

Influence of Particles Shape, Size and Uniformity of sands on the Void Ratio Range and Consequently on the Dynamic Penetration Tests Results

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ABSTRACT

It has been discussed earlier in the 20th & 21st centuries that the particles shape, size, uniformity, and distribution have great influences on geotechnical properties such as the void ratio range ($e_{\max} - e_{\min}$) consequently on the geotechnical tests results. Although this is very well known, a minor progress for studying their effects on the void ratio range has been carried out as a parametric study. this is due to three reasons; first: These parameters are deeply integrated with other parameters such as the relative density. second: There are insufficiency in the most common used techniques to differentiate between these parameters; e.g. in the sieve analysis test, there is failure to differentiate between the real size of particles and the shape effect. Third: The overview has shown that there is no certain quantitative descriptor used in the geotechnical field for the particles shape. This paper presents a literature review on such parameters and initially proposes some solutions for the problem by several techniques such as the Image analysis which seem to be promising tool. correlation between the old and new techniques are also proposed and finally a majority of correlations of $(N_1)_{60}$ with such geotechnical parameters are discussed.

RÉSUMÉ

aux 20^e et 21^e siècles, l'influence de la forme, la taille, l'uniformité et la distribution des particules sur les propriétés géotechniques telles que l'étendu de vide des vides ($e_{\max} - e_{\min}$) et par conséquent sur les résultats des essais géotechniques, a été démontré. bien que, des études paramétriques mineures de leurs effets sur l'étendu de l'indice des vides ont été mené. Cela est dû à trois raisons; premièrement: Ces paramètres sont étroitement liés à d'autres paramètres tels que l'indice de densité relative. Deuxièmement: Il ya une insuffisance dans les techniques les plus utilisées pour différencier ces paramètres; par exemple le tamisage de permet pas de distinguer entre la taille réelle des particules et de l'effet de forme. Troisièmement: l'absence de mesures quantitatives pour approcher les forme de particules le domaine de la géotechnique. Cet article présente une revue de la littérature de ces paramètres et propose plusieurs solutions telles que l'analyse de l'image qui semblent être un outils prometteurs. La corrélation entre les anciennes et les nouvelles techniques sont également proposées ainsi que quelques corrélations entre $(N_1)_{60}$ et ces paramètres géotechniques ont été discutées.

1 INTRODUCTION

In fact particles shape, size, uniformity and distribution exert significant influence on the technical properties of the sand material (e.g. Santamarina and Cho, 2004; Mora and Kwan, 2000). this reason was the motivation for a lot of researchers to study the effect of such parameters on the void ratio, porosity, void ratio range, internal friction and permeability (e.g. Skempton, 1986; Cubrinovski and Ishihara, 1999, 2000; Menq, 2003; Miura et al., 1997, 1998; Rouse et al., 2008; Shinohara et al., 2000).

It is worth to mention that particles size, distribution and uniformity are involved in soil classification and evaluation of the internal friction angle as in most of the international specifications and guidelines (e.g. ASTM 2010 and AASHTO classification system), with largely ignorance to the particle's shape effect (e.g. Skredkommisionen, 1995 and Rodriguez J. M. et al., 2013). However investigations shows that sieving analysis which is the most used method to determine the particle size distribution has deviations when particle shape is involved; the average volume of the retained particles on

any sieve varies considerably with the particle's shape (Lees, 1964a, 1964b and Fernlund, 1998). For this reason image analysis is taking advantage over the traditional sieving technique moreover it's time, effort and cost saving (Andersson, 2010). Grading curves using image analysis is not standardized yet but after good results in the practice, new methodology for the soil grading curves could be developed including the shape effects (Andersson, 2010 and Rodriguez J. M. et al., 2013).

