Evaluation of equivalent cycle liquefaction concept based on T_xSS test results

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ABSTRACT
The concept of equivalent number of cycles is that, an irregular earthquake load can be replaced by uniform cycles that have the same damage effect on the soil to trigger liquefaction. In this paper, uniform and non-uniform cyclic strain-controlled tests have been carried out using the new combined triaxial simple shear (T_xSS) apparatus on different saturated reconstituted samples of Ottawa C-109 and Baie Saint-Paul sands. An empirical expression developed to estimate the buildup of pore pressure, R_u that has been later used in this study as damage metric for computing the equivalent number of cycles, $n_{eq\gamma}$. The estimated $n_{eq\gamma}$ based on the experimental results has been compared successfully with those computed by the well-established P-M and R-N damage hypotheses.

RÉSUMÉ
Le concept de nombre de cycles équivalent est que, une charge irrégulière d'un tremblement de terre peut être remplacée par des cycles uniformes qui ont le même effet ou dégâts sur pouvant déclencher la liquéfaction. Dans cet article, des essais en déformation contrôlée cycliques uniformes et non uniformes ont été effectuées en utilisant le nouvel appareil de cisaillement simple triaxial (T_xSS) combiné sur différents échantillons de sol reconstitués saturés (Sable Ottawa C-109 et un sable Baie Saint-Paul). Une expression empirique élaborée pour estimer l'accumulation de la pression interstitielle, R_u qui a été plus tard utilisé dans cette étude comme indice de dommages métrique pour le calcul du nombre de cycles équivalent, $n_{eq\gamma}$. Le $n_{eq\gamma}$ estimée sur la base des résultats expérimentaux a été comparé avec succès avec ceux calculés par les bien établies P–M et R–N hypothèses de dommages.

1 INTRODUCTION

When fully saturated soils are subjected to cyclic loading in undrained conditions their stiffness and strength decrease with the progress of loading (i.e., increasing the number of cycles, N). The cyclic degradation is one of the most important damaging phenomena in soil dynamics known as liquefaction (e.g., Castro and Poulos 1977). This phenomenon has occurred in almost all large earthquakes and results in sand boil on the ground surface. The tendency of soil to contract results in pore pressures buildup and decreasing its effective stresses. At the extreme case, soil can completely lose the stiffness and strength then liquefy. Due to the damages in structures results from liquefaction, many investigations have been done to understand this phenomenon and developing procedure for evaluation its potential.

In order to facilitate the assessment of liquefaction potential of a soil deposit, several applicable approaches (e.g., Seed and Idriss1971; Whitman 1971) have been introduced to convert irregular earthquake motions to equivalent damage number of uniform cycles. There are several logical reasons for converting an earthquake motion to an equivalent series of uniform cycles. The main reason is that one set of laboratory test data can be used to evaluate any number of earthquake patterns (Seed et al. 1975). There are three main approaches to evaluate the liquefaction potential; namely stress, strain, and energy-based approaches. The very pioneer work of liquefaction evaluation is the stress-based approach by prof. Seed and his colleagues at University of California at Berkeley in their continuous efforts to study the liquefaction potential for the Cooper Generation Station, Nebraska in 1967 (Annaki and Lee 1977). This procedure is mostly empirical and based on laboratory and field observations. In fact, the cyclic behavior and pore pressure buildup of cohesionless soils is affected more fundamentally by the cyclic shear strain than the cyclic shear stress (e.g., Dobry et al, 1982; Vucetic and Dobry, 1988; Vucetic, 1994; Hsu C-C, 2002).

The aim of the present experimental study was to investigate the conversion of a non-uniform earthquake motion to equivalent number of uniform sinusoid strain cycles, $n_{eq\gamma}$. The conversion was based on the results of strain controlled cyclic triaxial simple shear tests (T_xSS) carried out on reconstituted specimens of Baie Saint-Paul and Ottawa C-109 sands. Excess pore pressure has been considered as damage metric throughout this study. Also, the equivalent number of uniform strain cycles has been calculated by using P–M and R–N hypotheses and compared with experimental results.

