Numerical modelling of the Guelph permeameter test using the complete unsaturated hydraulic functions of the soil

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ABSTRACT
A Guelph permeameter test is modelled using SEEP/W software using the full unsaturated property functions of six soils. The results of the tests are analysed using Glover's equations, the Guelph Richards single head analysis and the Guelph Richard's simultaneous equations analysis. The results are compared to the known values of the hydraulic conductivity of each soil. The accuracy of the Glover analysis was between -46% to 240%, the accuracy of the Guelph permeameter single head analysis was between -82% to 39% and the Guelph permeameter simultaneous equations approach yielded negative values of K_{fs}.

RÉSUMÉ
Un essai de perméabilité de Guelph a été modélisé à l'aide du logiciel SEEP/W en utilisant les courbes complètes de propriétés non saturées de six sols. Les résultats de la modélisation ont été analysés à l'aide de la méthode de Glover, de la méthode à charge unique de Guelph-Richards et de la méthode de résolution simultanée des équations de Guelph-Richards. La méthode de Glover a donné des valeurs entre -46% et 240% de la réelle valeur de la conductivité hydraulique saturée, la méthode à charge unique de Guelph-Richards a donnée des valeurs entre -82% et 39% de la valeur réelle alors que la méthode par résolution d'équations simultanée de Guelph-Richards à donné des valeurs négatives de K_{fs}.

1. INTRODUCTION

The saturated hydraulic conductivity of a soil is an important property which engineers strive to determine as correctly as possible. Projects involving the upper layers of unsaturated soil, or vadose zone, such as the design of drainage system, the design of leaching fields or the testing of cover layers in landfill sites, often imply extensive testing, area wise. The cost and time associated with the collection and laboratory testing of soil core samples may be prohibitive for such projects. A fast and economic method of determining the soil hydraulic conductivity must be used in such situations.

The Guelph permeameter is one method that corresponds to both these criteria's. The equations associated with the method have been improved many times over the years, yet they still rely on simplifying assumptions that reduce the accuracy of the obtained results.

Using a numerical code that considers the full saturated- unsaturated functions of a soil such as SEEP/W, it is possible to model a Guelph permeameter test. The numerical modelling was done for six soils with known k(u) and θ(u) functions. The flow rates obtained numerically have been used to compute the values of Kfs using the Glover analysis, (Zangar, 1953), the Guelph Richards analysis of simultaneous equations (Reynolds et al. 1985) and the Guelph Richards single head analysis (Elrick et al. 1989).

2. THEORY

The Guelph permeameter method consists of maintaining a constant hydraulic head in a small radius well previously bored in the soil layers being investigated, as pictured in Figure 1. Upon reaching complete " field saturation" or "satiation" in the vicinity of the well, the infiltration rate is measured and used to calculate the soil's satiated hydraulic conductivity, commonly referred to as K_{fs}. This calculation can be accomplished using the most recent method; the Guelph Richards single head analysis, or the simultaneous equations approach, or the Glover solution which was not developed specifically for the Guelph permeameter method but has been used (Amoozegar, 1989) to solve Guelph permeameter problems.

The development of the Guelph permeameter methods of analysis is based on previous analysis methods of the constant head well permeameter method, a method which like the Guelph permeameter relies on the application of a constant head in a shallow well and was described by Reynolds et al. (1983). The main improvement the Guelph permeameter method has brought to this type of shallow well, unsaturated zone testing method is the addition of a Mariotte cell to maintain a constant head in the shallow well, bringing about an important reduction in time and water quantity required to perform a test.

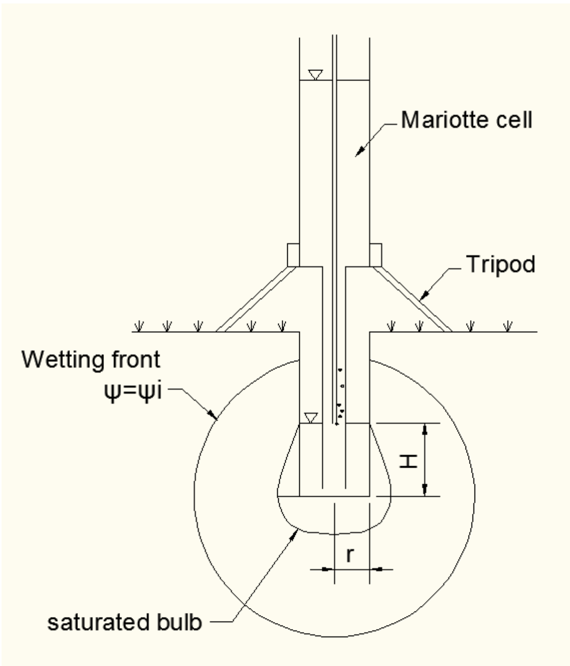

Figure 1 : Schematics of the Guelph permeameter and water occurrences in the infiltration zone

