



## 3D modeling laterally loaded battered piles in sand

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### ABSTRACT

A series of three-dimensional finite difference analyses have been carried out to investigate the lateral capacity of battered piles under lateral loads. Analyses were performed in homogenous sandy soils. Numerical results show that the lateral capacity of the battered piles is influenced by the value and the sign of the pile batter angle as well as the sand density. For negative batter angles, when lateral load acts opposite to the direction of pile inclination, the lateral capacity of the battered piles increases substantially with batter angle and with soil density. However, for positive batter angles, when the lateral load acts in the direction of the pile inclination, the lateral capacity is slightly decreased to moderately increased all dependent to the value of batter angle and the sand density.

### RÉSUMÉ

Une série d'analyses tridimensionnelle en différences finies a été réalisée pour étudier la capacité latérale de pieux inclinés soumis à des charges latérales. Les analyses ont été effectuées pour des sols sableux homogènes. Les résultats numériques montrent que la capacité latérale des pieux inclinés est influencée par la valeur et le signe de l'angle d'inclinaison du pieu ainsi que de la densité du sable. Pour des angles d'inclinaison négatifs, où la direction de la charge latérale est opposée à celle de l'inclinaison du pieu, la capacité latérale des pieux inclinés augmente considérablement avec l'angle d'inclinaison et avec la densité du sable. Cependant, pour des angles d'inclinaison positifs, où la direction de la charge latérale est la même que celle de l'inclinaison du pieu, la capacité latérale est légèrement diminuée à modérément augmentée tout dépendant de la valeur de l'angle d'inclinaison et de la densité du sable.

## 1 INTRODUCTION

When a soft or loose soil extends to a considerable depth, piles are generally used to transmit vertical and lateral loads to the surrounding soil. Piles are used as foundations for high rise buildings, high retaining walls, offshore structures, etc. and are normally subjected to high lateral loads. In these situations, piles should control both vertical and lateral movements. Vertical piles are used in foundations to carry vertical loads and small lateral loads. When the horizontal load per pile exceeds the value suitable for vertical piles, battered piles are used in combination or not with vertical piles.

The use of battered piles along with vertical piles in the pile-soil system increases the overall efficiency. Depending on their direction of inclination with lateral loads, piles are called "Pile battered reverse", if the lateral load acts opposite to the direction of pile inclination (negative batter angle), and "Pile battered forward", if the lateral load acts in the direction of the pile inclination (positive batter angle) (Zhang et al 2002).

The work on batter piles is very little as compared to vertical piles. During the last few decades, several researchers have studied the behavior of batter piles using both laboratory tests and theoretical studies. Model tests were performed on piles to determine the effect of the batter on pile load capacity (Murthy 1965; Meyerhof and Ranjan 1973; Awad and Ayoub 1976; Hanna and Afram 1986; Veeresh 1996; Zhang et al. 1999). The effect of batter on deflections has been investigated by Kubo (1965) and Awad and Petrasovits (1968) from tests on model piles in sand.

Laboratory tests were carried out on single and group batter piles. Using instrumented model battered pile installed in sand, Murthy (1965) developed relationships between vertical and battered piles. The batter angles varied within 0 to  $\pm 45$  deg range. These relationships offered the same results as given by typical experiments performed by Alizadeh and Davisson (1970). Ranjan et al. (1980) inferred from observations that "Pile battered reverse" offer more resistance than "Pile battered forward". However, Lu (1981) conducted experiments on laterally loaded piles and observed a satisfactory criterion for determining design pile loads. They inferred observations because the lateral capacity of pile is zero for a "Pile battered forward" and maximum for a "Pile battered reverse", indicating that the upper layer soil support in a negative batter is enormous. Hanna and Afram (1986) present an experimental investigation on the ultimate shaft resistance of battered piles. The model piles were pushed in medium dense sand deposits at different inclination up to 30 deg with respect to the vertical and tested under axial compression loads. From the experimental results, it was found that the total shaft resistance decreases with increasing the pile inclination. They related the reason of this reduction to the reduction of the average mobilized angle of friction between the pile shank and sand taking into account the vertical earth pressure distribution. Based on the centrifuge test results and data reported in the literature, Zhang et al. (1999) reported that the lateral capacity increases over plumb piles were 4, 14, 24, and up to 50 % in very loose, loose, medium-dense, and dense sands, respectively, at negative angle batter ( $-14^\circ$ ). In contrast, the lateral capacity decreases over plumb piles were 4, 5, 15, and up

to 35 %, respectively, at positive angle batter ( $14^\circ$ ). Zhang et al. (1999) concluded that the effects of pile batter were significant in medium dense and dense sands, but minor in very loose and loose sands.

