Numerical Modelling Issues Associated with the 2D and 3D Analysis of Tunnel Excavation and Support within Weak Rock Masses

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ABSTRACT
In spite of the gradual development of 3D analysis packages utilizing finite element models or finite difference algorithms for stress-strain calculations, 2D analysis is still used as the primary tool for tunnel behaviour and tunnel support analysis and design. 2D finite element analysis or analytical convergence confinement solutions, for example, depend on independence between the ground reaction curve and the support resistance. In addition, the longitudinal displacement profile, prior to support is assumed to be independent of the support effect. Also it is assumed that non-isotropic stresses and non-circular geometries can be handled in the same way as circular tunnels in isotropic conditions. The process involves generating a ground reaction curve (internal tunnel wall resistance versus tunnel closure) and calibrating this using a standardized longitudinal displacement profile (LDP). This paper examines the validity of these assumptions associated with weak rock masses and the error inherent in these extensions to 2D tunnel analysis. Anisotropic stresses and lagged (staged) excavation present a particular problem. Solutions are proposed for support LDP's in simplified conditions.

RÉSUMÉ
Malgré de le développement des programmes d'analyse 3D qui utiliser les modèles de « finite element » ou « finite difference » pour les calculs de stress, pour la plupart, la 2D analyse est encore employée comme l'outil de choix pour l'analyse du comportement de tunnel et du tunnel soutiennent pour l'excavation d'un tunnel. L'analyse en deux dimensions par les finité éléments ou les solutions analytiques de confinement de convergence, par l'exemple, sont prédiquer sur la notion de l'indépendance entre la courbe de réaction de terre et la résistance de soutien. De plus, on assume que le profil longitudinal de déplacement, avant l'installation de soutien, est indépendant de l'effet de résistance du soutien. En outre, on suppose que des stresses non-isotropes et les géométries non-circulaires peuvent être manipulés comme les tunnels circulaires en conditions isotropes. Le processus inclus la fabrication d'une courbe de réaction de terre (résistance interne de mur de tunnel contre la fermeture de tunnel) et de calibrer ceci avec un profil de déplacement longitudinal. Cet article examine la validité de ces prétentions et de l'erreur inhérente à ces prolongements à 2D analyses de tunnel. Stresses anisotropes et aussi l'excavation par étapes présents des problèmes particulier. On propose des solutions pour le profil de déplacement longitudinal de soutien en conditions simplifiée.

1 INTRODUCTION

Tunnelling is an inherently three-dimensional process. The advancing tunnel face creates a complex three-dimensional stress path as explored by Eberhardt (2001) and also generates a three dimensional bullet-shaped zone of plasticity in soft rock. This developing plasticity or yielding zone, combined with the elastic closure of the surrounding rock mass creates a wall displacement profile that is non-linear, develops partially before the advancing face and continues for a number of tunnel diameters before equilibrium conditions are achieved. This profile, known as the longitudinal displacement profile (LDP) is a function of tunnel radius and the extent of the ultimate plastic radius. This relationship is explored in detail by Vlachopoulos and Diederichs (2009) for the axisymmetric condition (tunnel geometry and stress). This paper examines issues related to two dimensional modelling, both as plane strain and as axisymmetric configurations and the relationships to true three-dimensional effects. In order to accurately simulate the loading of support or the effects of sequential excavation, the two dimensional model must capture the pre-face conditions, the state of displacement and plasticity at the face and the subsequent development of deformation and yielding as shown in Figure 1.

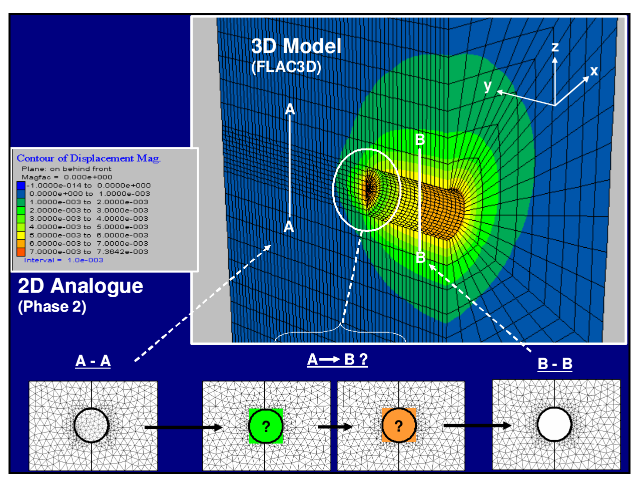


Figure 1. Highlights the requirement to develop 2D analogues to a fundamentally 3D phenomenon. Must take into consideration the strength issues provided by the faces that are associated with successive excavation in a 2D sense. It is important to examine these issues as two dimensional modelling is still very much state of practice for tunnel engineering analysis (Hoek et al., 2008).