2.1 THE GLOVER ANALYSIS

The Glover analysis (Zangar, 1953) of the flow in a shallow well is based on the solving of the Laplace equations, as applied to water flow in a shallow well and takes the form:

$$K_{fs} = CQ/\left(2\pi H^2\right)$$

[1]

Where, K_{fs} (m/s) is the parameter we are attempting to estimate; the field saturated hydraulic conductivity, Q (m^3/s) is the flow of water necessary to maintain a constant head of water H (m) in the well, and C is a shape parameter associated primarily with the geometric parameters of the well which must be calculated using equation 2:

$$C = \sinh^{-1}\left(\frac{H}{r}\right) - \left(\frac{r^2}{H^2}+1\right)^{1/2} + \frac{r}{H}$$

[2]

Where H (m) is the constant hydraulic head in the well as defined previously and r (m) is the radius of the said well.

2.2 THE GUELPH RICHARDS ANALYSIS

The Guelph Richards analysis (Reynolds et al. 1985), offers a solution to the Richards equations (Richards, 1931) applied to the flow of water from a shallow well located in the vadose zone. It considers not only gravitational and pressure induced flows, but also allows for a capillary component of flow. The resulting equation takes the form:

$$K_{fs} = \frac{CQ}{2\pi H^2 + C\pi r^2 + 2\pi\frac{H}{\alpha^*}}$$

[3]

Where K_{fs}, Q and H retain the same definition as stated above, and α^*, is defined as:

$$\alpha^* = \frac{K_{fs}}{\phi_m},$$

[4]

Where Φ_m is the matric flux potential (m^2/s) as defined by Gardner (1958).

$$\phi_m = \int_{\psi_i}^{0} K(\psi)\,d\psi$$

[5]

Where Ψ_i (m) represents the initial suction present in the soil.

The Guelph Richards analysis, contrary to previous analyses applied to shallow well infiltration problems, took into account the unsaturated properties of soils, namely its capillary potential. In the denominator of equation 3, the first term on the left represents the portion of flow attributable to water pressure in the well, the middle term represents the portion of flow attributable to gravity through the base of the well and the third term on right represents the portion of flow due to capillary action of the soil. This analysis method takes into account the fact that the short period of time used to reach a stable infiltration flow only produces a condition of satiation in the very close vicinity of the borehole, therefore, the dryer soil around the satiated zone impacts the total flow exiting the well. Note that equation 3 contains two unknowns. Two methods have been proposed to surpass this difficulty: the simultaneous equations approach (Reynolds et al. 1985) and the single head approach (Elrick et al. 1989).

The values of the shape factor coefficient, C, are provided in a figure (Soil Moisture Corp. 2008) which the operator can use to select the appropriate C value for the well conditions. Later on, Zhang et al. (1998) have developed equations that allow plotting of these numerical results, facilitating computerized interpretation of Guelph Permeameter tests. The equations 6, 7 and 8 are respectively applicable to sands, silts and clays and unstructured clays.

$$C_1 = \left(\frac{H/r}{2.074 + 0.093 H/r} \right)^{0.75}$$
[6]

$$C_2 = \left(\frac{H/r}{1.992 + 0.091 H/a} \right)^{0.68}$$
[7]

$$C_3 = \left(\frac{H/r}{2.102 + .118 H/r} \right)^{0.655}$$
[8]

These equations are represented in Figure 2 below.