On the other hand, the importance of studying such parameters for sandy soils due to the knowledge that the mechanical characteristics of the coarse clean grained materials (clean sand and gravel) are mostly controlled by grain size as well as grain size distribution, and particles shape, while the characteristics of fine grained soils are governed by fine minerals as well as water content. e.g. Cubrinovski and Ishihara (1999, 2002) clearly pointed out that two sand samples with identical fine content as well as two clean sands can show remarkably different stress-strain characteristics. For that reason they suggested that the void ratio range ($e_{\max} - e_{\min}$) and the relative density (D_r) may be the most appropriate parameters to describe

the behavior of sands. Also Studying soil parameters affects the standard penetration tests count blows (N) is a unique challenge for geotechnical engineers (Daniel et al., 2003). meanwhile effects of grain size distributions, uniformity, and particles shape of sand soils on the SPT count blows (N), are poorly studied as a parametric study, however the engineering experience shows that wide variations in (N) values can occur as between different clean sands due to such parameters (Ghali et al. 2014).

The objective of the present study is to highlight a spot upon the deficit to determine the real effect of each of the above mentioned parameters on the void ratio range for sand soils under certain relative density. This deficit is due to insufficiency of the present techniques to differentiate between such parameters and separate it's interaction. Also This paper initially proposes some solutions for the problem by several techniques such as the Image analysis which seems to be promising tool. correlation between the old and new techniques are also proposed and finally a majority of correlations of $(N_1)_{60}$ with such geotechnical parameters are discussed depending on previous work done by Ghali et al. 2014.

2 BACKGROUND INFORMATION

A lot of previous research works have been gathered as shown in table 1 in order to give more explanation about the concerned parameters integrations and there affects on the void ratio range and the SPT count blows (N).

2.1 Effect of particles shape on the void ratio range and some geotechnical parameters:

Wentworth, (1922a) and Rodriguez J. M. et al., (2013) stated the fact that particles shape is a result of different transportation conditions (e.g. distance, temperature, moisture changes, etc.) for various mother rock mineralogical natures (e.g. hardness, rock formation ,etc.). Also most of the resent researchers (e.g. Michell & Soga, 2005; Arasan et. al, 2010; Rodriguez J. M. et al., 2013) agreed to define the particle shape as shown in Figure 1.

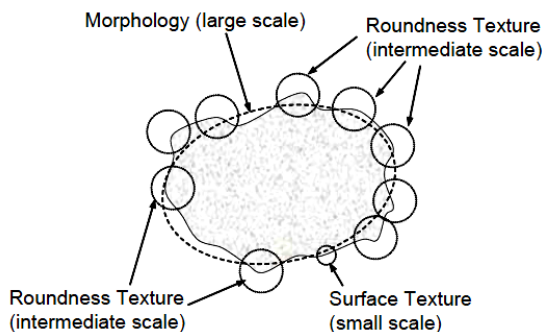


Figure 1. Particle shape qualitative description (after Michell & Soga, 2005).

Depending on this agreement and in order to state a certain qualitative description for the particles shape, Figure 1 can be summarized as in Table 2.

Table 3 also illustrates some definitions for the shape quantities mentioned by different authors.

Table 1. Sample of the studied references.

| Reference | Studied Parameter | Description |
|-------------------------------------|-------------------|--|
| Santamarina and Cho, 2004 | Particles shape | Sphericity and roundness |
| Skredkommisionen, 1995 | Particles shape | Guidelines for the particles shape |
| Mitchell and Soga, 2005 | Particles shape | Form, roundness and surface texture of particles |
| Arasan et al., 2010 | Particles shape | Form, roundness and surface texture of particles |
| Rodriguez J.M. et al., 2013 | Particles shape | Quantitative description for the shape |
| Miura et al., 1997-1998 | Angularity | Void ratio range with angularity |
| Cubrinovski and Ishihara, 1999-2000 | D_{50} | Void ratio range with mean grain size |
| Daniel et al., 2003 | D_{50} | Void ratio range with mean grain size |
| Menq, 2003 | C_u | Void ratio range with uniformity coefficient |
| Fernlund, 2005 | Particles shape | Orthogonal image analysis |
| Zeidan et al., 2007 | Particles shape | Image analysis |
| Ghali et al., 2014 | $(N_1)_{60}$ | Correlations with D_{50} , C_u and A_{2D} |