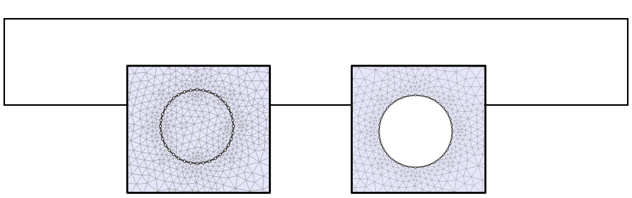
The basic premise of 2D tunnel modelling is that the tunnel boundary moves (normally inwards) progressively as the tunnel face passes the model section. Ultimately a stable tunnel closure is reached (for elasto-plastic analysis without strain softening and without ground surface interaction). This inward displacement of the tunnel boundary can be simulated by replacing the “rock” inside the tunnel with an outward pressure (initially equivalent to the in situ pressure) and reducing this internal pressure to zero over a number of model steps.

2 MODELLING APPROACHES

For the purposes of this investigation, Phase2 (Rocscience Inc., 2004) was used for the 2D numerical analysis and FLAC3D (Itasca, 2005) was used for the 3D numerical analysis. Phase2 utilizes the implicit Finite Element Method (FEM) while FLAC3D employs the Finite Difference Method (FDM) in its determinations. Phase 2 models utilized both plane strain and 2D axis-symmetric modelling techniques. Both of these numerical modelling programs are widely used in the rock mechanics community for design purposes as well as to capture the behaviour of a tunnel (i.e. stress re-distributions and displacements) associated with tunnel excavation.

As the effect of an excavation in a rock mass is clearly a 3D phenomenon, the ensuing deformations cannot be simulated directly in 2D finite element plane strain analysis. 2D, axi-symmetric modelling does replicate 3D effects for very simple cases (circular geometry and isotropic material and stress). In 2D plane strain, the progressive displacement of the tunnel boundary must be recreated in accordance with the appropriate linear displacement profile. If done correctly, this will capture the progressive development of loads and displacements in tunnel geometries and in support elements that respond in the radial plane (liners and bolts for example, but no forepoles and face support). The LDP is recreated implicitly in 2D plane strain. The methodologies commonly employed in current design practice for 2D modelling to mimic real 3D effects are (Figure 2):

- Straight Excavation;
- Field Stress Vector / Average Pressure Reduction;
- Excavation of Concentric Rings; and,
- Face De-stressing (with or without softening).



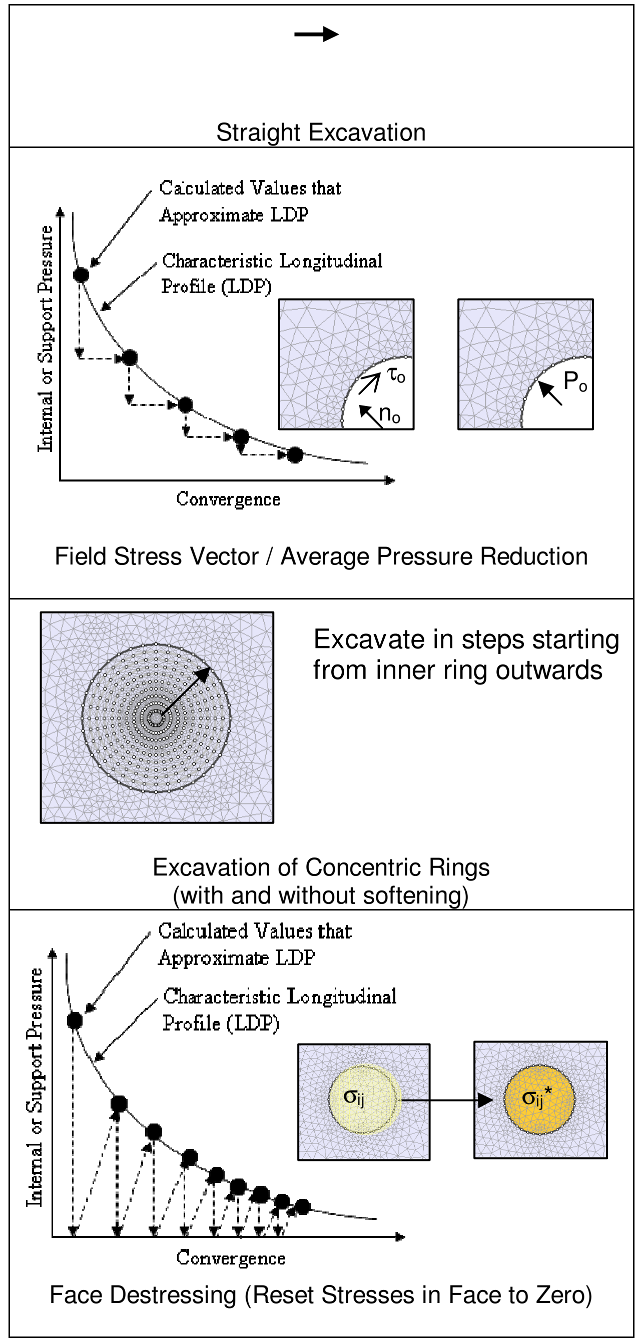


Figure 2. Methods or advanced strategies used in 2D Numerical Analysis in order to approximate the uniquely 3D behaviour associated with rock tunnel excavation.