2 THE USED T_XSS APPARATUS AND THE TESTED MATERIAL

Cyclic triaxial simple shear test (T_xSS) apparatus, used in this study, was designed to provide confining pressure to the soil specimen during simple shearing by a non-uniform wave form. Baie Saint-Paul and Ottawa C-109 sands were used in this study with their properties summarized in Table 1. The grain-size distribution curves of the two sands are shown in Fig. 1. Wet tamped preparation method was used to prepare reconstituted soil specimens in unreinforced rubber membrane. The soil specimen was 76 mm in diameter and 25 mm height. Moist sand was placed in three layers and every layer compacted to the desired density. After saturation, with a Skempton's B value greater than 0.97, the sample was isotropically consolidated to an initial stress ratio of $k_o = (\sigma'_h/\sigma'_v) = 1$, where σ'_v and σ'_h are the effective vertical and horizontal stresses, respectively. It is observed that the consolidation increases relative density, I_d. After consolidation, the specimens were subjected to cyclic shear strain under undrained condition until initial liquefaction has been occurred. Initial liquefaction is defined throughout this study as the excess pore pressure, $R_u = u/\sigma'_{co}$, of 0.9, where u is the residual pore pressure; σ'_{co} is the initial effective confining pressure, as shown in Figs. 2b and 3b.

Table 1: Physical properties of the used sands

Soil properties	Baie Saint-Paul sand	Ottawa sand C-109
G_s	2.78	2.67
I_d %	55	20
$e_{max.}$	0.91	0.82
$e_{min.}$	0.598	0.5
e	0.7375	0.756
ρ_{max} (Kg/m^3)	1745.4	1780
ρ_{min} (Kg/m^3)	1457.4	1467
ρ (Kg/m^3)	1600	1520
C_u	2.25	1.75
C_c	1	1.016
D_{50}	0.15	0.4

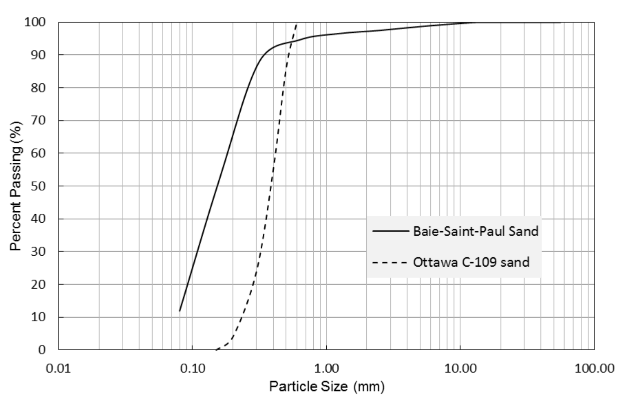

Figure 1. Grain size distribution of the used sands

3 EXPERIMENTAL RESULTS AND DISCUSSION

A series of undrained cyclic strain-controlled tests on Ottawa C-109 and Baie Saint-Paul sands under isotropic stress conditions and at different values of γ_{cyc} is conducted until initial liquefaction has been occurred. The sample preparation method and the testing procedure mentioned above were almost the same in all tests. All tests have been carried out at the initial effective confining pressure of 26 and 75 kPa for Ottawa C-109 and Baie Saint-Paul sands, respectively, and at the same loading frequency of 1 Hz.

Typical responses of Baie Saint-Paul and Ottawa C-109 sands to cyclic loading are presented in Figures 2 and 3, respectively. Figs. 2a and 3a show the variations of cyclic stress ratio CSR, defined as the amplitude of the applied cyclic shear stress ($\tau_{cyc.}$) divided by the initial effective confining stress (σ'_{co}), with the cyclic shear strain, γ_{cyc}. CSR- γ_{cyc} hysteric loops rotate towards the γ axis with the increase in the number of cycles, and the bounded area representing the dissipated energy decrease with further cycles. Figs. 2b and 3b show that the increase of the excess pore pressure, R_u, with strain cycles results in exponential decay of cyclic stress ratio (CSR). Figures 2c and 3c, show that the reduction in shear stiffness of the soil due to the act of cyclic shear strain is associated with an increase in the vertical deformation of soil structure ε_v.

Cyclic shear strain γ_{cyc} in every test is plotted against the number of cycles required to cause liquefaction N_{liq} in Fig. 4. The (γ_{cyc}-N_{liq}) curve presented in Fig. 4 is an analogy to the well-known (S-N) curve introduced by Palmgren (1924) and Miner (1945) for metal fatigue studies, where S is uniform cyclic shear stress applied on metal component, N is number of cycles to cause a specific degree of strength deterioration (Annaki and Lee 1977). The P-M damage hypothesis is later adapted by Seed et al. (1975a) for soil liquefaction assessment in the form of (CSR-N_{liq}) curve. In this study, the (γ_{cyc}-N_{liq}) curve was adopted as an analogy of the (CSR-N_{liq}) curve adopted by Seed et al. (1975a) and others. Figure 4 illustrates that the N_{liq} increase with the decrease in cyclic shear strain γ_{cyc}. It can be seen that the liquefaction resistance (γ_{cyc}-N_{liq}) curve for Baie Saint-Paul sand is greatly higher than the corresponding curve of Ottawa C-109 sand. This difference may be attributed to the difference in the initial effective confining pressure, physical properties as well as particle characteristics of the two sands.

