Numerical modelling technique to predict the load versus settlement behavior of single piles in unsaturated coarse-grained soils

Mohammed Al-Khazaali[1,2] and Sai K. Vanapalli[1]
[1]Department of Civil Engineering, University of Ottawa, Ottawa, ON, Canada
[2]Building and Construction Engineering Department, University of Technology, Baghdad, Iraq

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ABSTRACT
The load-settlement behavior of three model piles with 38.3, 31.75, and 19.25 mm base diameter tested with different capillary suction values of 0, 2, and 4 kPa in two coarse-grained soils are presented. A simple finite element analysis technique using PLAXIS 2D is proposed to estimate the load versus settlement behavior of the model piles using the information of the predicted stiffness and the shear strength behavior of unsaturated soils derived from the information of saturated soil properties and the soil-water characteristic curve (SWCC). There is a good comparison between the numerical modeling results and the experimental results. The results of the study highlight the contribution capillary suction towards load-settlement behavior of pile foundations in coarse-grained soils. The proposed numerical modeling methodology is encouraging for implementing the mechanics of unsaturated soils into engineering practice.

RÉSUMÉ
Le comportement charge-tassement de trois modèles de pieux avec des diamètres de 38.3, 31.75, et 19.25 mm mis à l'essai sous des succions capillaires de 0, 2, et 4 kPa dans un sol à grains grossiers est présenté. Une technique d'analyse par éléments finis simple est proposée pour estimer le comportement mécanique des modèles de pieux en utilisant les informations de la rigidité prédite et le comportement de la résistance au cisaillement des sols non saturés provenant de l'information des propriétés des sols saturés et de la courbe de rétention d'eau (CRE). Il y a une bonne comparaison entre les résultats de modélisation numérique et les résultats expérimentaux. Les résultats de l'étude mettent en évidence la contribution de la succion capillaire sur le comportement de chargement/déformation des fondations des pieux dans des sols à grains grossiers. L'approche numérique proposée est encourageante pour l'application de la mécanique des sols non-saturés dans la pratique du génie.

1 INTRODUCTION

In engineering practice, the bearing capacity and settlement behavior are the two key parameters that govern in the design of shallow and deep foundations. Conventionally, design of the foundations is carried out extending the principles of saturated soil mechanics. The soil above the groundwater table is however typically in a state of unsaturated condition. Georgiadis et al. (2003) stated that conventional foundations design procedures ignore the influence of suction in the unsaturated zone; the soil in this zone is in a state of saturated condition or completely dry.

Recent studies have shown that the bearing capacity of shallow and deep foundations increase significantly due to the contribution of capillary stress or matric suction (Georgiadis et al. 2003, Mohamed and Vanapalli 2006, Sun 2010, Vanapalli and Taylan 2012, Chung and Yang 2014, and Sheikhtaheri 2014). In other words, ignoring the influence of capillary stress or matric suction leads to underestimation of the bearing capacity of foundations.

In many scenarios, it is the elastic settlement which is the governing factor in the design of foundations placed in coarse-grained soils. The modulus of elasticity is typically assumed to be constant within a homogeneous soil layer regardless of the location of groundwater table. Recent studies suggest that the modulus of elasticity increases due to the contribution of matric suction. The variation of the modulus of elasticity with respect to suction can be represented as a functional relationship which can be estimated from the soil-water characteristic curve (SWCC) and the modulus of elasticity of the saturated soil (Oh et al. 2009). Studies by Oh and Vanapalli (2011) and Byun et al. (2013) suggest that ignoring the influence of capillary stress or matric suction may lead over estimation of the settlement of foundations.

In this paper, finite element analysis (FEA) is undertaken using the commercial software, PLAXIS 2D to simulate the load versus settlement behavior of three model piles extending elasto-plastic constitutive model. The estimated settlement versus load behavior are compared with those from model pile test results performed in two coarse-grained soils at three different matric suction values (i.e. 0, 2, and 4 kPa). The soil shear strength and modulus of elasticity for unsaturated conditions were derived using the semi-empirical models proposed by Vanapalli et al. (1996) and Oh et al. (2009), respectively. There is a good comparison between the numerical modeling results and the experimental results.