Larger contrasts exist between the results obtained throughout these experimental studies of lateral response of battered piles under lateral loads. In fact, the value of batter angle as well as the direction of piles inclination are the underlying factors that control the battered piles behavior.

In other hand, little research work has been reported on numerical studies of battered piles. For example, Mroueh and Shahrour (2009) used a 3D finite element analysis to study the response of battered piles subjected to inclined pullout loads. It is shown that the pullout capacity of battered piles is affected by the pile's inclination and the load's inclination.

In view of the above stated issues, this paper describes and discusses the results of a series of 3D finite difference FD analyses using the commercial program *FLAC<sup>3D</sup>* (Itasca 2009) carried out to investigate the effects of pile batter on the lateral capacity as well as internal forces of battered piles installed in sandy soils. Initial analyses were based on the validation of the proposed numerical model. Further analyses were then carried out to investigate the influence of several parameters (the batter angle and the soil density) on the lateral response of battered piles.

## 2 FINITE DIFFERENCES MODELLING

### 2.1 Finite differences mesh and boundary conditions

The 3D FD program *FLAC<sup>3D</sup>* (Itasca 2009) was employed to study the behavior of battered piles under lateral loading. Full 3D geometric models were used to represent the coupled pile-soil system. Taking advantage of symmetry, only half of the actual model was built, thus significantly reducing the computational effort. Figure 1 shows the general layout and meshing of the FD half model used for the pile-soil system. A floating battered pile with a diameter,  $B$  and a length,  $L$  was embedded in a soil stratum with total thickness corresponding to  $L\cos\beta+6B$ , where  $\beta$  is the batter angle (Fig. 2). The pile-soil system was meshed with 8-node brick elements, and the soil elements are fairly small adjacent to the pile and gradually increase in size as they move away from it. The soil element size was kept uniform at 0.5 m in the vertical direction. The total mesh size was extended to a horizontal distance of  $16B$  from the center of the pile. This distance was decided after performing a number of initial trial analyses with several horizontal distances until the displacements and stresses of the pile did not change significantly with further increasing of the distance. All displacements were restrained at the bottom of the meshes while those at the vertical "external faces" were fully fixed in the x- and y-directions. The symmetry face were fixed against displacement normal to the symmetry plane, but were free to move on the surface of the plane. The top and bottom of the pile were set as displacement and rotation free.

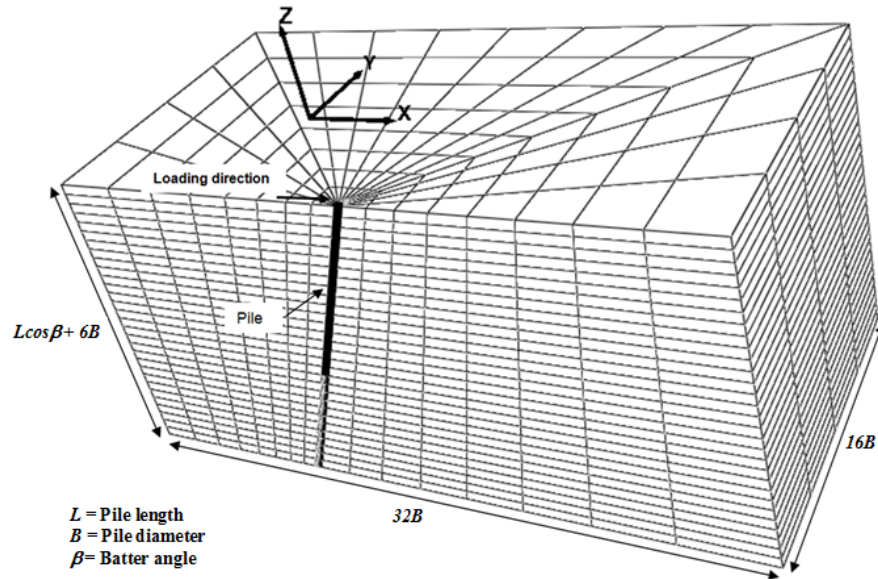


Figure 1. Typical mesh used for the 3D finite differences analyses (example corresponding to  $L = 10$  m,  $B = 1.0$  m and  $\beta = -25.5^\circ$ ).

