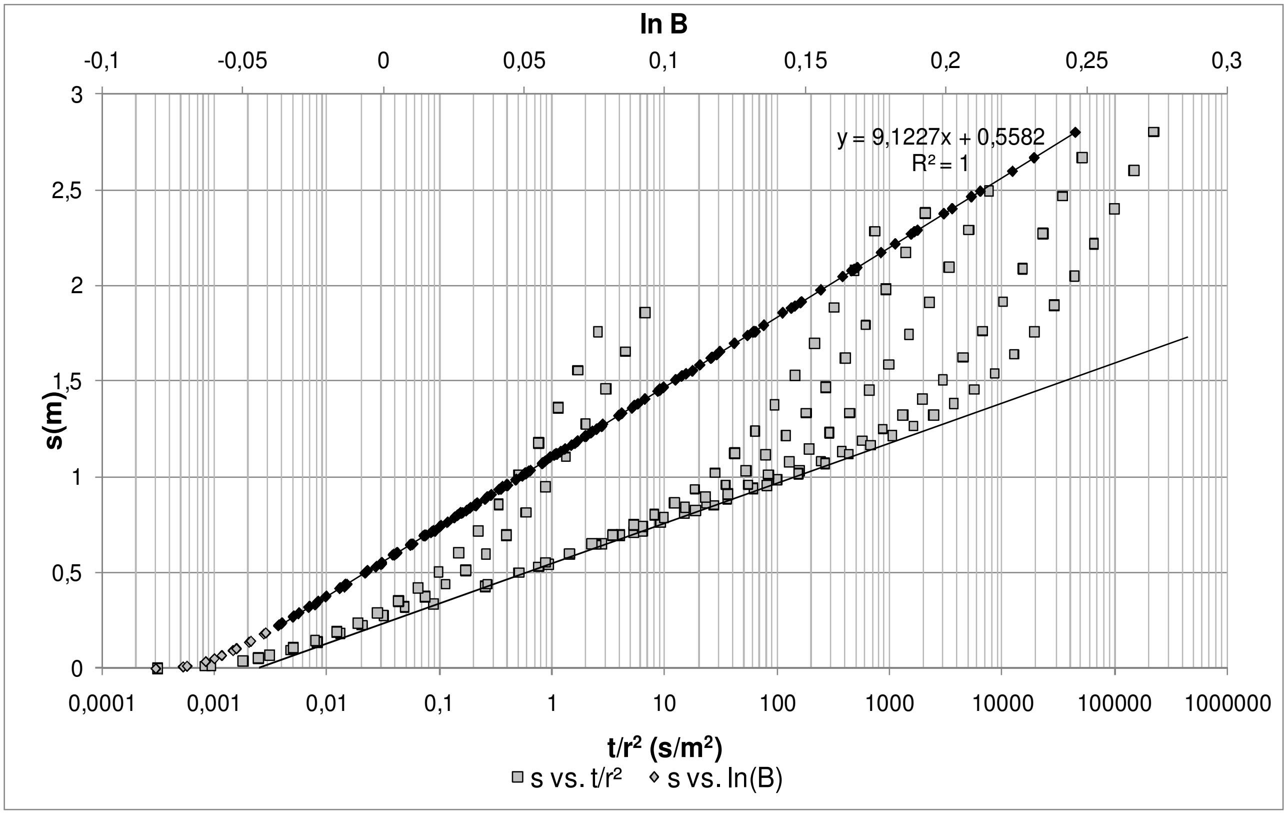

Figure 3. Comparison of the new method for many piezometers with Cooper-Jacob s vs. t/r² method

3.2.4 Residual drawdown vs. *C*

The plot on Figure 4 presents, for the same two piezometers as before, the residual drawdown *s*' vs. *C* and vs. *t*/*t*'.

When using the new method, the plot is a straight line independent from the distance between the pumping well and the piezometer. This line approximately reaches the origin, according to Equation 15.