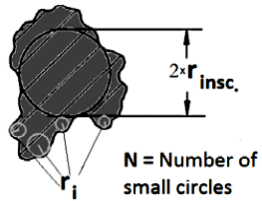
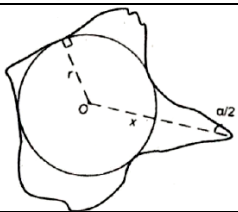
Table 2. Particles shape descriptions (modified from Rodriguez J. M. et al., (2013))

| Scale | Representative | Qualitative Description |
|--------------------|-----------------------------|-------------------------|
| Large Scale | Particle form size | Sphericity-Elongation |
| Intermediate Scale | Particle corners conditions | Roundness-Angularity |
| Small Scale | Particle surface texture | Smoothness-Roughness |

Also many researchers (e.g. Santamarina and Cho, 2001, 2004 & Cho et al., 2006 and Rosé et al., 2008) have been studied the influence of the roundness of the particles on the void ratio range as well as the internal frictional strength for different sands as illustrated in

Figures 2 and 3, while Bareither et al., 2008 tried to involve the effect of D_{50} into the relation as shown in Figure 4.

Table 3. Some shape descriptions as well as figures, relations, and references.

| Reference | Description | Relation | Figure |
|---|-------------------------------------|--|---|
| Mitchell and Soga, 2005 | Roundness 2D (R) | $R = \frac{\sum r_i / N}{r_{insc}}$ |  |
| Miura et al., 1997 And Das et al., 2012 | Angularity 2D (A _{2D}) | $A_{2D} = \sum \left[(180^\circ - \alpha^\circ) \left(\frac{x}{r} \right) \right]$ |  |

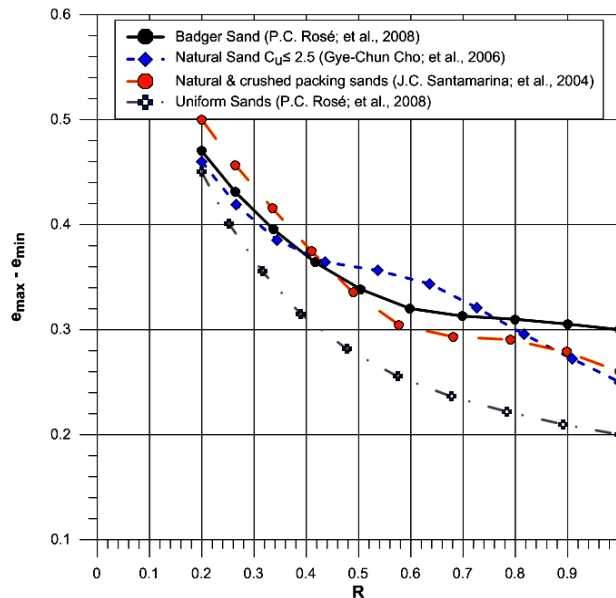


Figure 2. Influence of roundness on void ratio range for different sands (modified after some researchers as illustrated).

Miura et al. (1997, 1998) used another descriptor for particles shape called two-dimensional angularity (A_{2D}) to measure and quantify the grain shape as shown in Figure 5. Also in Figure 6, Ghali et al., (2014) presented the influence of A_{2D} on ($e_{max} - e_{min}$) under constant C_u and D_r including the effect of D_{50} .