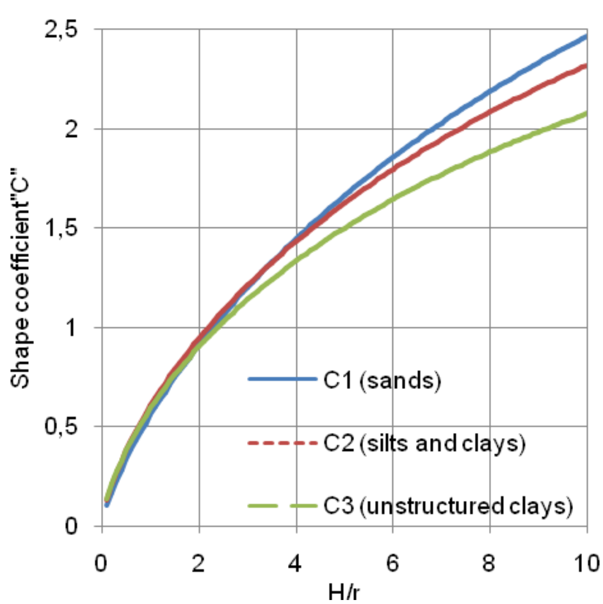

Figure 2: Graphical representation of the C value equations (after Zhang. 1998).

2.2.1 SIMULTANEOUS EQUATIONS APPROACH

Equation 3 was originally developed to be solved simultaneously. To achieve this, the operator performs two tests, using two different heads, H_1 and H_2. This inevitably yields two different values of Q and C, to be used to obtain two different equations 3, to be solved simultaneously for K_{fs} and α^*.
These methods are considered suitable under certain conditions, but authors (e.g., Reynolds et al. 1989.) have reported obtaining negative values for K_{fs} and Φ_m (which can be calculated once alpha is known). These negative values have been attributed to soil heterogeneities in area comprised between H_1 and H_2. These can be soil fractures, plant roots, unusual macropores or the presence of stones to name a few.

2.2.2 THE SINGLE HEAD APPROACH

The single head approach came in response to the negative values of K_{fs} and Φ_m that were obtained using the simultaneous equations approach. It proved to be a method to consistently eliminate this type of occurrences.
The single head approach consists, just like the simultaneous equations approach, in performing two tests at different heads. This yields two equations of the form of equation 3, with still two unknowns, K_{fs} and α^*. If each of these equations is to be solved independently, one of the two unknown must be evaluated in another manner. Elrick et al. (1992) suggest evaluating the parameter α^* on site, according to the soil texture. Recommended values of α^* are presented in Table 1.

Table 1: Recommended α* values for single head analysis (from Elrick et al. 1992)

Soil type	α^*
Coarse sands and highly structured soils (m^{-1})	36
Most structured soils and medium and fine sands (m^{-1})	12
Unstructured fine-textured soils (m^{-1})	4
Compacted clays (m^{-1})	1

3. NUMERICAL MODELLING

Many experiments were done using the Guelph permeameter apparatus and the different methods proposed, be it in the laboratory, in the field or by numerical methods (Reynolds et al, 1987, Mohanty et al. 1994). This particular project aimed at testing the different interpretation methods through the modeling of a Guelph Permeameter test. Modeling has many advantages when it comes to evaluating a field test method such as the Guelph permeameter test. The first advantage is that perfect saturation can be considered. Many authors who have obtained field K_{fs} values smaller than those obtained with other methods (Mohanty et al. 1994) have explained these differences by the incomplete saturation of the "saturated bulb". This incomplete saturation is mainly due to entrapped air in pore space, and leads to reduced values of K_{sat}. It is now widely accepted that the Guelph permeameter method does not yield the "true" K_{sat}, but rather a "satiated" or "field saturated" value of K. The second advantage one may find in evaluating such a method using numerical modeling is that time is eliminated as a factor. Where it may take hours or days to reach a stabilization of the infiltration flow in the field (fine grained soils for example), the numerical modeling software allows the operator to replicate a test lasting months in a computation requiring a few minutes. This allows the experimenter to test a variety of field conditions and soil types in a limited amount of time.

Numerical modeling does however have its limits and for that reason, the software chosen and its usage must be properly reasoned. The numerical modeling software we chose to use is SEEP/W (Geo-Slope, 2007). This numerical modeling software is one that completely solves Richards' equations (Richards. 1931). In doing so, it ensures that the phenomenon occurring in the unsaturated zone will be represented correctly. The evaluation of the numerical code's response in saturated and unsaturated conditions has been extensively tested and the many details of these tests can be found in Chapuis et al. (2001). The software was chosen for this particular project because its behaviour under unsaturated conditions has been judged adequate.