### 2.2 Soil model

As only the undrained behavior of the soil was being considered, it was deemed sufficient to use a total stress model for the soil. This model is implemented into *FLAC<sup>3D</sup>*

as a modification of the Mohr–Coulomb failure criterion and requires the following six parameters: mass density ( $\rho$ ), cohesion ( $c$ ), friction angle ( $\phi$ ), dilatancy angle ( $\psi$ ), elastic bulk modulus ( $K$ ) and elastic shear modulus ( $G$ ).

### 2.3 Pile model

The pile is considered as linear-elastic material. Three parameters are required to define the pile material behavior. These parameters are the mass density ( $\rho_p$ ), the elastic bulk modulus ( $K_p$ ) and the elastic shear modulus ( $G_p$ ).

### 2.4 Pile-soil interface

The pile-soil interface was simulated using the basic concept of the Coulomb friction model, which relates the maximum allowable shear stress (friction) across an interface to the contact pressure between the contacting bodies. The shear strength was defined with zero cohesive strength and 2/3 of the friction angle for sandy soils. Separation is able to cause a significant increase in displacements and therefore the interface elements are allowed to separate if tension develops across the interface and exceeds the tension limit of the interface. Once gap is formed between the pile-soil interfaces, the shear and normal forces are set to zero (Itasca 2009).

In the current study, involving nonlinear analysis, high pile-soil interface stiffness is assigned to minimize the contribution of pile-soil interface elements to the accumulated pile displacements. According to the results of trial numerical analyses conducted to identify an appropriate stiffness value, a value of  $10^8$  Pa/m for both  $k_n$  and  $k_s$  was found to be sufficient to ensure that no additional deflections were attributed to the pile due to the deformation of the springs representing the interface. The use of such considerably higher values is tempting as it could be considered as more appropriate, but in that case the solution convergence would be very slow. In that way, the interface elements behave practically as a slider with a rigid/plastic behavior.

### 2.5 Numerical modelling procedures

The model is first brought to an equilibrium stress-state under gravitational loading before the installation of the pile. In the next stage of analysis, the model is brought into equilibrium after the installation of the pile. The installation is modeled by changing the properties of the pile zones from the properties representing the soil material to those representing the pile material. It is helpful to know the addresses of the grid-points at the pile, to facilitate both the loading of the pile and the monitoring of the pile response. The pile top is then displaced laterally for a deflection of  $0.25B$ . This is accomplished by applying a horizontal velocity at the pile top. The modeling of the pile installation process is rather complicated, so that pile is assumed to be in a stress-free state at the beginning of the analysis, and the effect of the pile installation is ignored in the analysis.

## 3 VALIDATION OF THE NUMERICAL MODEL

Before describing the numerical results on the influence of batter angle on the lateral response of battered piles, the applicability of the adopted model was verified by predicting the pile load test data from a published case

study. Lateral load tests on battered piles were conducted in a centrifuge at 45g (Zhang et al 1999). These tests simulated prototype square piles with 0.43 m wide and 13.7 m long founded in loose and dense sand. Five pile inclinations were modelled:  $7^\circ$  and  $14^\circ$  at negative pile batter, plumb, and  $7^\circ$  and  $14^\circ$  at positive pile batter (Fig. 2). The lateral load was applied at the pile head. The free length corresponding to the distance between the point of lateral load application and the ground surface, is 2.14 m. The Young's modulus of the model aluminum was 73.1 MPa. The soil used in the study was mixed sand with average particle diameter of 0.23 mm. The sand layer was prepared by dry pluviation through three rectangular sieves (U.S. standard sieve No. 14) which were stacked on top of the rectangular sample container. Two sample densities were prepared for the tests: (1) loose sand and (2) dense sand. The dry unit weights corresponding to these relative densities were  $14.05 \text{ kN/m}^3$  and  $14.50 \text{ kN/m}^3$ , respectively. Originally, the internal friction angles of the sands were  $34.5^\circ$  and  $37.1^\circ$ , respectively (Zhang et al 1999).