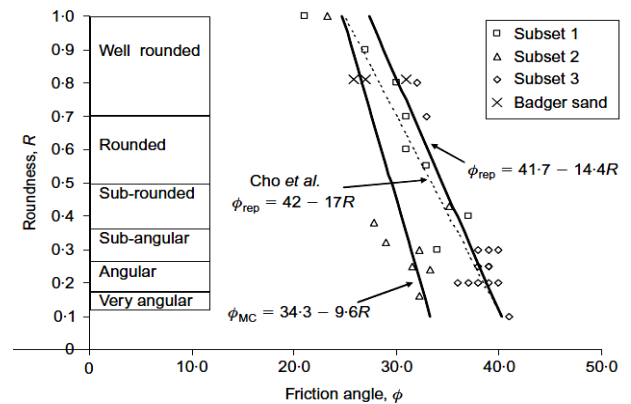


Figure 3. Influence of roundness on the internal friction for uniform sands (after Rosé et al., 2008).

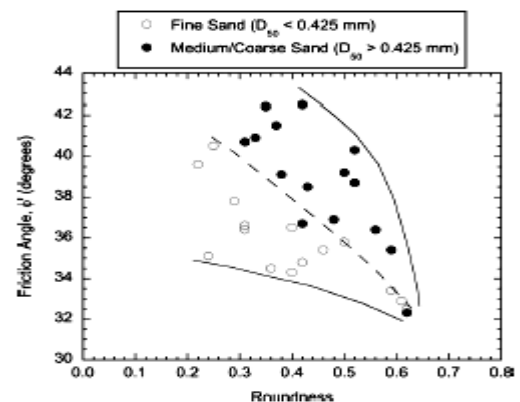


Figure 4. Relation between ϕ' and Roundness for fine sands (after Bareither et al., 2008).

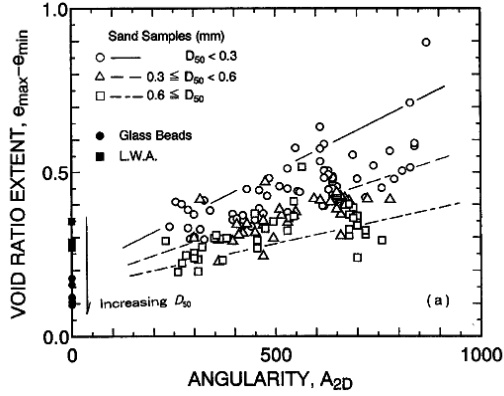


Figure 5. Variation of void ratio range with angularity of grains (after Miura et al., 1997).

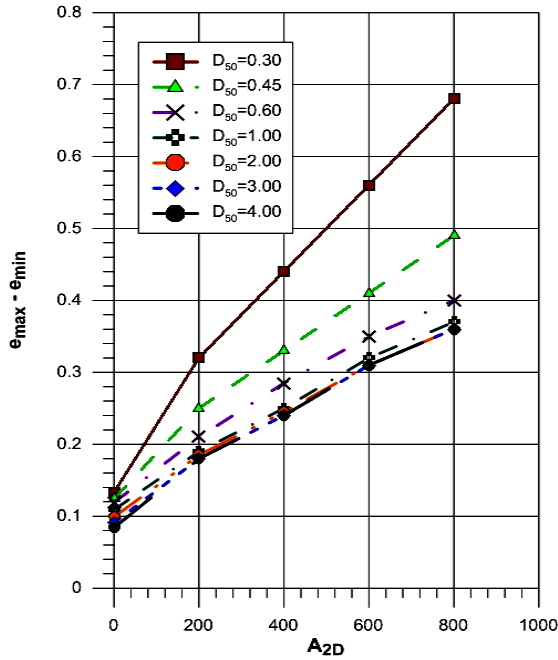


Figure 6. Influence of A_{2D} on $(e_{\max} - e_{\min})$ at $C_u = 2.5$ (after Ghali et al., 2014).

2.2 Effect of mean grain size on the void ratio range:

Cubrinovski and Ishihara (1999, 2000) proposed a relation between $(e_{\max} - e_{\min})$ and D_{50} as follows:

$$(e_{\max} - e_{\min}) = 0.23 + \frac{0.06}{D_{50}} \quad [1]$$

Meanwhile Ghali et al., (2014) included the effect of the uniformity into the relation between $(e_{\max} - e_{\min})$ and D_{50} under constant A_{2D} and D_r , Figure 7.