The axisymmetric model consisted of a volume of 2 meters high by 3 meters in radius. The borehole was located in the symmetry axis of the volume. The borehole consisted of a 5 cm radius hole bored to a depth of 50 cm. The initial water table was set at 50 cm from the bottom of the volume. Thus, the distance between the bottom of the borehole and the water table respected the so-called "deep water table conditions". The soil volume was subdivided in 19 regions (plus that corresponding to the borehole). Each region was given a nodal spacing that corresponded to the expected importance of the phenomenon in this region. For example, the regions in immediate contact with the borehole were given a nodal spacing of 1 cm in both horizontal and vertical directions, while regions located under the water table where given a more relaxed spacing. This allowed the model to be kept to a minimal nodal number, thus reducing the computation time while retaining the precision needed in regions where important changes were expected. The model contained 13 167 nodes. The model was run in both a steady state mode and in a transient mode. The steady state mode served to evaluate the steady state conditions of the problem and the steady state flow while the latter allowed to observe the evolution of the test over a period which might be more representative of the time spent in the field to perform a test. The transient model was run for a period of six hours and results were saved at 10 preset time values.

The use of the SEEP/W software in an unsaturated mode requires the user to input soil property data into the software. So that it became possible to observe the behaviour of the method with regard to soil type, six soils where chosen, ranging from silt to coarse sand. For each of these soils, a realistic water retention curve (WRC) was established, ensuring that the air-entry value (AEV), the water-entry value (WEV) and the residual water contents were representative of the soil type. The six WRC used are presented in Figure 3 below.

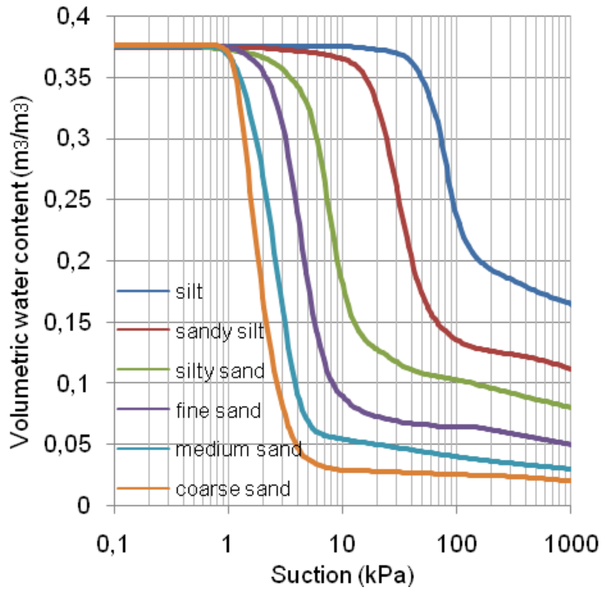

Figure 3: Water retention curves (WRC) of the six soils used for the modelling.

From these WRC, it became possible to estimate the hydraulic conductivity functions with regard to the pore water pressure (u_w). To do this, the Fredlund and Xing's method, already incorporated into the software was used. This enabled us to obtained full unsaturated properties curves for all soils. The calculation of the matric flux therefore became possible through the implementation of equation 4 and yielded the information necessary to perform our analyses. This information is summarized in Table 2. For each soil, the test was modeled using a hydraulic head (H) in the borehole of 5, 10 and 15 cm, which allowed observing the effect of this parameter which is usually left to the discretion of the operator.

Table 2: Basic properties of the tested soils as calculated from the soil property curves

Soil	K_{sat} (m/s)	ϕ_m (m^2/s)	$\alpha*$ (m^{-1})
Silt	5,79E-07	6.34e-7	0.91
Sandy silt	3,81E-06	3.87e-6	0.99
Silty sand	4,73E-05	2.7e-5	1.75
Fine sand	1,40E-04	4.92e-5	2.84
Medium sand	4,14E-04	8.72e-5	4.75
Coarse sand	7,81E-04	1.31-4	5.98

4. RESULTS AND DISCUSSION

The principal hypothesis contained in the resolution of borehole permeameter problems is that a steady infiltration rate has been attained when the flow measurement is made. The time required to reach this steady-state infiltration rate depends on the author but is usually about 15 to 30 minutes. Plotting the infiltration rate versus time in the transient numerical model has allowed us to notice that the infiltration rates decline rapidly but do not seem to stabilize during the 6 hours that represented the test duration. The steady-state model provided the true steady-state infiltration rate which can be compared to the infiltration rate measured at 1100 seconds, typical test duration. The differences are evidenced in Table 3:

Table 3: Percentage (%) difference between steady-state infiltration rate and infiltration rate measured at a time *t* of 1100 seconds.