4 PORE PRESSURE MODELS USED IN CONVERTING EARTHQUAKE MOTION TO UNIFORM CYCLES.

The using of excess pore pressure ratio, R_u, as damage metric for computing n_{eq} is more applicable for liquefaction evaluations (e.g., Carter et al. 2013). However, several researchers didn't consider the buildup of pore pressure in the saturated soil during the application of the earthquake motion in their converting methods because its limitations in computing n_{eq}, (Haldar and Hochaimi 1984). It is observed that the excess pore pressure ratio can reach the unit, i.e. R_u=1, prior to the end of earthquake shaking. So the subsequent motions do not contribute in computing n_{eq} (Carter et al. 2013). To overcome this limitation, the amplitude of the ground motions can be scaled so that R_u = 1 at the end of shaking (Ishihara and Nagase, 1988; Wer-Asturias, 1982).

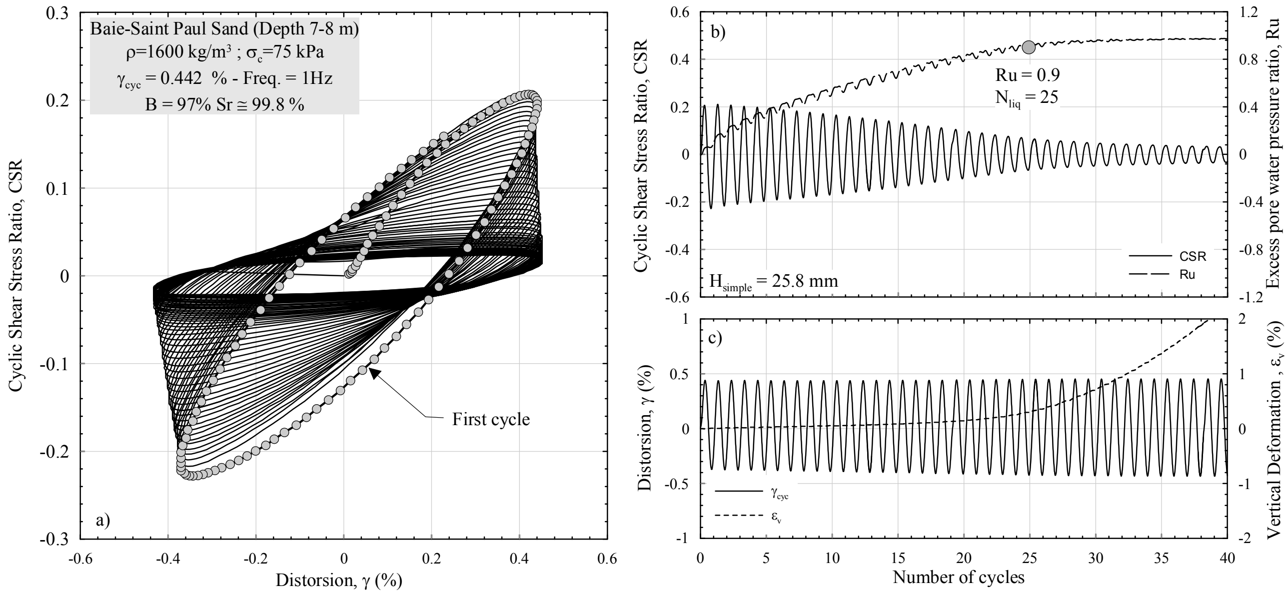

Figure 2. Typical records of T_xSS test on Baie Saint-Paul sand

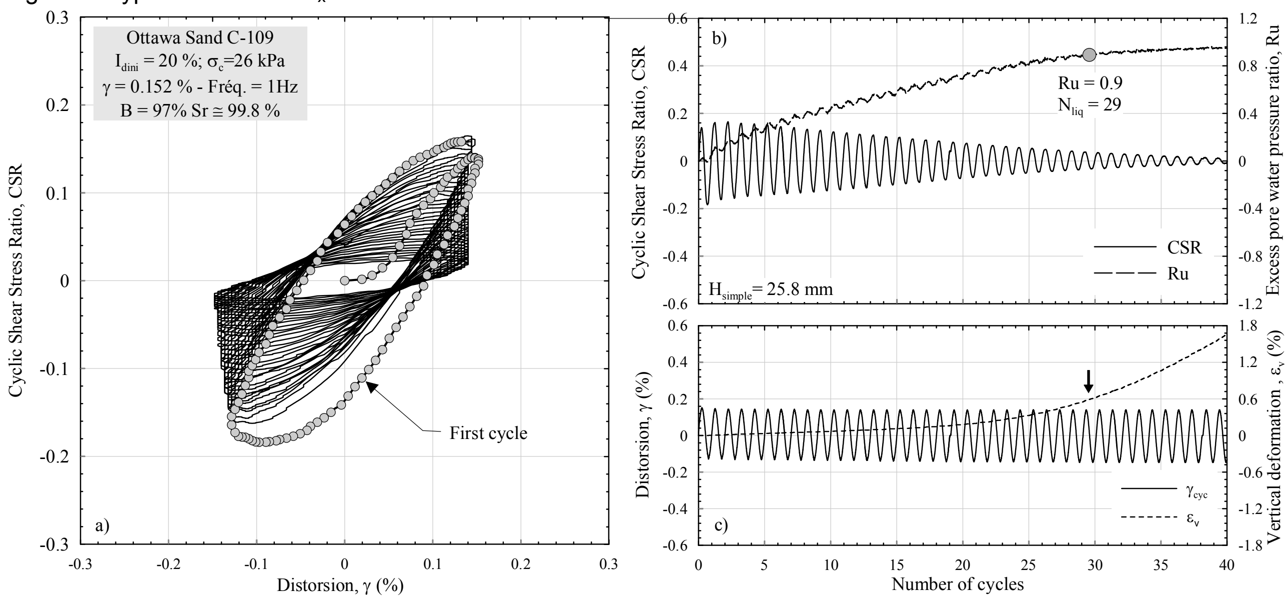

Figure 3. Typical records of T_xSS test on Ottawa C-109 sand

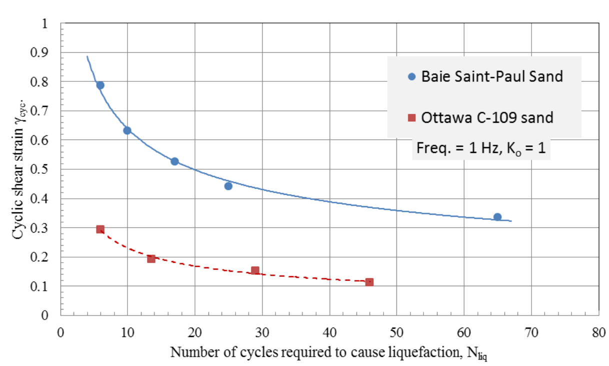

Figure 4. Variation of cyclic shear strain, γ_{cyc} with N_{liq} for Ottawa C-109 and Baie Saint-Paul sands

In the literature, many empirical models have been introduced for computing R_u as a function of cyclic ratio, R_n. Where $R_n = n/N_{liq}$, and n is the number of loading cycles. For example, Seed et al. (1975b) proposed a simple linear relation between excess pore pressure buildup and R_n (Wer-Asturias 1982). Another non-linear stress-based relation first proposed also by Seed et al. (1975b) and simplified later by Booker et al. (1976), is given by:

$$R_u = \frac{2}{\pi}\sin^{-1}[(R_n)^{\frac{1}{2\theta}}] \qquad [1]$$