2.1 Average Pressure Reduction (Convergence-Confinement Method)

Convergence-confinement analysis or the stress relief method (Panet 1995, 1993; Carranza-Torres and Fairhurst 2000; Duncan-Fama 1993 and others) is a widely used tool for preliminary assessment of squeezing potential and support requirements for circular tunnels in a variety of stress states and geological conditions. An internal pressure (p_o), initially equal to the in-situ stress is applied on the inside of the excavation boundary. The pressure is incrementally relaxed until the excavation boundary condition is effectively zero normal stress. The extent of plastic yielding and thereby, the boundary deformation is calculated at each stage of the process. The result is a continuous representation of the deformation-internal pressure relationship for the tunnel given a particular material strength, deformability, dilation and stress state. The internal pressure is, not a direct representation of real effects, however, it is a substitute for the effect of the gradual reduction of the resistance due to the effect of the distancing supporting tunnel face. The internal pressure that is coupled with a given boundary displacement is a measure of the amount of support resistance required to prevent further displacement at that point in the progressive tunnelling model. The stress is applied normal to the inner boundary and idealizes the progressive stress state. Also referred to as the load step method, as there is an incremental reduction of tunnel boundary tractions that simulate advance.

2.2 Field Stress Vector

In cases where the initial stresses are not isotropic, the boundary pressure in the convergence-confinement technique must be replaced with a traction vector with shear (τ_o) and normal (n_o) components is applied to each tunnel boundary element to replace the in-situ stress acting on the element plane pre-tunnel. In this technique, in terms of the Ground Reaction Curve (GRC) (i.e. convergence versus internal support pressure), there is an incremental reduction (dashed line) of tunnel boundary tractions that simulate progressive tunnel excavation advance. This technique has been recently incorporated into Phase2 and will be used here.

2.3 Concentric Disks of Excavation Method

This is an outdated method that excavates the tunnel cavity in stages concentrically from the centre of the tunnel to the outer boundaries of the desired tunnel diameter (or shape). This can be seen in Figure 2 for a circular tunnel. Each excavation disk that is nulled in this system of excavations represents a different stage of tunnel advancement. In a 3D sense, the excavation of the central disk represents a weakening of the material ahead of the excavated face while the final ring that is excavated represents the open cavity and passing of the face past that location. This method can also be combined with softening or distressing of the material whereby on would reduce the Modulus of Elasticity (E_i) of the core material from its original value, E. This method is still used in practice but will not be discussed further here.

2.4 Face Replacement or Destressing

Plane strain simulation of tunnel advance in this method involves the replacement of the tunnel core with unstressed, elastic material during each step. The tunnel boundary is allowed to converge during the subsequent model step until the stresses re-establish in the tunnel core and a temporary equilibrium is reached. The face is then replaced again and the process is repeated. In this way the tunnel works its way down the pressure-displacement (ground-reaction) curve in a series of steps. This method was favoured in the past as the stress-vector technique was difficult to incorporate manually into a model. These two methods will be compared here.
The face replacement method can be made more efficient by progressive softening of each successive core replacement. Softening the face on its own will not create a response as the model functions on the basis of stress equilibrium (resetting the stiffness does not create a force imbalance in the model and therefore no direct response). Softening combined with face replacement (or distressing) results in an efficient excavation sequence simulation.

3 APPLYING THE LONGITUDINAL DISPLACEMENT PROFILE

The Longitudinal Displacement Profile (LDP) is one of the three basic components of the convergence-confinement method. A characteristic LDP diagram indicates that there is an amount of axial displacement at some distance ahead of the face (i.e. a zone of influence prior to excavation of the core beyond the face) and at certain distance behind the face that the amount of displacement approaches a constant value (Carranza-Torres and Fairhurst, 2000). As shown by Vlachopoulos and Diederichs (2009) the normalized LDP (d/d_{max} vs X/R_t) is a function of the ultimate plastic radius.

The first step in the analysis process is to determine the maximum plastic radius via a simple plane strain analysis of the unsupported tunnel or through an analytical solution such as that given by Carranza-Torres and Fairhurst (2000). Next, the longitudinal deformation profile can be calculated using the methodology of Vlachopoulos and Diederichs (2009). Alternatively, an axisymmetric model can be used for this purpose, facilitated by the assumed isotropic stresses and circular profile. A longitudinal deformation profile for an unsupported tunnel is developed as shown by the solid line ("Disp. vs Location") in Figure 3.

A 2D finite element plane strain analysis was then applied to the full face construction sequence (unsupported). The technique of progressive face replacement (distressing) described in the previous section was applied in this case. At the end of each stage in the 2D model, the tunnel wall will have moved a certain distance.