Soil	H=5cm	H=10cm	H=15cm
Silt	6.91	8.61	10.05
Sandy silt	8.10	10.12	18.58
Silty sand	7.55	9.52	11.16
Fine sand	7.45	2.34	2.80
Medium sand	1.20	1.31	0.92
Coarse sand	0.34	0.09	0.00

The general trend observed from this table is that the veracity of stating that steady state infiltration rate is reached after several minutes (18 here) depends primarily on the soil type. In coarser soils infiltration rates after 1100 seconds were very close to the true steady-state infiltration rates, while in finer soils differences reached percentages up to 20%.

The Guelph Richards's single head analysis yielded consistently underestimated K_{fs} values, when the $\alpha*$ value was calculated based on equations 4 and 5, while the results obtained using $\alpha*$ values estimated from Table 1, showed either an overestimation or an underestimation of the K_{fs} values. These results are evidenced in Table 4.

Table 4: Percentage (%) error displayed between K_{fs} values estimated from the Guelph Richard's single head analysis and the actual K_{sat} values for H = 0.05m at t = 1100 seconds.

Soil	α^* calculated	α^* Estimated from table 1.
Silt	-82.38	38.59
Sandy silt	-81.83	33.08
Silty sand	-77.65	-3.71
Fine sand	-74.43	-27.88
Medium sand	-73.49	-20.68
Coarse sand	-72.00	-29.46

While the error associated with estimated α^* values was consistently smaller than the error associated to calculated α^* values, the variability in error associated with the calculated α^* values was less important. This indicates the need to verify the choice of the alpha value in the situation where the operator wishes to rely on Table 1. In this particular case, the full unsaturated curves of the tested soils were known to us, yet choosing α^* values proved to be difficult, primarily because the four soil types that are available to choose from are fairly general.

Consider the silty sand as an example; when choosing an alpha value from Table 1, the operator will have to classify the soil as falling in one of the two following categories: "structured soil, medium and fine sands" or "unstructured fine soils" having respective α^* values of 12 and 4. (The calculated value for this soil was 1.75). The choice of the correct α^* value greatly impacts the result of the analysis; the value of 12 yields K_{fs} = 4.55e-5m/s (a -3.71% error) while the value of 4 yields K_{fs} = 2.14 e-5 m/s (a -54.82 % error). This example emphasizes the impact the selected α^* value can have on the final K_{fs} estimate, considering that the selection can be hard to make in the field, on a production basis.

The influence of the head was less obvious in the Guelph Richards's single head analysis as shown in Table 3. The tested silt for example yielded an increasing error with increasing head, while the opposite was observed for soils like medium sand. Fine sand on the other hand, exhibited no relationship between error and applied head. From these results, it is impossible to conclude if applied head has an impact on the precision of the Guelph Richards's single head analysis.

The same cannot be said for the Glover's method. The applied head has an important impact on the results of the Glover's method. More important heads have consistently produced smaller values of K_{fs}, as evidenced in Figure 4.

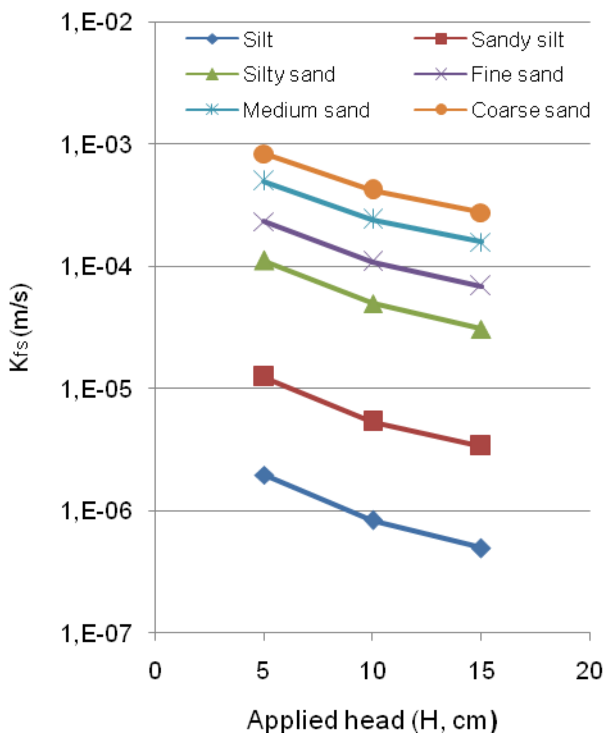

Figure 4: Values of K_{fs} estimated using the Glover's method for heads of 5, 10 and 15 cm.