2.3 Effect of particles uniformity on the void ratio range:

Menq (2003) proposed a relationship between the void ratio range $(e_{\max} - e_{\min})$ and C_u for granular soils tested in US and Japan as:

$$(e_{\max} - e_{\min}) = 0.35 * \left(\frac{1}{C_u} \right) + 0.21 \quad [\text{For } 2 \leq C_u \leq 200] \quad [2]$$

Also Ghali et al., (2014) presented the effect of the uniformity on $(e_{\max} - e_{\min})$ at constant A_{2D} , D_{50} and D_r , as illustrated in Figure 8.

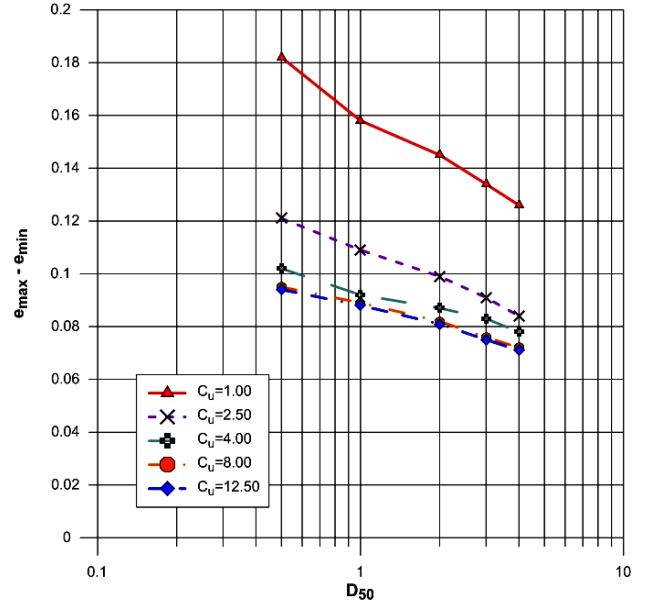


Figure 7. Influence of D_{50} on $(e_{\max} - e_{\min})$ at $A_{2D} = 0.00$ (after Ghali et al., 2014).

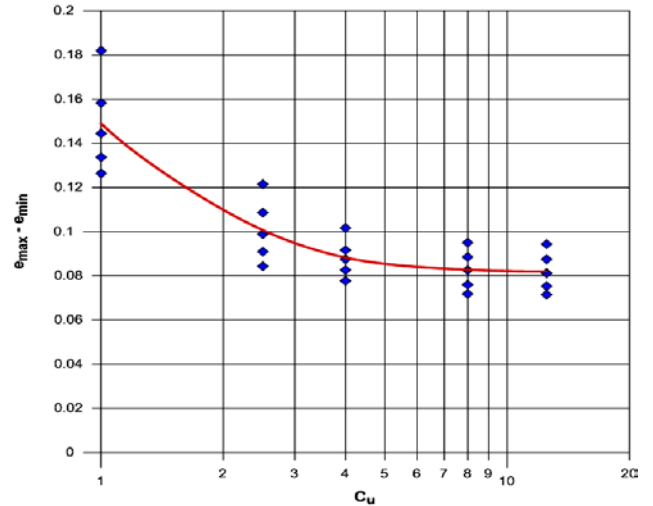


Figure 8. Influence of C_u on $(e_{\max} - e_{\min})$ at $A_{2D} = 0.00$ (after Ghali et al., 2014).