4 BOUNDARY CONDITIONS AND 2D METHOD COMPARISON

The comparisons that follow in the rest of this paper were conducted using supported and unsupported simulations

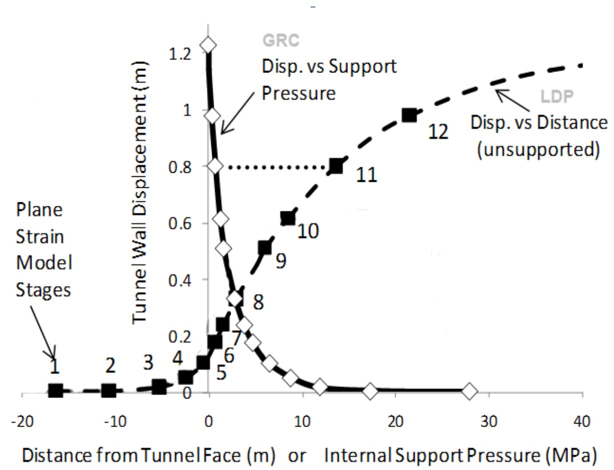


Figure 3. Ground reaction curve, “Disp. vs Support Pressure” and corresponding longitudinal displacement profile “Disp vs Distance (unsupported)”. Normalized plastic radius R_p/R_t = 8 in this example. Point symbols and number ID’s represent corresponding stages in plane strain model (related symbols are linked horizontally between two curves as shown for stage 11 by dotted line).

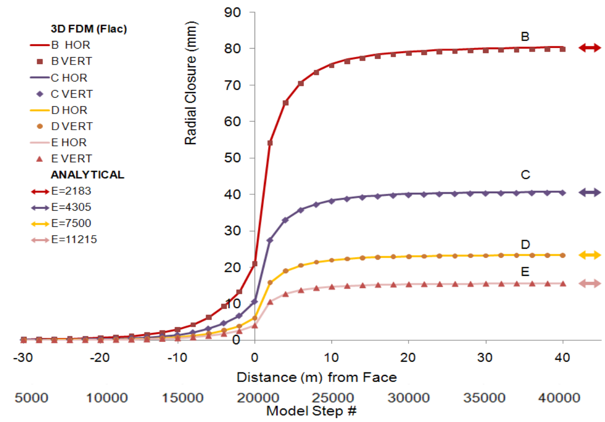


Figure 4. Elastic LDP results calculated using FLAC3D circular tunnel models (isotropic stresses and elastic properties as per Table 1. Comparison with 2D analytical solution for final displacement is shown.

with elastic and elastic-perfectly plastic models (Mohr-Coulomb constitutive model within FLAC3D and Phase2). The materials and input parameters were selected in order to span the spectrum of the ratio of rock mass strength to in situ stress and strain considerations. The suite is similar to that used in Vlachopoulos and Diederichs 2009. The parameters or properties associated with each material B1, C1, D1 and E1 are located in Table 1. As can be seen, materials B1, C1, D1 and E1 have a p_o/UCS_{RM} (in-situ pressure to rock mass uniaxial compressive strength) ratios of 8, 6, 4 and 2 respectively. Mohr-Coulomb equivalent properties and rock mass strengths were estimated as per Hoek et al. (2002) and the elastic moduli were estimated based on Hoek and Diederichs (2007).

4.1 Boundary Conditions

It is important to establish the influence of 2D boundary conditions to ensure that valid comparisons can be made. Two options are explored here - fixed displacement (=0) outer boundary conditions some distance away from the excavation and free boundary conditions with an in-situ boundary traction applied. The results of the FLAC3D analysis are also compared here with the analytical solution for displacements into a circular tunnel (as per Brady and Brown 1993 for example). The comparison is shown in Figure 4 illustrating that the boundary conditions and mesh accuracy is acceptable for the 3D models.

Figure 5 illustrates the comparison between the FLAC3D LDP’s, normalized with respect to the respective analytical solution for maximum elastic closure, with the 2D solutions based on fixed boundary conditions 6 12 and 32 radii from the tunnel and with free boundary conditions 12 radii from the tunnel. The latter is exact (coincident with the analytical solution as is the FLAC3D

Table 1. Parameters used for 2D and 3D model comparisons

Parameter \ Material	B1	C1	D1	E1
p_o/σ_{crm}	8	6	4	2
σ_{ci} (MPa)	35	35	50	75
m_i	7	7	7	7
ν	0.25	0.25	0.25	0.25
γ(MN/m^3)	0.026	0.026	0.026	0.026
E_i	19212	19249	27630	21567
p_o (MPa)	28	28	28	28
GSI	35	45	48	60
m*	0.687	0.982	1.093	1.678
s*	0.0007	0.0022	0.0031	0.0117
a*	0.516	0.508	0.507	0.503
E_{rm} (MPa)	2183	4305	7500	11215
σ_{crm} (MPa)*	3.5	4.7	7	14
c (MPa)*	1.100	1.753	2.145	3.259
φ	21.50	23.71	27.05	33.40

*Values obtained using Rocklab software based on Hoek et al. 2002.