The diminishing trend in the calculated value of K_{fs} does not necessarily tend towards the actual value of K_{sat}. In fact, the results were at times higher and at times lower that the actual values. The error observed ranged from overestimation by 240% to underestimation by 46%, depending on the soil and head applied.

The Glover's method does consider only flow due to water pressure, i.e. gravity flow through the base while capillary action of the soil is neglected (Elrick et al. 1992.). This leads to errors more important in situations where the head of water is small. Take for example the case where the head of water applied is of 5 cm, in a 5 cm radius well. The importance of the area where gravity flow acts becomes more important relative to the water pressure infiltration area than say a situation where the applied head would be greater. If only the results obtained when H was either 10 or 15 cm are considered, we observe that the Glover analysis produces values which are within -46% to 70% of error with respect to the actual K_{sat} values. This error is smaller than that obtained using the Guelph Richard's single head analysis, allegedly a more accurate method.

The K_{fs} values computed using the Glover's method are presented in Table 5. Computing the K_{fs} values using the steady state infiltration rates proved to minimally reduce the error in the calculated values.

Table 5: Computed values of K_{fs} (m/s) using the Guelph Richards's single head analysis at t = 1100 seconds.

Soil	H=5cm	H=10 cm	H=15cm
Silt	1.02e-7	8.86e-8	8.52e-8
Sandy silt	6.92e-7	6.26e-7	6.20e-7
Silty sand	1.06e-5	9.65e-6	9.07e-6
Fine sand	3.39e-5	3.1e-5	2.90e-5
Medium sand	1.10e-4	9.86e-5	9.24e-5
Coarse sand	2.19e-4	1.97e-4	1.82e-4

Finally, the Guelph Richards's simultaneous equations approach produced negative unusable results for K_{fs}. Many authors have reported negative values of K_{fs} when using this method but had attributed these occurrences to the presence of roots, soil fractures, heterogeneities of macropores, (Elrick et al., 1989). All these elements were absent from our numerical model, yet the method yielded negative values, as presented in Table 6.

Table 6: Computed values of K_{fs} (m/s) using the Guelph Richards's simultaneous equations at t = 1100 s

Soil	H=5 et 10cm	H=10 et 15 cm	H=5 et 15cm
Silt	-6.83e-7	-4.84e-7	-2.93e-7
Sandy silt	-4.18e-6	-2.63e-6	-1.14e-6
Silty sand	-2.44e-5	4.82e-5	-1.61e-5
Fine sand	-4.12e-5	-5.82e-5	-1.61e-5
Medium sand	-6.05e-4	7.90e-5	-1.32e-5
Coarse sand	-2.54e-3	-1.61e-3	-5.19e-4

5. CONCLUSION

The modelling of a Guelph permeameter test was executed using SEEP/W software and the full unsaturated property functions of six soils. The results of the tests were analysed using Glover's equations, the Guelph Richards single head analysis and the Guelph Richard's simultaneous equations analysis. The results were compared to the known values of the hydraulic conductivity of each soil. The accuracy of the Glover analysis was between -46% to 240%, the accuracy of the Guelph permeameter single head analysis was between -82% to 39% depending on whether the α^* parameter was calculated from soil property curves or estimated using the soil texture, and the Guelph permeameter simultaneous equations approach yielded negative values of K_{fs}.

From these results, we can see that the Glover analysis and the Guelph permeameter analysis yielded values of K_{fs} with important errors. These types of errors had already been reported in literature, but were attributed to improper saturation of the soil surrounding the well, due to entrapped air during the saturation process. In the case of the Guelph permeameter simultaneous equations approach, the occurrence of negative values had been reported in literature, but attributed to the presence of soil heterogeneities in the zone between the two applied heads. We must realize that there is some other source of error in these analyses, because the software used does not simulate entrapped air (saturation is therefore perfect), and soils heterogeneities were absent from the model, yet important errors in the final estimations remained. This leads to think that some of the assumptions made in the development of these methods are inaccurate and lead to these errors.

The Guelph permeameter method remains an acceptable means to obtain economically and rapidly an estimate of the soil's hydraulic conductivity. The results can give an idea of the order of magnitude of this property to the operator, which for many preliminary studies might be sufficient. On the other hand, for projects where accurate determination of field hydraulic conductivity is necessary such as when public health, safety and the environment are at stake, the professional engineer should be aware of the limitations of the methods and equipments at his disposal and make decisions accordingly.

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