Where n is number of loading cycles in a stress-controlled cyclic test , and θ is an empirical constant depends on the soil type and test conditions and can be set equal to 0.7 as suggested by Green et al. (2000).

One of the most commonly used is a strain-based model first proposed by Martin et al. (1975) and simplified by Byrne (1991). This model compute the volumetric strain of dry sand subjected to strain-controlled cyclic loading. Green et al. (2000) introduced energy based model (GMP), Eq.2, to compute excess pore pressure ratio, R_u, by using the energy dissipated per unit volume of soil, W_s, which equal to the area bounded by stress-strain hysteric loop, (Green et al. 2000).

$$R_u = \sqrt{\frac{W_s / \sigma_c^{'}}{PEC}} \qquad [2]$$

where; PEC is pseudo energy capacity.

In this study, an empirical expression, Eq.3, which relates excess pore pressure ratio, R_u, for uniform strain controlled tests to the liquefaction number, N_{liq}, and material parameter, r.

$$R_u = 0.9\,[R_n]^r \qquad [3]$$

According to Green and Lee (2006), the material parameter, r, can be determined from laboratory tests. Figure 5 shows that there is a good agreement between the computed R_u by applying Eq. 3 and the measured data from laboratory tests on both Baie Saint-Paul and Ottawa C–109 Sands. Figure 6 shows the calibration of material parameter, r, at frequency = 1 Hz following Green and Lee (2006). It is observed that the material parameter, r, is a function of the strain level and greatly change from material to other. Material parameter will be used to estimate cumulative damage for uniform and non-uniform strain tests, as will be later discussed, by applying R-N fatigue hypothesis (Richart and Newmark, 1948).