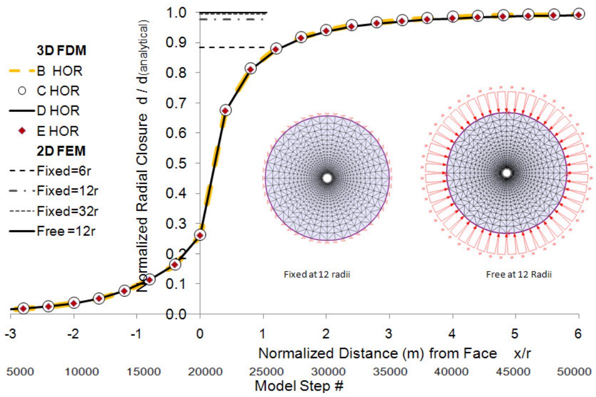


Figure 5. Elastic LDP results calculated using FLAC3D circular tunnel models (isotropic stresses and elastic properties as per Table 1. Comparison with 2D analytical solution for final displacement is shown.

results. Since the kinematic control of a free boundary is more difficult when the tunnel is not circular, fixed 2D boundaries at 16 to 20R are used for the rest of this work.

Finally, it is necessary to compare the normalized LDP's from the FLAC3D analyses with the Axisymmetric models used in this chapter and compare both to accepted analytical formulations for the longitudinal displacement profile. It can be seen from Figure 5 that the normalized LDP's from the FLAC analysis are independent of elastic modulus. Figure 6 shows that this normalized profile is coincident with the analytical formulation by Unlu and Gercek (2003). The axisymmetric analysis with a fixed boundary at 30R shows good correlation.

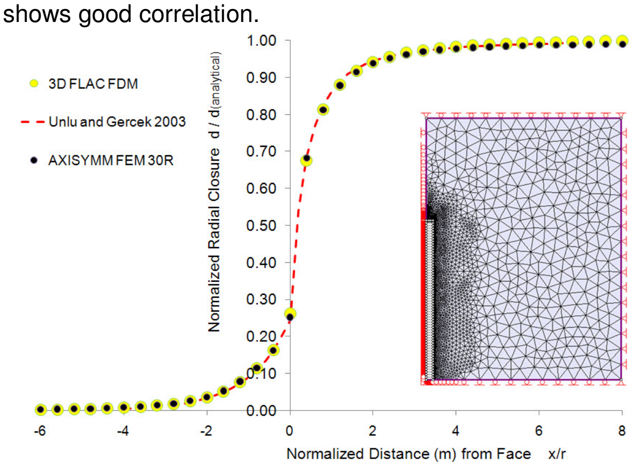


Figure 6. Elastic LDP results calculated using FLAC3D circular tunnel models (isotropic stresses and elastic properties as per Table 1.) Comparison with 2D axisymmetric solution and analytical result.

4.2 3D and Axisymmetrical LDP's

The plastic LDP's for the FLAC 3D models are now compared with the equivalent axisymmetric 2D results in Figure 7 illustrating that they are acceptably coincident. The semi-analytical LDP function proposed by Vlachopoulos and Diederichs 2009 was based on axisymmetric modelling. In Figure 8, the FLAC 3D results are compared with this function. The developing plastic radii from the FLAC3D models are also shown in this figure. The LDP functions are based on the final value of R_p/R_t.

4.3 Comparison of 2D Plane Strain Methodologies

An idealized 2D model with a circular tunnel, 6-noded triangular elements arranged in an expanding radial grid with fixed boundaries at 32R from the tunnel, and isotropic stress conditions is initially used to compare the Ground Reaction Curves generated using the stress-vector (pressure) method and the face-replacement (modulus) method described in Section 4. In this comparison, 20 steps are used to regenerate the GRC. In order to provide similar load/displacement steps, the "modulus" method is executed first and the internal pressure increments from this analysis are used as input

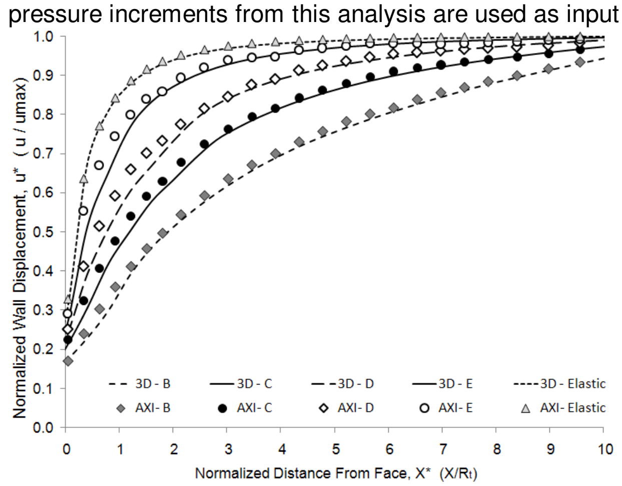


Figure 7. Plastic LDP results calculated using FLAC3D circular tunnel models Comparison with 2D axisymmetric solution.