2.4 Effect of the void ratio range on the SPT results $(N_1)_{60}$:

Daniel et al. (2003b) summarized data gathered by several researchers (e.g. Skempton (1986), and

Cubrinovski and Ishihara (1999, 2000)) at a given σ'_v of 98 kPa by the equation:

$$\frac{N_1}{D_r^2} = 9 * \left(0.23 + \frac{0.06}{D_{50}} \right)^{-1.7} \quad [2]$$

Meanwhile, Kulhawy and Mayne, (1990) collected some field and laboratory data for SPT blow count $(N_1)_{60}$ and the square of the relative densities (D_r^2) of both normally consolidated and overconsolidated sands and presented the following empirical equation:

$$\frac{N_1}{D_r^2} = 60 + 25 \log D_{50} \quad [3]$$

On the other hand Ghali et al., (2014) simulated a laboratory SPT in order to perform a parametric study on such concerned geotechnical parameters and the following relations have been reached:

$$N - f_{(n_0)} = f_{(D,U,A)} \cdot f_{(N,OCR)} \cdot f_{aging} \cdot D_r^2 \quad [4]$$

where

$f_{(n_0)}$ is a function representing the SPT-N values for the soil at loosest state ($D_r = 0.0$), which is very small N value.

and

$f_{(D,U,A)}$ is a function of D_{50} , A_{2D} and C_u , developed under $\sigma'_v = 98$ kPa and $K = K_0$ as follows:

$$f_{(D,U,A)} = x_1 * \frac{e^{\frac{-1.7 A_{2D}}{1000} * [\log(D_{50} + 1)]^{1.7} * C_u^{0.485}}}{D_{50}^{(0.078 \log C_u - 0.289)}} \quad [5]$$

where $[x_1 = 5000 \text{ for } D_{50} < 1.0 \text{ mm}]$ and $[x_1 = 150 \text{ for } D_{50} > 1.0 \text{ mm}]$ Also for well graded sandy soils of $C_u \geq 8$, it is very acceptable to be substituted in Equation 5 equal to 8.

and

$f_{(N,OCR)}$ is a function of σ'_v and OCR. as follows:

$$f_{(N,OCR)} = \left(\frac{1 + 2K}{1 + 2K_0} \right)^{x_2} * \left(\frac{\sigma'_v}{98} \right)^{0.5} \quad [6]$$

where $[x_2 = 0.45 \text{ for } K < K_0]$ and $[x_2 = 0.55 \text{ for } K > K_0]$

And f_{aging} is a function of aging in accordance with B.M. Das et al. (2012):

$$f_{aging} = 1.2 + 0.05 \log \left(\frac{\text{Time(years)}}{100} \right) \quad [7]$$

2.5 Image analysis::

Grading curves for coarse grained particles are determined usually by the sieve analysis depending on

the shape of particles while for fine grained particles usually determined by sedimentation analysis depending on the weight of the particles.

Image analysis for coarse grained particles mainly depends on simulating the particle's volume and with assuming that all particles have the same specific gravity, grading curves for coarse particles depending on the weight of the particles can be obtained. In addition the image analysis is fast, economic, and can be easily automated.

For all the above mentioned reasons, Image analysis can be considered as practical method for the soil shape classification.

Table 4 summarizes some of the previous works and the development of such analysis.

3 PARTICLES SHAPE EFFECTS

A series of experimental simulations for the SPT on natural disturbed clean sand samples as well as artificial (glass beads) representing sand sizes were carried out in order to determine $(N_1)_{60}$ values and compare the results with the predicted values from Equations 2,3, and 4.

In order to apply Feret diameter and two ortho-images techniques, three randomly selected samples (total of 15 grams) were taken from every tested samples. Two images were taken for each individual particle in the horizontal and vertical projections in order to predict the Major (a), intermediate (b), and minor (c) diameters for an ellipsoidal shape has the same volume of the original particle then an equivalent diameter (D_{equ}) can be determined in accordance with the following equation:

$$D_{equ} = \sqrt[3]{a.b.c} \quad [8]$$

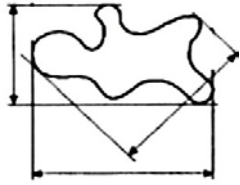
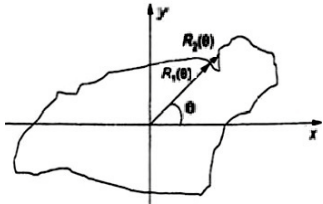
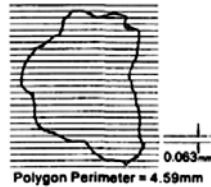

Grading curves for the tested samples were developed using sieve analysis as well as Feret diameter technique and two Ortho-images technique as illustrated in Table 5 and Figure 9.