5 COMPUTING OF CUMULATIVE DAMAGE AND EQUIVALENT NUMBER OF CYCLES.

Several approaches have been proposed for computing cumulative damage of metal component subjected to uniform and non-uniform load as well as computing the equivalent number of cycles, n_{eq}. The most widely used approach is the Palmgren–Miner (*P–M*) and Richart-Newmark (*R-N*) cumulative damage hypothesis. Details about these two approaches are provided below:

5.1 Computing of cumulative damage by P-M and R-N hypotheses

P-M hypothesis was first introduced by Palmgren in 1924 and further developed by Miner (1945). It is considered the basis for engineering problem dealing with non-uniform load. It is intended for high cycle fatigue conditions, large number of cycles – low amplitude, wherein the loaded material remains in the elastic range. The basic assumption is the energy applied during any stress cycle has an accumulative linear damage effect on the material, Eq. 4, and independent of sequencing of load in the time history, (e.g., Annaki and Lee 1977; Green and Terri 2005b).

$$D = \sum_i^m \frac{n_i}{N_i} = \sum_i^m \frac{\omega_i}{W_i} \qquad [4]$$

where; n_i = number of cycles in the load history having a given peak stress amplitude S_i; N_i = number of stress cycles to failure at stress level S_i; ω_i = absorbed work after n_i cycles; W = absorbed work at failure. In the late 1960s to early 1970s, Prof. Seed and his colleagues adapted *P–M* hypothesis, with slight modifications to compute $n_{eq\tau}$, for site response analysis and for evaluating soil liquefaction potential (e.g., Seed et al. 1975a; Annaki and Lee 1977). Green (2001) introduced adjustment to *P-M* method, which underlies the Green–Mitchell energy based liquefaction evaluation procedure to overcome shortening of P-M hypotheses, linear accumulative damage and constrains to the elastic stage, and to alleviate the influence of soil softening on the computed dissipated energy, (Green and Terri 2005b).

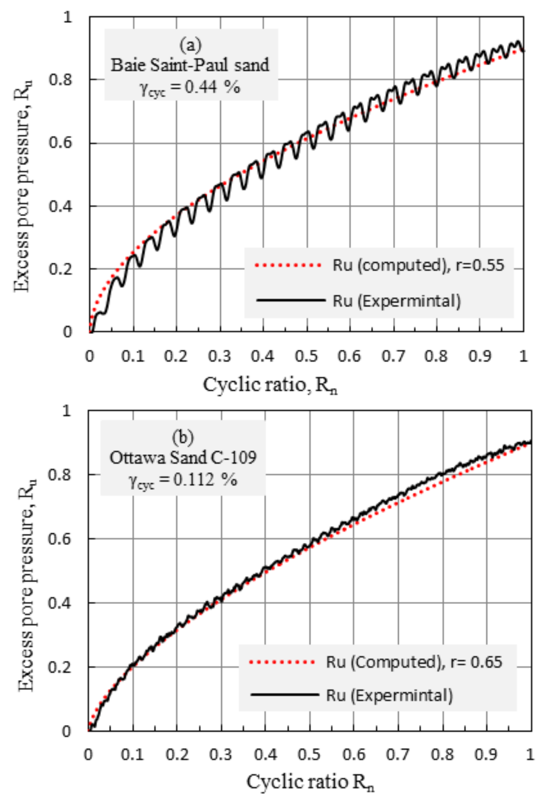

Figure 5. Validation of the proposed relation.

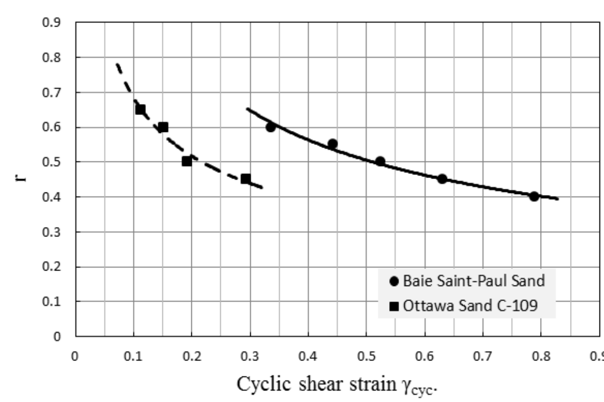

Figure 6. Calibration of the material parameter, r.