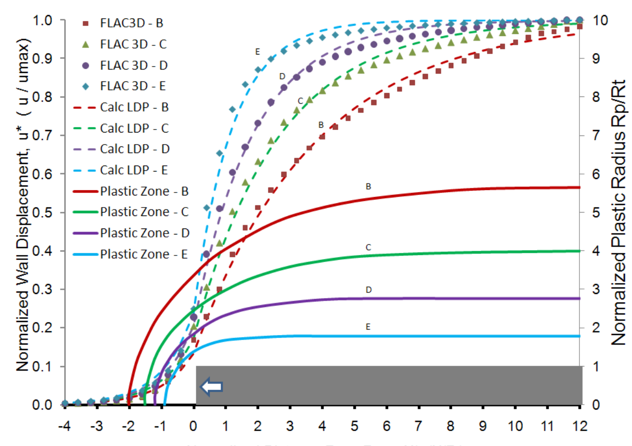


Figure 8. Plastic LDP results calculated using FLAC3D circular tunnel models) Comparison with 2D analytical

(calc) solution from Vlachopoulos and Diederichs 2009). Development of plastic radii in FLAC3D models is shown.

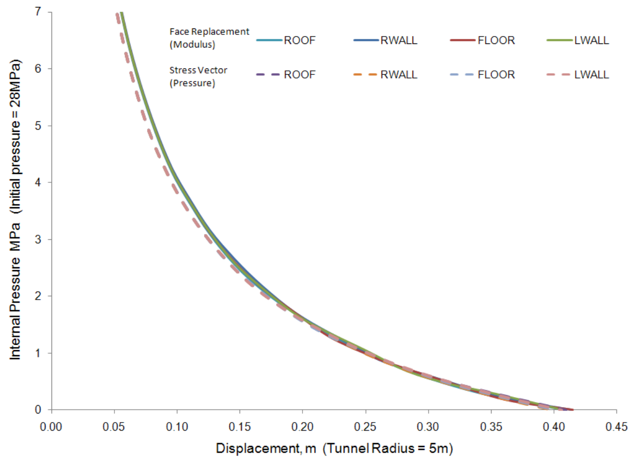


Figure 9. Comparison of Ground Reaction Curve for idealized circular tunnel plane strain analysis.

into the “pressure” method. The “internal normal pressure” is queried at the tunnel boundary after each stage. The displacements are given directly. Figure 9 shows that the process is not sensitive to the method used.

Next, the same comparison is made using a more practical grid (randomly generated - 3 noded, boundaries at 32R). for both the circular and horseshoe geometries. Results in Figures 10 shows that the “pressure” method is less sensitive to element type (more deviation between roof, floor, wall) in the circular case, and both are subject to deviations caused by non-ideal geometry (in the case of the horseshoe). It is important to keep this level of inherent error in mid when evaluating the effects of support, stress ratio, sequencing, etc. The average convergence-confinement (GRC) results are compared for two shapes and two methods in Figure 11.

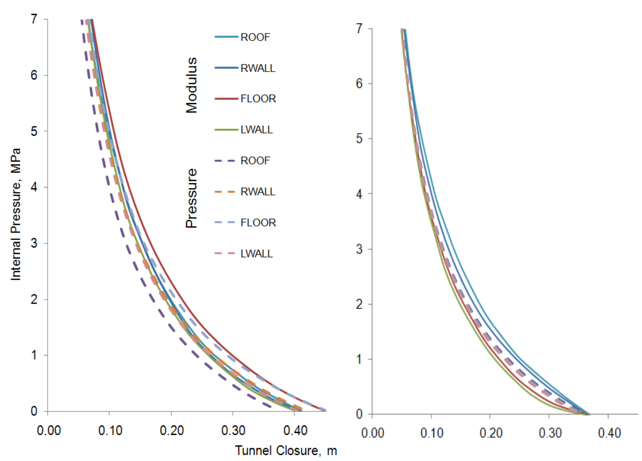


Figure 10. Comparison of GRC’s generated using 2D plain strain analysis, Left: horseshoe tunnel. Right: circular tunnel. Isotropic stress field = 28MPa, material C from Table 1. “Modulus” refers to the face replacement method while “Pressure” refers to the stress vector method.

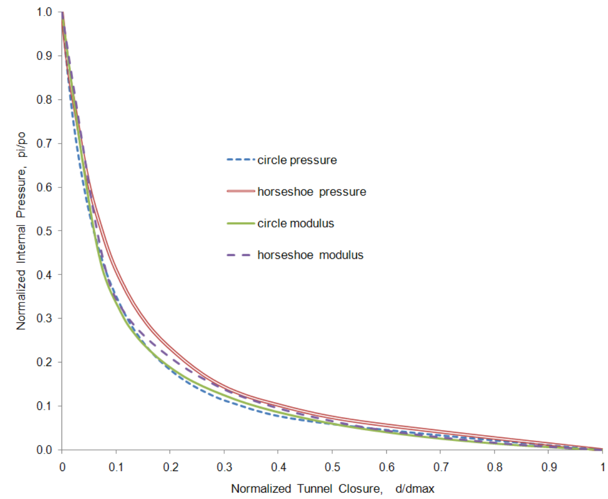


Figure 11. Comparison of average GRC’s generated using 2D plain strain analysis using two methods: “Modulus” refers to the face replacement method while “Pressure” refers to the stress vector method.