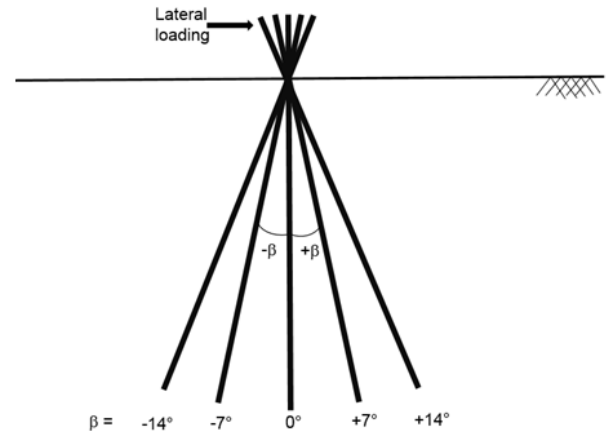


Figure 2. Layout of single battered pile tests.

The comparisons between the FD predictions and the reported data, corresponding to battered piles in the loose and dense sands, are shown in Fig. 3. The 3D numerical results are fully consistent with the experimental results obtained by Zhang et al (1999). Hence, it could be concluded that the numerical scheme adopted in the present investigation is capable of modeling the soil-pile interaction under lateral load, and for several batter pile angle.

## 4 PARAMETRIC STUDIES

*FLAC<sup>3D</sup>* was used to perform a series of analyses on steel-battered piles embedded in sandy soils, and subjected to pure lateral loads. The primary objective of these analyses is to study the influence of typical parameters on the lateral capacity of battered piles. Due to the abundant number of parameters involved, this study focuses on a selected number of parameters. These parameters include batter angle ( $\beta$ ), and relative density of sandy soil. Table 1 presents the dimensions and the

material proprieties of the used piles. Geometrical and material properties of piles are extracted from Handbook of Steel Construction (CISC 2011). In the other hand, soil properties considered are regrouped in Table 2.

Table 1. Pile dimensions and material properties.

| Pile details                               |       |
|--------------------------------------------|-------|
| Outside diameter $B$ (mm)                  | 406.4 |
| Thickness $T$ (mm)                         | 6.3   |
| Length $L$ (m)                             | 10.0  |
| Type of pile                               | Steel |
| Young's modulus $E_p$ (MPa)                | 78000 |
| Mass density $\rho_p$ (kg/m <sup>3</sup> ) | 6260  |
| Poisson's ratio $\nu_p$                    | 0.29  |

The response of battered piles under lateral load was analysed for several batter angles ( $\beta$ ) ranged from  $-25.5^\circ$

to  $25.5^\circ$  (these values correspond to the limit values where the model is stable in the numerical analysis with respect to the adopted pile dimensions). The numerical results are presented and discussed in the following section.

## 5 RESULTS AND DISCUSSIONS

Figures 4a-d show the influence of a batter pile angle ( $\beta$ ) on the lateral capacity of battered piles installed in sandy soils. Each plot in Fig. 4 corresponds to different state of sand density including very loose, loose, dense, and very dense. It is appeared from Fig. 4 that the lateral capacity of 'Piles battered forward', is not much affected compared to a vertical pile ( $\beta = 0^\circ$ ).