In 1948, R-N hypothesis was developed by Richart-Newmark (*R-N*) hypothesis. Green and Lee (2006) proposed the use of the R-N fatigue hypotheses for computing number of equivalent strain cycles, $n_{eq\gamma}$, for evaluating seismic compression. R-N hypothesis is applicable for both high and low cycle fatigue analyses, low number of cycles – high amplitude. Using R-N is more applicable than P-M hypotheses because the first one takes into account the sequence of the amplitudes of peaks in load history. Green and Lee (2006) used the Martin et al. (1975) procedure, which simplified later by Byrne (1991), to calibrate the material parameter, r.

Volumetric strain is commonly used as the damage metric to compute equivalent strain number $n_{eq\gamma}$ for strain time histories obtained from the site response analyses. However, in this study, the excess pore pressure ratio (R_u) was used as damage metric for computing $n_{eq\gamma}$. New empirical expression was introduced to calibrate the material parameter, r, for Ottawa C-109 and Baie Saint-Paul sands as presented in Fig.6. It was observed from Fig. 6 that the material parameter, r is a function of the load amplitude. More specifically, the material parameter, r, exponentially decreases with the applied shear strain, $\gamma_{cyc.}$, and consequently with the pore pressure buildup, R_u. The time history of the measured excess pore pressure ratio, R_u is compared to the computed damages by P-M and R-N hypotheses for uniform and non-uniform cyclic shear strain applied to Baie Saint-Paul Sand in Figs. 7 and 8, respectively. For the non-uniform cyclic strain presented in Fig. 8, peak-between-mean crossing count method has been used to determine number of peaks that are in time history. In this method only largest peak between successive crossings of the mean is counted (Green and Terri 2005a). The cumulative damages have been computed by using R-N hypothesis according to Eq. 5. Equation 5 is also employed to compute the cumulative damages according to P-M hypothesis by setting $r_1=r_2=...= r_m=1$ (Green and Lee 2006).

$$D_1 = \left(\frac{n_1}{N_{liq\,1}}\right)^{r_1} \quad [5.a]$$

$$D_2 = \left[(D_1)^{1/r_2} + \left(\frac{n_2}{N_{liq\,2}}\right)\right]^{r_2} \quad [5.b]$$

$$D_m = \left[(D_{m-1})^{1/r_m} + \left(\frac{n_m}{N_{liq-m}}\right)\right]^{r_m}$$

[5.c]

where, D_1 is the damage induced in the first sequence of n_1 cycles having shear strain amplitude γ_1 and; D_2 is the damage induced by the next sequence of n_2 cycles having amplitude γ_2; etc. D_m is the cumulative damage for non–uniform load history after m cycles in the load history. $N_{liq.1}$, $N_{liq.2}$, ….., $N_{liq\text{-}m}$ are number of cycles required to cause liquefaction corresponding to regular cyclic strain γ_1, γ_2, …., γ_m, respectively, which calculated from (γ_{cyc} - N_{liq}) curve, as shown in Fig. 4. r_1, r_2, …, r_m are the material parameter corresponding to γ_1, γ_2, …., γ_m, respectively, as shown in Fig.6.

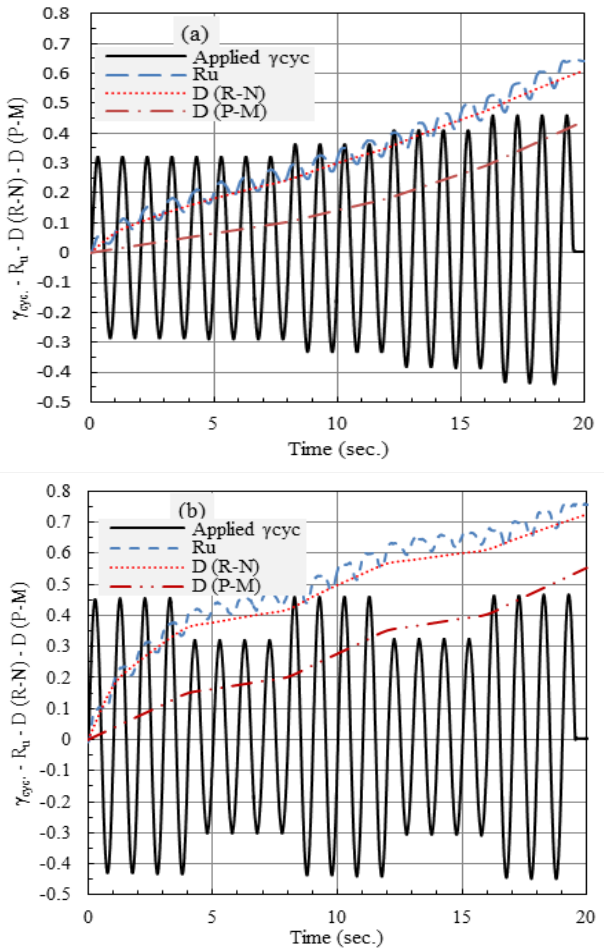

Figure 7. Relation between applied γ_{cyc}, R_u, cumulative damage computed by P-M and R-N hypotheses for Baie Saint-Paul Sand.