Without considering the interfering well, the plot of *s*' vs. *t/t*' presents a straight part also independent from the distance r_p, but which does not reach the origin. Moreover, the last points (low values of *t/t*') are not on the straight line.

We can observe on both plots an increase of *s'* at the far end of the recovery phase, due to the predominance of the interfering well. However, the last points are still superimposed on the straight line when using the new method. The time *t/t'* tends in fact to 1 and the multiplication of this factor to the Q_p^th^ power by the value $(t+t_0)/t_0$ to the Q_i^th^, which tends to the infinite, implies an increase of *C* at the end of the recovery. As both *s*' and *C* increase, the points still fall on the straight line.

3.2.5 Extended drawdown method

This method cannot be applied if an impermeable boundary has been reached during the pumping phase. The problem is the same in the presence of an interfering well, as the behaviour of the drawdown curve is similar. In fact, calculating the value of s_p-*s*' gives only negative values with the Cooper-Jacob *s* vs. *t* straight line.

As a consequence, the new method cannot be compared with the Johnson one. Thus, only the plots of Pz4 and Pz6 obtained with the new method are shown on Figure 5.

We can see on Figure 5 that the method gives straight lines as soon as Cooper-Jacob approximation is verified.

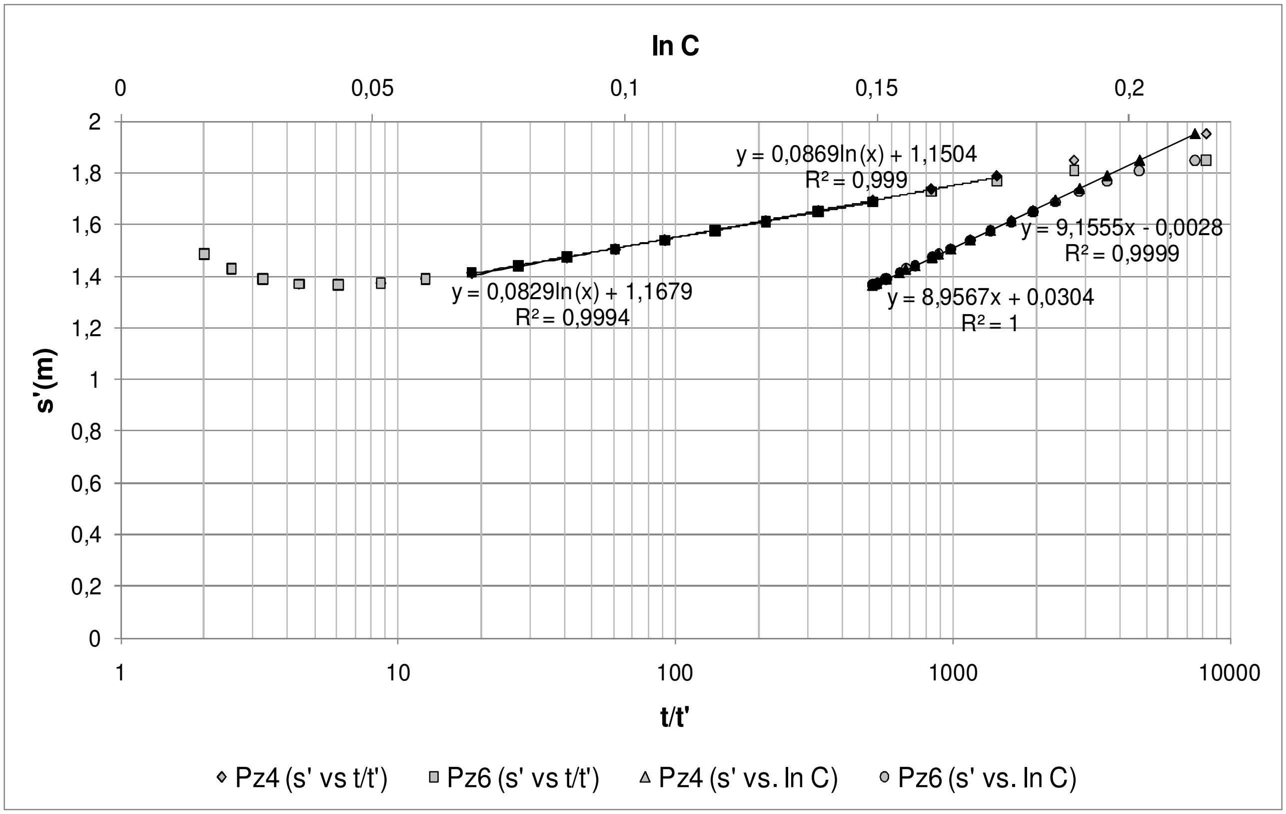

Figure 4. Comparison of the new method during the recovery with the Theis s vs. t/t' method

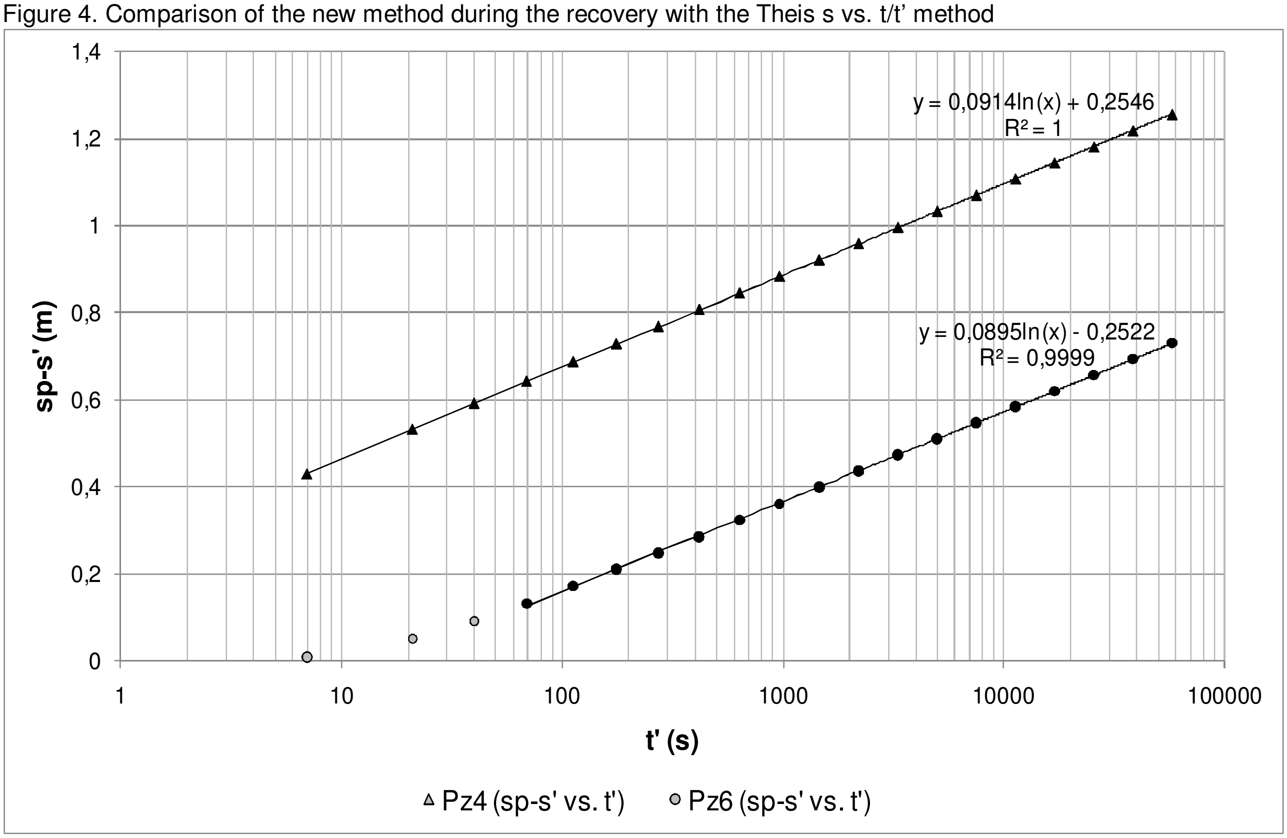

Figure 5. Extended drawdown method for Pz 4 and Pz6.