Figure 9 Also shows that Feret diameter technique gives grading curves close to the sieve analysis meanwhile curves predicted by the two ortho-images technique show variation in D_{50} and small variation in C_u values. Also it has been noticed, as illustrated in Figure 10, that with increasing of A_{2D} , the corrected diameter using two ortho-image technique decreases than the diameter obtained by sieve analysis or Feret diameter technique.

Comparative study was performed on a high quality standard penetration tests results gathered by Skempton A. W., 1986. the purpose of this study is to illustrate the reliability of equations 2, 3, and 4 for predicting N_1 values. Also in order to include the effect of the shape in equation No. 4 all diameters were considered similar to Feret diameters and corrected in accordance with Figure 10.

Figure 11 shows that the relations which take into considerations the effect of particles shape give close results to the real tests.

Table 4. Some previous works for the image analysis.

| Reference | Analysis Name | Description | Schematic figure |
|----------------------------|---------------------------|---|--|
| Janoo, 1998 | Feret Diameter | Longest distance between two parallel tangents |  |
| Bowman et al., 2001 | Fourier Technique | Simulating the profile for individual particles |  |
| Hyslip and Vallejo, 1997 | Fractal Dimensions | Computing the perimeter by dividing the image into segments |  |
| Fernlund, 2005 | Orthogonal image analysis | Two ortho-images to simulate the volume of individual particles |  |
| Lanaro and Tolppanen, 2002 | Laser scan | 3D scanning for rocks from several faces using certain reference points on the rock's surface | |

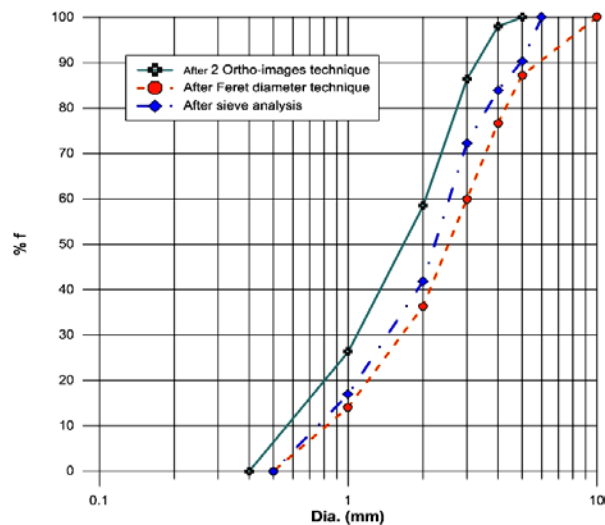


Figure 9. Grading curves of a tested sample using different techniques of analysis.

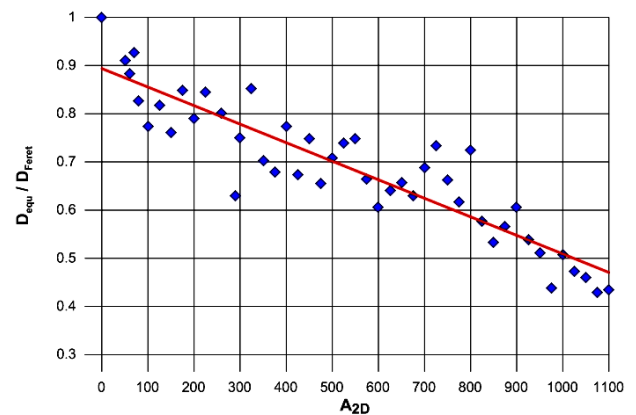


Figure 10. Variation of $(D_{\text{equ}} / D_{\text{Feret}})$ with A_{2D} for ellipsoidal and roller shaped particles.