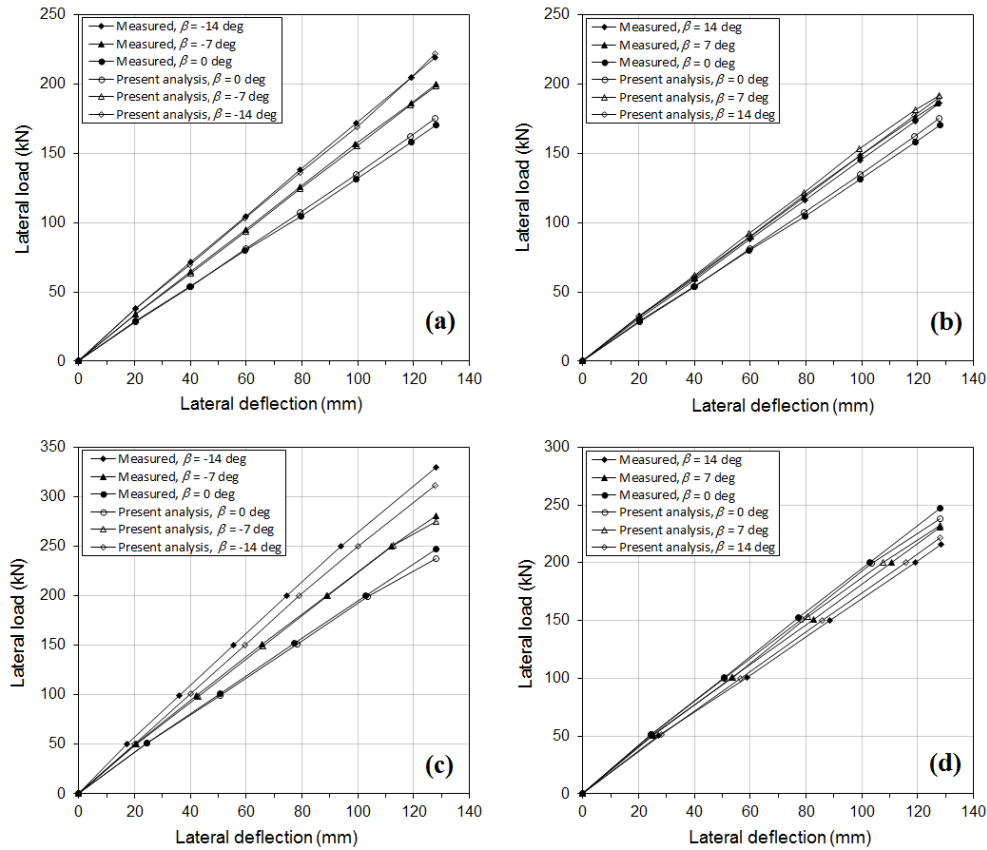


Figure 3. Comparison of FD predicted results with test data of Zhang et al (1999): (a) loose sand-negative angle, (b) loose sand-positive angle, (c) dense sand-negative angle and (d) dense sand-positive angle.

Table 2. Model parameters of sandy soil used in parametric study.

| Soil type  | Mass density<br>$\rho$ (kg/m <sup>3</sup> ) | Shear modulus<br>$G$ (MPa) | Bulk modulus<br>$K$ (MPa) | Undrained shear<br>strength $c_u$ (kPa) | Angle of<br>friction $\phi$ ( $^\circ$ ) |
|------------|---------------------------------------------|----------------------------|---------------------------|-----------------------------------------|------------------------------------------|
| Very loose | 1600                                        | 4.6                        | 10.0                      | 0                                       | 26                                       |
| Loose      | 1800                                        | 7.7                        | 16.7                      |                                         | 30                                       |
| Dense      | 2000                                        | 19.2                       | 41.7                      |                                         | 36                                       |
| Very dense | 2200                                        | 26.9                       | 58.3                      |                                         | 42                                       |

In the case of very loose sand, the lateral capacity is slightly increased for  $\beta$  of 25.5° and not significantly changed for  $\beta$  of 12.5°. In the case of very dense sand, the lateral capacity is relatively decreased for

$\beta$  of 12.5° and 25.5°. For the 'Piles battered reverse', the lateral capacities are considerably increased for  $\beta$  of -12.5° and -25.5° in all sand state considered.