The results of the study presented in this paper are of interest for practicing engineers to understand the contribution of matric suction on the load-settlement behavior of pile foundations in coarse-grained soils. The proposed numerical modeling methodology is of considerable promise for implementing the mechanics of unsaturated soils in the conventional geotechnical engineering practice.

2 BACKGROUND

2.1 Pile Capacity and Shear Strength of Coarse-Grained Unsaturated Soil

Pile capacity is conventionally calculated taking account of the contribution that arises from the shaft and end bearing resistance. In saturated soils, pile capacity is related to the saturated shear strength, τ_{sat} (Eq. [1]), which is a function of effective stress, (σ - u_w), effective cohesion, c' and effective angle of internal friction, ϕ'.

$$\tau = c' + (\sigma - u_a) \tan \phi' \quad [1]$$

where σ = normal stress, and u_w = pore water pressure.

The shear strength equation for saturated soils (i.e. Eq. [1]) needs to be modified to describe the shear strength of unsaturated soils, τ_{unsat}, using two independent stress state variables, namely; net normal stress, (σ - u_a) and matric suction, (u_a - u_w) (Fredlund et al. 1978) (Eq. [2]).

$$\tau_{unsat} = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad [2]$$

where u_a = air pore pressure, and ϕ^b = friction angle due to the contribution of matric suction when (σ - u_a) is held constant.

The shear strength of unsaturated soils increases in a linear fashion up to the air-entry value (AEV), where ϕ^b = ϕ'. The soil is in a state of saturated condition up to the AEV. As the matric suction increases, the soil transits from a saturated to an unsaturated condition. During the process of desaturation, the wetted area of contact of water between the soil particles along which the suction transmits decreases significantly. Due to this reason, there is a nonlinear increase in the shear strength and the ϕ^b value is less than ϕ' (Vanapalli 2010). Figure 1 shows a 3D extended non-linear Mohr-Coulomb failure surface for an unsaturated soil.

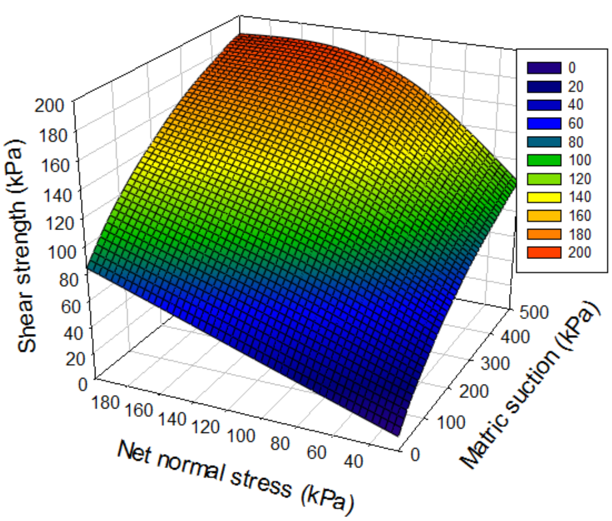


Figure 1. Extended non-linear Mohr-Coulomb failure surface

For measurement of the shear strength parameters of unsaturated soils elaborate testing equipment is required, which is expensive. In addition, trained personnel are required for conducting these tests which are also time consuming. Vanapalli et al. (1996) and Fredlund et al. (1996) proposed a model for estimating the shear strength of unsaturated soils using the saturated shear strength parameters and the SWCC as a tool as shown in Eq. [3].

$$\tau_{unsat} = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) S^{\kappa} \tan \phi' \quad [3]$$

where S = degree of saturation, and κ = fitting parameter.

Based on a large database, Garven and Vanapalli (2006) proposed an empirical equation to estimate the value of κ as a function of plasticity index, I_p (Eq. [4]). For non-plastic soils (i.e., I_p = 0), κ = 1.

$$\kappa = -0.0016(I_p^2) + 0.0975(I_p) + 1 \quad [4]$$

In Eq. [3], the contribution of matric suction towards the shear strength can be expressed as apparent cohesion, c using the relationship in Eq. [5] (Figure 1).

$$c = c' + (u_a - u_w) S^{\kappa} \tan \phi' \quad [5]$$

In this study, this concept has been utilized to predict the apparent cohesion of unsaturated sand and incorporate it into the Mohr-Coulomb model to estimate the load-settlement behavior of the model piles in unsaturated soils.