5 LIMITATIONS OF THE 2D, LDP BASED SIMULATION OF 3D TUNNELLING

This section will summarize a series of investigations to determine the limitations of 2D FEM modelling to simulate 3D tunnel advance using the LDP approach outlined in Section 4.

5.1 Excavation Shape

Figure 12 compares the plastic zone development and the associated LDP’s for the circle and horseshoe tunnels under hydrostatic stress. Two tunnel strength/stress ratios are used here. This comparison, combined with Figure 11 demonstrates that within the limits of error inherent in the FEM analysis, the LDP-Plane Strain analysis procedure outlined in Section 4 is valid for non-circular shapes, even if the LDP is based on the correlated LDP functions of Vlachopoulos and Diederichs (2009) for circular tunnels. The validity of this approach is likely reduced as the aspect ratio of the non-circular opening increases.

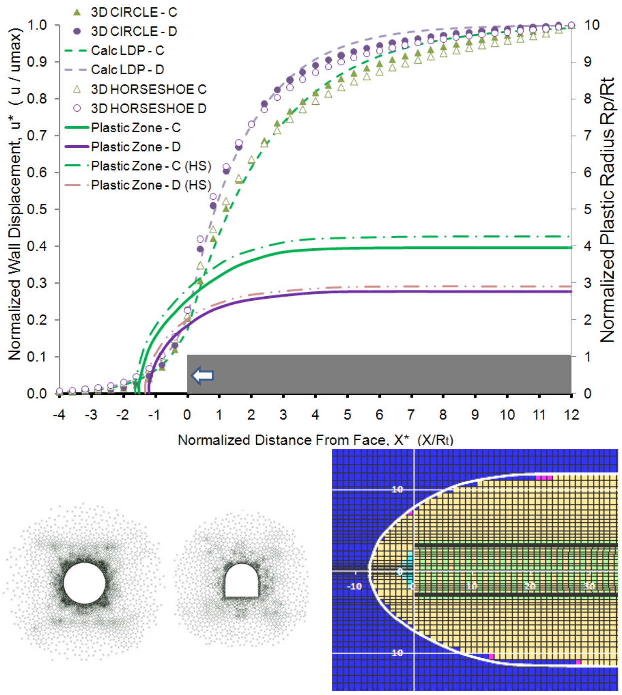


Figure 12. Comparison of LDP's for circular and horseshoe tunnels. Bottom: Plastic zones are shown for the two tunnel shapes and material C. Plastic zone for FLAC 3D (circle) is shown in long section.

5.2 In-Situ Stress Ratio

The LDP procedures developed for 2D modelling are based on an isotropic stress field (k=1). A brief examination is performed here to determine whether this is a significant limitation for the approach. Figure 13 represents results for material C under a horizontal stress ratio of 1.5 (28 MPa vertical stress). The normalized deformation profiles for the walls for the roof / floor are different. The stress ratio axial to the tunnel has a minimal impact. Using the LDP function derived for isotropic stress (Vlachopoulos and Diederichs 2009) does not work for either horizontal or vertical plastic radius. However, the LDP derived for the case k=1 does seem to follow the deformation profile for the vertical direction (direction of maximum yield).

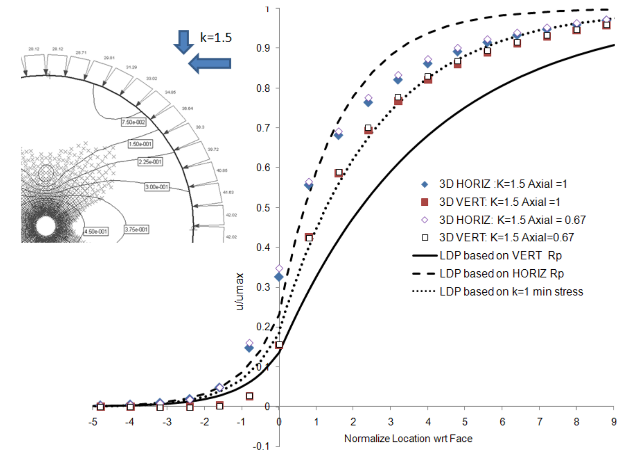


Figure 13. Comparison of 3D LDP's with derived functions based on R_p. k=1.5

A same comparison was conducted with k=0.67 (same vertical stress). Here again it was shown that the LDP's are different for different directions even though each LDP is normalized to itself. The LDP for the case of k=1 best approximates the deformation profile for the horizontal direction (maximum yield). This is consistent with the previous example.

5.3 Sequential Excavation

One assumption that is generally accepted in practice is that once a 2D sequenced model is calibrated based on the LDP for a single excavation phase, each subsequent stage can use the same sequence of face replacement or pressure reduction to simulate the 3D advance (of a bench after a top heading for example). The convergence confinement approach (analytical or in 2D plane strain) assumes that the excavation stages are independent. Through a series of stepped excavation stages, it was shown that this may not truly be the case for tunnel excavation as the approaching bench softens the ongoing response of the initial top heading excavation (as the tunnel face moves on).