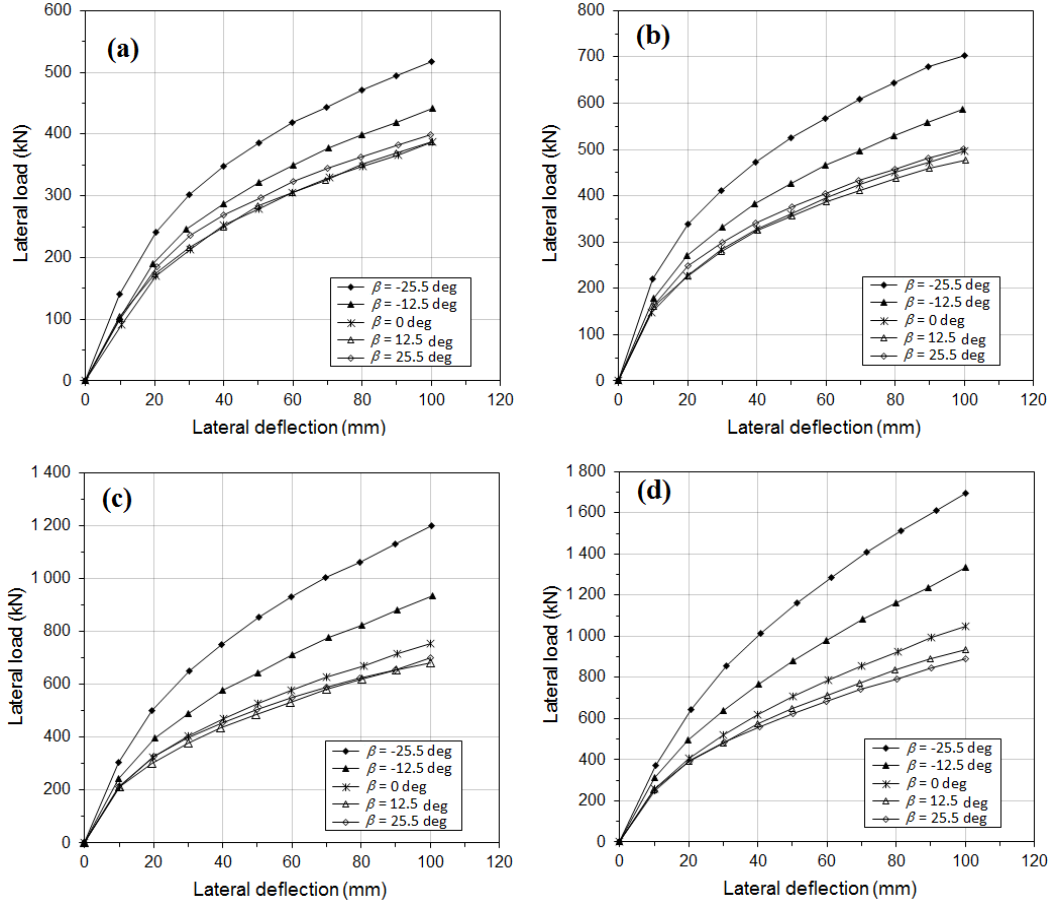


Figure 4. Lateral load–lateral deflection curves of battered piles: (a) very loose sand, (b) loose sand, (c) dense sand, and (d) very dense sand.

The variation of the ratio of lateral capacities of battered piles relative to vertical pile with the battered pile angle is portrayed in Fig. 5 for different state of sand densities. The general trends in Fig. 5 indicate that the lateral capacities of battered piles depend on both the pile batter angle and the sand density. For positive batter angle, the dependence of the battered pile capacity of the pile batter is minimal especially for very loose and loose sands. This result confirms that proposed by Meyerhof and Yalcin (1994). For  $\beta$  from 0 to 18°, the lateral capacity is not significantly changed relative to the corresponding capacity of vertical pile in very loose and loose sands. For dense and very dense sands, the lateral capacity of battered piles however, decreases by almost 8 % and 11 %, respectively. For  $\beta$  more than 18°, the lateral capacity increases by 6 % and 4 % in very loose and loose sands, respectively, and decrease by 5 % and 11 % in dense and very dense sands, respectively. For negative batter angle, the dependence of the battered pile capacity on the pile batter becomes more significant with increasing sand relative density. The percentages of the increases in

lateral capacities are 14, 18, 22, and reach 24 % for  $\beta$  of -12.5°. For  $\beta$  of -25.5°, the increases are 39, 45, 58 and reach 61 % in the very loose, loose, dense, and very dense sands, respectively.

The reason for the considerable increase in the lateral capacity for 'Piles battered reverse' and the little change for 'Piles battered forward' has been examined by plotting the stress state (Mohr circle) of a soil element adjacent to the pile and at a depth of 3 m. The major ( $\sigma_1$ ) and the minor ( $\sigma_3$ ) principal stresses corresponding to stress state of the soil element at 0.25B lateral deflection of the battered piles are plotted in Fig. 6a for  $\beta$  of -25.5°, -12.5° and 0° and in Fig. 6b for  $\beta$  of 0°, 12.5° and 25.5°.