2.2 Pile Settlement and Modulus of Elasticity of Unsaturated Coarse-Grained Soil

Determination of pile foundation settlement is complex due to soil disturbance associated with the installation process and the uncertainty of soil properties along the shaft and under the tip of a pile (Vesić 1977). Poulos (1989) stated that pile settlement is governed by the ratio of the modulus of elasticity of the pile section, E_P to modulus of elasticity of the soil, E_S, ratio of modulus of elasticity of the bearing stratum, E_b to modulus of elasticity of the soil, E_S and pile length to diameter ratio L/d. Since pile stiffness is very high (i.e., pile compressibility is small in comparison to soil compressibility), in a homogeneous soil layer, modulus of elasticity of the soil would be the major factor that controls the elastic settlement.

Oh et al. (2009) analyzed load-settlement data of model footing tests for three different sands for different suction values. Their study showed that the modulus of elasticity linearly increases in the boundary effect zone (the zone in which the soil is in a state of saturated condition). In transition zone (in which the soil desaturates), the modulus of elasticity increases non-linearly up to a certain suction value and then starts decreasing. Finally, in the residual zone, the modulus of elasticity converges to a constant value which is almost the same as that of saturated condition. Based on these observations, Oh et al. (2009) proposed a semi-empirical equation to estimate the variation of modulus of elasticity with respect to suction using the modulus of elasticity for saturated condition and the SWCC along with two fitting parameters, α and β (Eq. [6]).

$$E_{i(unsat)} = E_{i(sat)}\left[1+\alpha\frac{(u_a-u_w)}{(P_a/101.3)}(S^{\beta})\right] \quad [6]$$

where $E_{i(unsat)}$ is modulus of elasticity under unsaturated condition, $E_{i(sat)}$ is modulus of elasticity under saturated condition, P_a is atmospheric pressure (i.e., 101.3 kPa), and α and β are fitting parameters. For non-cohesive soils (i.e., I_p = 0), β = 1.

Table 1 summarizes values of the fitting parameter, α from the model footings for three different types of sands.

Table 1. Fitting parameter α for model footing tests on three different sands (Oh et al. 2009)

Sand Type	Footing size	α
Coarse-grained	100 mm × 100 mm	1.5
	150 mm × 150 mm	2.5
Sollerod	22 mm × 22 mm	2.5
Lund	22 mm × 22 mm	0.5

3 TESTING PROGRAM

A series of model pile load tests were performed in two different coarse-grained soils for three matric suction values (i.e., 0, 2, and 4 kPa).

3.1 Soil Properties

Two sandy soils were used in this study; namely, Unimin. 7030 sand and Industrial sand, which are referred to as Soil #1 and Soil #2, respectively in this paper. The basic soil properties of the two sandy soils are presented in Table 2. Direct shear tests were conducted to determine the effective shear strength parameters of soils (i.e., c′, and ϕ′) and the pile-soil interface angle, δ′. The grain-size distribution curves for the two sands are shown in Figure 2. Tempe cell apparatus was used to determine the SWCCs of both sands at OMC. Figure 3 shows the SWCCs for both the soils.

3.2 Testing Methodology and Equipment Details

The load settlement behavior of three model piles (i.e. 38.3, 31.75, and 19.25 mm in diameter) in two different coarse-grained soils (i.e. Soil #1 and Soil #2) were investigated for three suction values (i.e. 0, 2, and 4 kPa). The tests were carried out in a specially designed tank of 300 mm in diameter, 700 mm in height, and 8 mm thickness. The sand was compacted in layers of 100 mm in the test tank using 1 kg hammer to achieve uniform density over the entire depth. The initial compaction water content was equal to the OMC. The model piles were placed at depth of 200 mm (i.e. embedded depth). Sand was then compacted around the pile shaft in two layers.