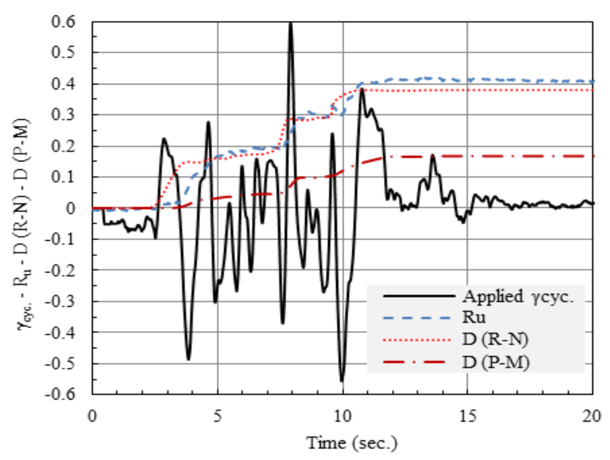

Figure 8. Relation between non-uniform shear strain γ_{cyc}, R_u, cumulative damage by P-M and R-N hypotheses for Baie Saint-Paul Sand.

A comparison of measured R_u values in cyclic triaxial simple shear specimens with damage computed using R-N hypotheses, Fig. 7, shows good agreement for both uniform and non-uniform applied cyclic shear strain. However, the use of P-M hypothesis is significantly under-predict cumulative damage in accordance with the results obtained by Green and Lee (2006). The slight difference between the computed damage by applying R-N hypothesis and the measured R_u in Fig. 8 may be attributed to the calibration of material parameter, r. More specifically, the calibration of the parameter r is done at cyclic strain frequency of 1 Hz while each cycle in the irregular time history has a different frequency. In order to get high degree of accuracy in the results, the material parameter should be correlated applying different frequencies. Generally, the computed damage computed by R-N hypothesis can be used in an acceptable degree of accuracy to predict the pore pressure buildup.

5.2 Converting non-uniform time history to equivalent uniform cycles

Converting an earthquake time history to equivalent number of cycles having an amplitude γ_{ref} can be done by equating the cumulative damage for both of them. Damage for regular amplitude loading can be computed as;

$$D = \left(\frac{n_{eq}}{N_{liq\ ref}} \right)^{r_{ref}} \qquad [6]$$

By equating the right hand sides of Eqs. 5.c and 6, n_{eq} according to R-N approach can be computed;

$$n_{eq} = N_{liq\ ref} \left[(D_{m-1})^{1/r_m} + \left(\frac{n_m}{N_{liq\ m}} \right) \right]^{r_m / r_{ref}} \qquad [7]$$

where r_{ref} and $N_{liq\text{-}ref}$ are the material parameter and the number of cycles having amplitude γ_{ref} which required to cause liquefaction, respectively. Also n_{eq} according to P-M approach can be calculated by setting $r_1 = r_2 = ... = r_m = r_{ref} = 1$:

$$n_{eq} = \sum_{i=1}^{m} \frac{N_{liq\ ref}}{N_i} n_i \qquad [8]$$

Tables 2 and 3 summarize, respectively the computing procedure of $n_{eq\gamma}$ with γ_{ref} = 0.33, 0.525, and 0.75 % for time histories presented in Figs. 7a. and 8 to cause R_u = 0.6 and 0.4, respectively. The $n_{eq\gamma}$ have been determined from Figs. 9 and 10 by considering the pore pressure buildup as damage metric. It is observed from Fig. 9 and Table 2 that there is a good agreement between the computed $n_{eq\gamma}$ by using both P-M and R-N hypotheses and that determined graphically, Fig. 9, to cause excess pore pressure (R_u=0.6). However, the path of pore pressure buildup is different according to the amplitude of applied cyclic shear strain. Although the computed $n_{eq\gamma}$ is equal in both the two hypotheses, the computed cumulative damage is different. As previously mentioned, the P-M hypothesis doesn't consider the sequence of loading. Converting non-uniform time history to equivalent number of uniform strain cycles, $n_{eq\gamma}$, having reference cyclic amplitude, γ_{ref} can be done with acceptable degree of accuracy by applying R-N method considering pore pressure buildup as damage metric.