Figure 6a illustrates that the negative batter angle  $\beta$  increases the major principle stress relative to that corresponding to the vertical pile under pure lateral load. The increase in the major stress then increases the mobilized shear strength,  $\tau_m$  of the soil according to:

$$\tau_m = \frac{\sigma_1 - \sigma_3}{2} \sin(90 + \phi) \quad [1]$$

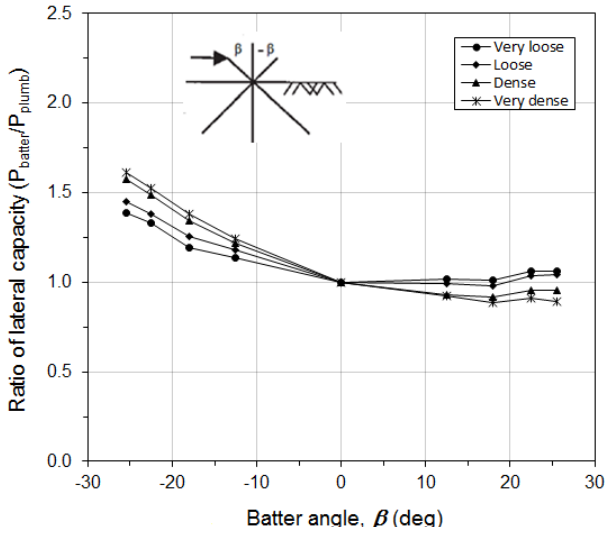


Figure 5. Influence of pile batter angle on lateral capacity of piles installed in sandy soils with different densities.

At the same lateral deflection of  $0.25B$ , Figure 6a confirms that the soil shear strength is reached for  $\beta$  of  $-25.5^\circ$ . More lateral deflection of pile is needed for  $\beta$  of  $-12.5^\circ$  and  $0^\circ$  to reach failure. On the other hand, when the batter angle  $\beta$  decreases from  $0^\circ$  to  $-25.5^\circ$ ,  $\sigma_3$  increases and therefore there will be a stress increment  $\Delta\sigma_{v0}$  (difference between  $\sigma_3$  corresponding to a vertical pile and  $\sigma_3$  corresponding to a battered pile with negative batter angle) due to the negative pile inclination. According to Zhang et al (2002), the stress increment  $\Delta\sigma_{v0}$  would cause considerable increase in the shear strength of the soil element. For the case of positive batter angle, when the batter angle  $\beta$  increases from  $0^\circ$  to  $25.5^\circ$ ,  $\sigma_3$  slightly decreases and therefore the  $\Delta\sigma_{v0}$  would cause a little decrease in the shear strength of the soil element (Fig. 6b). Therefore, the stresses from negative inclination of pile will cause a little decrease in the lateral capacity of the pile.

Considerable increase and decrease in the confining pressure in the soil in the vicinity of the battered pile inclined respectively by  $-12.5^\circ$  and  $12.5^\circ$ , and installed in very dense sand are shown in Fig. 6c. This increase or decrease in the confining stress of soil, then, increases or decrease the resistance of soil-pile interaction.

Figure 6d shows the stress paths of soil elements attached to the pile installed in very dense sand and at depths of 1.0 m and 2.5 m for both the cases of 'Pile battered reverse' inclined by  $\beta$  of  $-12.5^\circ$ , 'Pile battered forward' inclined by  $\beta$  of  $12.5^\circ$  and vertical pile ( $\beta = 0^\circ$ ). For all considered depths, Fig. 6d confirms that the soil element in the case of battered pile inclined by  $\beta$  of  $-12.5^\circ$  reached the failure surface earlier than that in the others cases corresponding of  $\beta$  of  $0^\circ$  and  $12.5^\circ$ . Moreover, the soil element located at 1.0 m reached the failure surface before the other soil elements at deeper depths due to the load transfer from the pile to the adjacent soil.