4 DISCUSSION

Through Tables 3 and 4, we can see that both *T* and *S* values are better estimated using the new method.

As mentioned in section 3.2.2, only one value of transmissivity has been obtained for a constant time, but two different storativity values (Table 3).

Even if it is a bit less accurate, the estimation of *T* is nevertheless quite good when the interfering well influence is neglected. The estimation of *S* depends also on the *x*-axis. When the drawdown is plotted vs. r_p, the estimation is really bad and the error increases with the considered time, whereas it is excellent when t/r^2 is the *x*-axis.

Table 3. Interpretation during the pumping phase with and without considering the interfering well influence

	Pz4		Pz6		t = 16 min		t = 7h 05min			
	s vs. A	s vs. t	s vs. A	s vs. t	s vs. r_p NM	s vs. r_p CJ	s vs. r_p NM	s vs. r_p CJ	s vs. B	s vs. t/r^2
T (m²/s)	8.70E-03	8.20E-03	8.72E-03	8.23E-03	8.74E-03		8.70E-03		8.72E-03	8.73E-03
S	4.45E-05	5.85E-05	4.08E-05	3.66E-05	4.20E-05	1.85E-05	4.60E-05	7.50E-09	4.32E-05	4.25E-05
Error T (%)	0.03	5.80	0.28	5.41	0.51		0.03		0.26	0.29
Error S (%)	5.95	39.39	2.79	12.88	0.08	56.05	9.44	99.98	2.87	1.28

NM: new method ; CJ: Cooper-Jacob

However, it is important to keep in mind that the results may have been biased by the fact that the theoretical values were known a priori. As a consequence, when more than one straight line could have been drawn, the better one was chosen. For example, the interpretation of Pz6 *s* vs. *t* plot alone could have yielded quite different results. This problem disappears with the new method as all the points fall on the straight line as long as a boundary is not reached.

Table 4. Interpretation during the recovery phase with and without considering the interfering well influence

	Pz4		Pz6		Pz4	Pz6
	s' vs. C	s' vs. t/t'	s' vs. C	s' vs. t/t'	sp-s' vs. t'	sp-s' vs. t'
T (m²/s)	8.69E-03	9.16E-03	8.88E-03	9.60E-03	8.71E-03	8.89E-03
S	X	X	X	X	4.38E-05	3.94E-05
Error T (%)	0.09	5.26	2.12	10.34	0.07	2.20
Error S (%)	X	X	X	X	4.36	6.17

The interpretation during the recovery phase should be better than during the pumping phase when the interfering well is neglected. The variation of drawdown induced by the interfering well is in fact smaller during the recovery phase than during the pumping phase (Figure 1). This is however not obvious when looking at the results, again because of the subjectivity of the interpretation.

The method will have to be validated with field data. In addition, because some piezometers as Pz4 give rather good estimations of *T* and *S*, it will be important to find criteria depending on t_0, Q_i and r_p in order to know whether the interfering wells can be neglected or not. These two aspects will be the object of future investigations.

5 CONCLUSION

A new method based on the Cooper-Jacob equation has been presented in order to interpret pumping tests disturbed by interferences from another pumping well.

The equations developed permit to obtain straight line plots on semi-logarithm paper, whose slopes and *y*-intercepts are related to the hydrodynamic parameters of the aquifer.

The presence of straight lines facilitates the interpretation of the pumping test data and yields more accurate results.

To use this method, the pumping time and rate of the interfering well must be known. However, its location and the equilibrium water level are not important parameters. The second can yet be determined if the first is known.

It is obvious that this method can be transposed to interpret a pumping test with more than one interfering well.

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