It is observed from Fig. 10 and Table 3 that there is a slightly difference between computed $n_{eq\gamma}$ from Eq.7 and that determined graphically and this may attributed to the material parameter calibration discussed above.

Table 2: Summary of computing procedure of $n_{eq\gamma}$ for non-uniform shear strain, Fig.7.a.

γ_{ref}	$N_{liq\text{-}ref}$	r_{ref}	$n_{eq}/N_{liq\text{-}ref}$	$n_{eq\gamma}$ (R-N)	$n_{eq\gamma}$ (P-M)	$n_{eq\gamma}$ (Fig.9)
0.33	65	0.601	0.44	28.5	28.6	29
0.525	18	0.505	0.375	6.8	8	6.5
0.75	7	0.442	0.32	2.3	3	2.8

Table 3: Summary of computing procedure of $n_{eq\gamma}$ for non-uniform load, Fig.8.

γ_{ref}	$N_{liq\text{-}ref}$	r_{ref}	$n_{eq}/N_{liq\text{-}ref}$	$n_{eq\gamma}$ (R-N)	$n_{eq\gamma}$ (P-M)	$n_{eq\gamma}$ (Fig.10)
0.33	65	0.601	0.2177	14.15	9.75	12.5
0.525	18	0.505	0.1629	2.93	2.7	3.2
0.75	7	0.442	0.1258	0.88	1.05	1.2

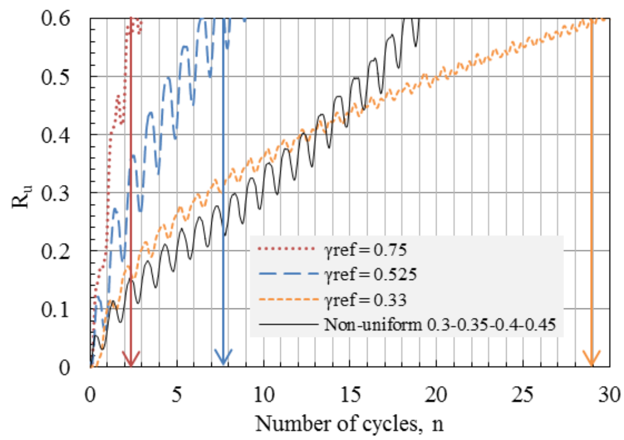

Figure 9. Pore pressure buildup and number of cycles required to cause R_u = 0.6 at Freq.=1 Hz.

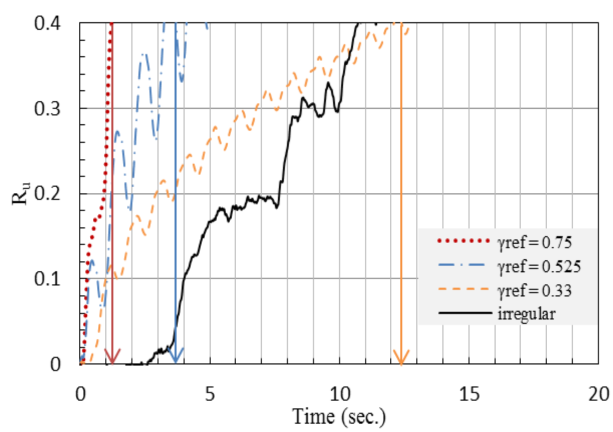

Figure 10. Pore pressure buildup and number of cycles required to cause R_u = 0.4 at Freq.=1Hz.

6 CONCOLUSION

By carrying out strain controlled cyclic T_xSS tests on soil specimen under undrained condition, a (γ_{cyc} – N_{liq}) curve is developed that has the potential to be used in liquefaction assessment as an alternative to (CSR-N_{liq}) curve. Pore pressure buildup in cohesionless soils under strain controlled tests can be estimated by empirical non-linear relation considering the material parameter, r, and N_{liq} of soil. Material parameter, r, was calibrated from the laboratory tests at frequency = 1 Hz by considering the pore pressure buildup as damage metric. Cumulative damages have been computed by both P-M and R-N hypotheses and compared to the excess pore pressure measured in the experiments. Using R-N hypothesis gives an accurate prediction of cumulative damage during cyclic loading more than P-M hypothesis. However, the computed $n_{eq\gamma}$ by using both hypotheses is roughly equal. Additionally, it is observed from studied tests that there is a good agreement between computed $n_{eq\gamma}$ by damage hypotheses and which determined graphically by considering pore pressure buildup as a damage metric.

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