The influence of the batter angle on the behavior of laterally loaded battered piles in sand soil was investigated by means of numerical modeling. The numerical models were conducted using the computer program *FLAC<sup>3D</sup>* and the model were verified using centrifuge model testing data. The verified numerical model was used to perform a parametric study considering different variation of batter angle and soil density to evaluate the lateral capacity of steel battered piles subjected to lateral loads. Based on the results of this parametric study, the lateral capacities of the battered piles in sandy soils under lateral loads are influenced by the both pile batter angle  $\beta$  and sand density. For  $\beta$  from 0 to  $18^\circ$ , the lateral capacities of 'Piles battered forward' are not significantly changed relative to the corresponding capacities of vertical pile in very loose and loose sands. However, for dense and very dense sands, these lateral capacities decreases by almost 8 % and 11 %, respectively. For  $\beta$  more than  $18^\circ$ , the lateral capacities increases by 6 % and 4 % in very loose and loose sands, respectively, and decrease by 5 % and 11 % in dense and very dense sands, respectively. In the case of 'Piles battered reverse', the lateral capacities are considerably increased with the increasing of both  $\beta$  and sand density. The percentages of these increases reach 39, 45, 58 and 61 % for  $\beta$  of  $-25.5^\circ$  in very loose, loose, dense, and very dense sands, respectively.

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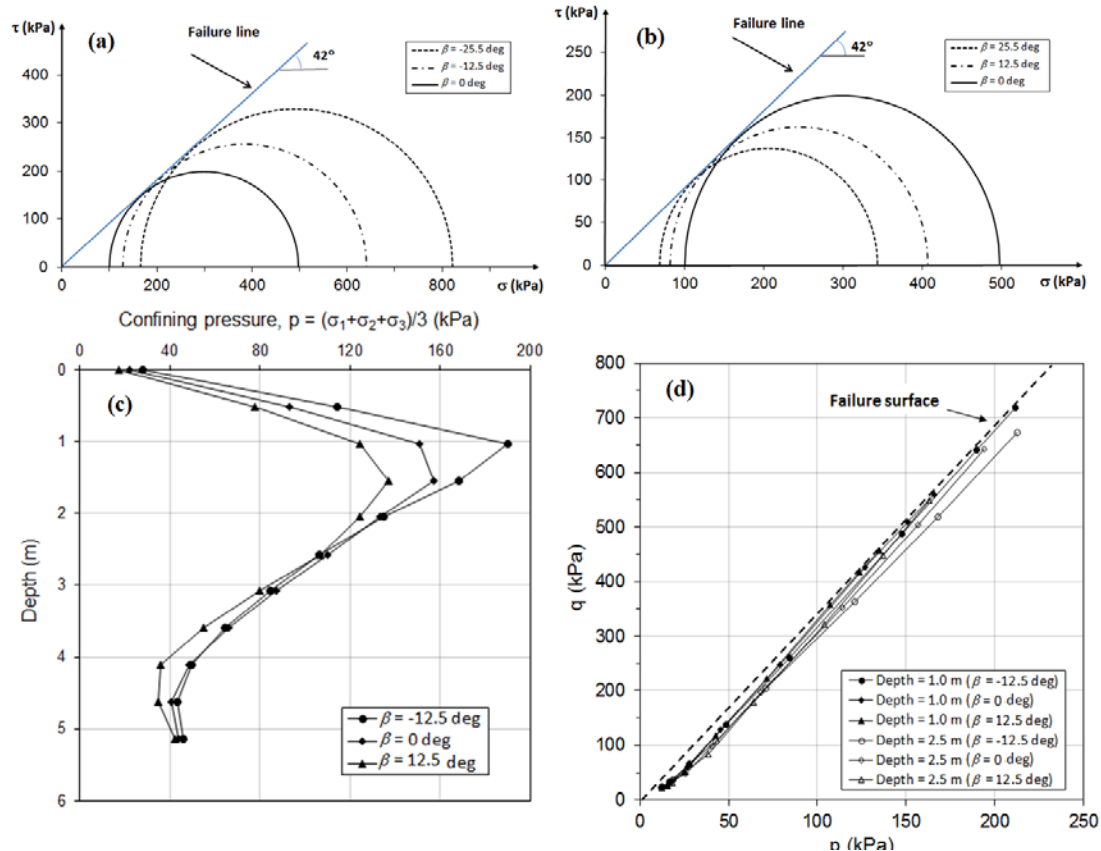


Figure 6. Analyses of battered pile, with several batter angle  $\beta$ , installed in a very dense sand: (a) Mohr circles of a soil element adjacent to a 'Pile battered reverse' and at a depth of 2 m, (b) Mohr circles of a soil element adjacent to a 'Pile battered forward' and at a depth of 2 m, (c) variation of the confining pressure along the pile stress paths of soil elements attached to the pile at different depths, and (d) stress paths of soil elements attached to the pile at different depths.

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