Table 5. Sample of the studied particles

| Horizontal projection image | Vertical projection image | D_{equ} (mm) | D_{Feret} (mm) |
|---|---|----------------|------------------|
|  |  | 3.64 | 6.77 |
|  |  | 2.89 | 3.66 |
|  |  | 1.64 | 2.29 |
|  |  | 1.34 | 2 |
|  |  | 0.84 | 1.03 |

4 ACCURACY AND RELIABILITY OF THE PRESENT CORRELATION

The usage of two ortho-image technique to predict the grading curves in order to predict the SPT N_1 -values using equation 4 gives acceptable and reasonable results.

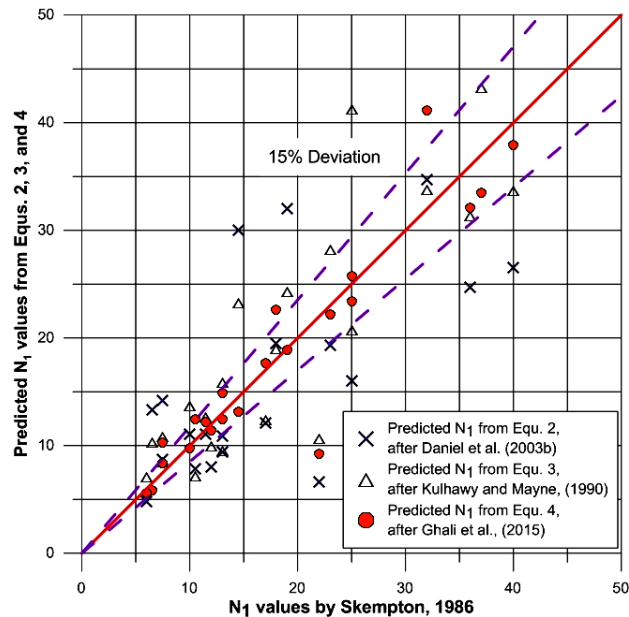


Figure 11. Comparative study between Equations 2, 3, and 4.

Also results were found to be matching with existing field tests with very small deviation which may be due to:

- The presence of fine grained soil content and water content which are not included in this study.
- Neglecting the effect of surface roughness of coarse grained particles.
- The resolution of the images, in addition to the very small error of determining the real particles volume using 2D images.

5 CONCLUSIONS

In this preliminary study and based on the data collected and analyzed from the present experimental simulations and also the Comprehensive studies with previous existing works, some conclusions has been reached as follows:

- The two ortho-image technique simulates volumes of the coarse particles, and by assuming that all particles have the same specific gravity, it can predict the grading curves matching with those determined by sedimentation analysis for fine grained soils.
- The Feret diameter technique may be considered as a good representative for sieve analysis results.
- The two ortho-image results show differences in D_{50} , C_u than the sieve analysis which means that by including the shape effect, many of the existing regular relations in the geotechnical field may be modified.
- With increasing of A_{2D} the corrected diameter using two ortho-image technique decreases than the diameter obtained by sieve analysis or Feret diameter technique.
- Correlations of N_1 which have been presented by Ghali et al., 2014 have been developed up on some parametric studies on D_{50} and C_u using fully spherical glass beads then the effect of A_{2D} has been added, hence it gives close results to the reality by using such parameters predicted from two ortho-image technique.
- The usage of A_{2D} in Equation 5 may considered sufficient to represent coarse particles forms and corners with neglecting the surface roughness.

ACKNOWLEDGEMENTS

The authors would like to thank Hydro-Québec and the NSERC for their financial support throughout this research project.

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