Prior to performing the test, the soil was saturated by increasing water table from the bottom of the tank to remove pore-air from the soils through the top surface. The water table was then lowered to achieve desired matric suction values, which were measured using conventional Tensiometers. An estimated time period of 24 - 48 hours was required to achieve equilibrium condition in the test tank with respect to the matric suction. Analyses were performed by using the concept of average matric suction value that corresponds to the matric suction value at the centroid of matric suction distribution profile between the base of pile and 1.5d (Figure 4).

Table 2. Basic soil properties of the two sandy soils

Soil Properties	Soil #1	Soil #2
Sand %	100	100
Void ratio, e	0.63	0.56
Angle of internal friction, ϕ′(°)	35.3	40.3
Effective cohesion, c′ (kPa)	0.0	0.0
Soil-Steel interface friction angle, δ(°)	24.2	33.1
Optimum water content, w_{opt} (%)	14.60	15.20
Max. Dry unit weight γ_{drymax} (kN/m^3)	16.80	17.70
Total unit weight, γ_{total} (kN/m^3)	18.60	19.90
Saturated unit weight γ_{sat} (kN/m^3)	20.40	20.80
Placed water content, w (%)	14.60	15.20
Placed unit weight γ_d (kN/m^3)	16.26	17.24

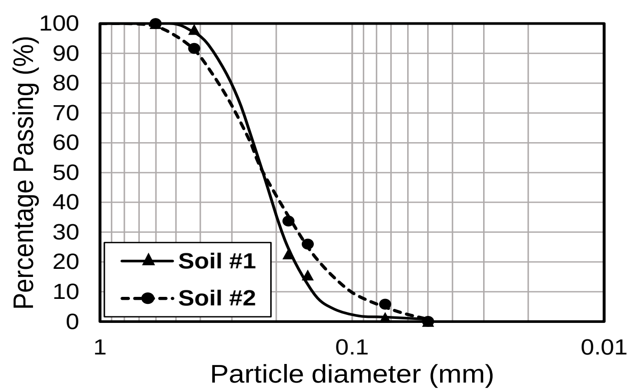


Figure 2. Grain size distribution of the selected soils (from Sheikhtaeri 2014)

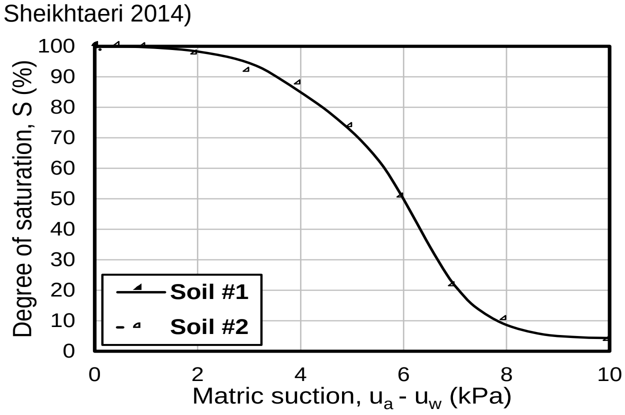


Figure 3. SWCCs for Soil #1 and Soil #2 (from Sheikhtaheri 2014)

Potts and Zdravković (2001) stated that, in practice, soil below a pile foundation is assumed to reach failure conditions when the settlement reaches 10% of pile diameter. They also added that the base capacity, however, keeps increasing and an ultimate value is reached at much higher settlement. In this study, the model pile was loaded statically up to a penetration of 20 mm (i.e., ultimate condition) with a constant penetration rate of 0.0025 m/s.

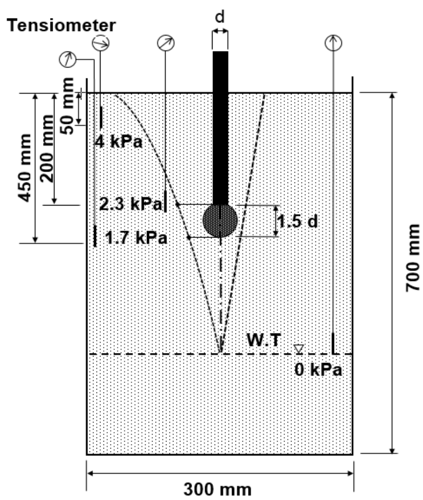


Figure 4. Variation of matric suction with respect to soil depth in the test tank under unsaturated condition of 2 kPa matric suction (modified after Sheikhtaheri 2014)