6 IMPLICATIONS

The primary purpose of convergence-confinement analysis using calibration via LDP's is to properly locate support installation. In squeezing ground, early installation of a liner can result in overloading of the liner and failure. Late installation will incur excess ground displacement and ground disintegration. It is therefore critical to properly "locate" the point of support installation within a staged 2D excavation model.

In conventional convergence-confinement analysis, the unsupported LDP is used to correlate the ground reaction curve (displacement vs internal pressure) or 2D analysis stages (face replacement or pressure reduction stages) with location along the tunnel. In the simple example in Figure 14, a 30cm concrete liner with 160mm steel sets

@ 1m spacing provide an estimated 15MN/m of hoop thrust capacity in a circular liner (Hoek and Brown 1980). The GRC analysis (as per Carranza-Torres and Fairhurst 2000) combined with the LDP of Vlachopoulos and Diederichs (2009), show that this liner, installed at 2m in a 5m radius tunnel with properties of material C, will have a factor of safety of 1.

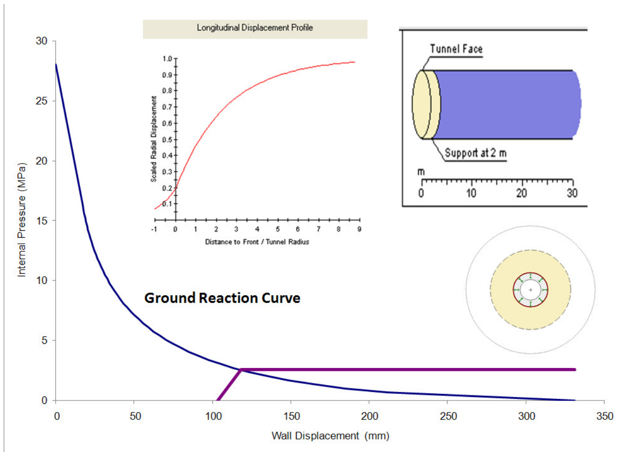


Figure 14. Ground reaction analysis of a lined tunnel (liner at 2m).

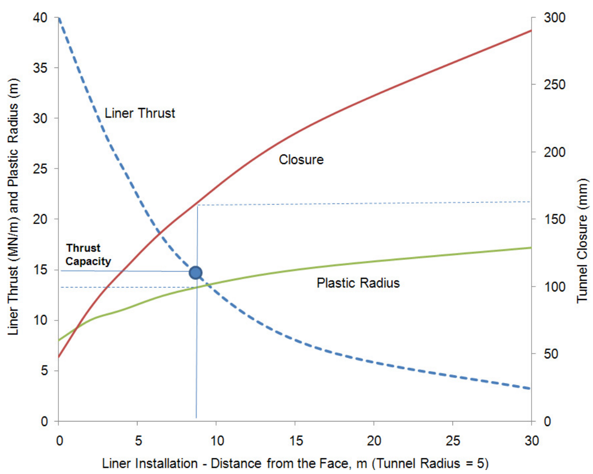


Figure 15. Summary of axisymmetric analyses of tunnel liner installed at different distances from the face.

Consider, however, the axisymmetric analysis results presented in Figure 15. In this analysis, the ground reaction prior to support is not independent of the support as in the previous analysis. A series of 8 analyses are summarized in this plot showing the final liner load versus the installation distance to the face. In addition, the ultimate plastic zone and tunnel wall displacement are also affected by installation distance. This analysis shows that the minimum distance between face and liner should be 8m and that the liner analyzed in the previous example (installed at 2m) would fail.

7 CONCLUSIONS

The conventional approach of 2D tunnel analysis, calibrating excavation stages with an LDP derived from simple 3D calculations based on an unsupported circular tunnel in isotropic stresses, has been examined in detail in this paper with the following conclusions:

- Boundary conditions are important to analysis of squeezing ground problems. Fixed boundaries should be a minimum of 10 radii from the tunnel or 3 plastic radii away from the plastic zone.
- For simple tunnel geometries, the 2D LDP and GRC is not sensitive to the choice of face replacement or pressure reduction technique but is sensitive to the step size (face too soft or pressure increment too great).
- Tunnel shape is not an important factor provided the aspect ratio of tunnel geometry is not extreme.
- Non-isotropic stresses render the standard LDP approach inaccurate. For mild values of stress ratio, k, some assumptions and adjustments can be made to make the approach practically viable.
- Sequenced excavation such as top heading and bench excavation poses a problem for the LDP approach unless the second excavation stage is distant from the first.
- A new LDP is required for stiff liners installed within 2 to 6 radii of the face. For installations closer than 2 radii, 3D analysis may be required.
- It is critical to correctly locate the installation step within a staged 2D modelling sequence to prevent overloading or excess deformations.

8 ACKNOWLEDGEMENTS

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