4 NUMERICAL MODELLING

Numerical analysis was carried out to simulate the load-settlement behaviors of three model piles for both saturated and unsaturated conditions using the commercial software, PLAXIS 2D. Drained condition was considered during the load-settlement simulation. The model boundaries extended to 0.7 m in depth and 0.15 m the outer boundary. The vertical boundaries were restrained in the horizontal direction, however, it was free in the vertical direction. The bottom boundary was restrained in both vertical and horizontal directions (Figure 5). The sand and the pile cluster were modeled using triangular elements with 15 nodes. The sand was modeled as an elasto-plastic material using Mohr-Coulomb model considering the dilatancy effect of the sand. The shear strength parameters (c', ϕ'), dilation angle (ψ), and modulus of elasticity ($E_{i(sat)}$) of the sands for saturated conditions were incorporated into the Mohr-Coulomb model. Dilation angle of the sand was estimated to be equal 10% of the effective angle of internal friction, ϕ' (i.e., 3.35^{o} and 4.03^{o} for Soil #1 and Soil #2, respectively) according to the recommendations of The Danish Code of Practice (D.S. 415-1984). A constant Poisson's ratio was used for both sands (i.e. $\mu = 0.334$) considering drained condition. The coefficient of earth pressure at rest condition, K_o was estimated using Eq. [7].

$$K_0 = 1 - \sin \phi' \quad [7]$$

The key parameter required for estimating the pile settlement is the modulus of elasticity of the soils. The modulus of elasticity of coarse-grained soil (e.g. sand) is significantly influenced by stress history, natural cementation, apparent cohesion due to matric suction and over consolidation ratio (Mohamed 2014). Janbu (1963) showed that there is a reciprocal relationship between modulus of elasticity value and confining stress. In the present study, the initial tangent elastic moduli for saturated condition were estimated based on the level of the confining stress and modulus of elasticity values used in previous studies for the same sand (Oh et al. 2009, Sun 2010, and Mohamed 2014). Table 3 shows the cohesion and initial tangent elastic modulus calculated using Eq. [5] and [6] considering the influence of matric suction.

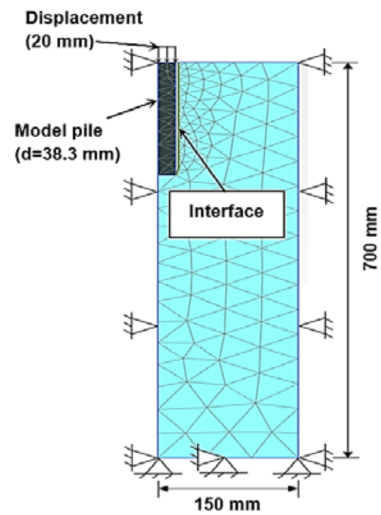


Figure 5. Details of the finite element model.

Table 3. Variation of apparent cohesion and initial modulus of elasticity with respect to matric suction

$(u_a - u_w)$ (kPa)	Apparent cohesion, c (kPa)		$E_{i(sat)}$ and $E_{i(unsat)}$ (kPa)	
	Soil #1	Soil #2	Soil #1	Soil #2
0	0	0	2000	4000
2	1.35	1.67	3900	7920
4	2.27	2.55	5200	10000

The fitting parameters, $\kappa = 1$ and $\beta = 1$ were used in Eq. [5] and Eq. [6], respectively since $I_p = 0$ (i.e. non-plastic soils). A value of $\alpha = 0.5$ provides good agreement with the measured load-settlement behavior, which is reasonable considering the diameters of the piles compared with size of model footings in Table 1. The values of degree of saturation, S were obtained from the SWCCs of the two sands (Figure 3). The model pile was modeled as a nonporous isotropic material using linear elastic model.

Only two parameters that are required to simulate the model pile; pile modulus of elasticity, E_P = 20 GPa and Poisson's ratio, μ = 0.15, were obtained from Potts and Zdravković (2001).

5 ANALYSES RESULTS

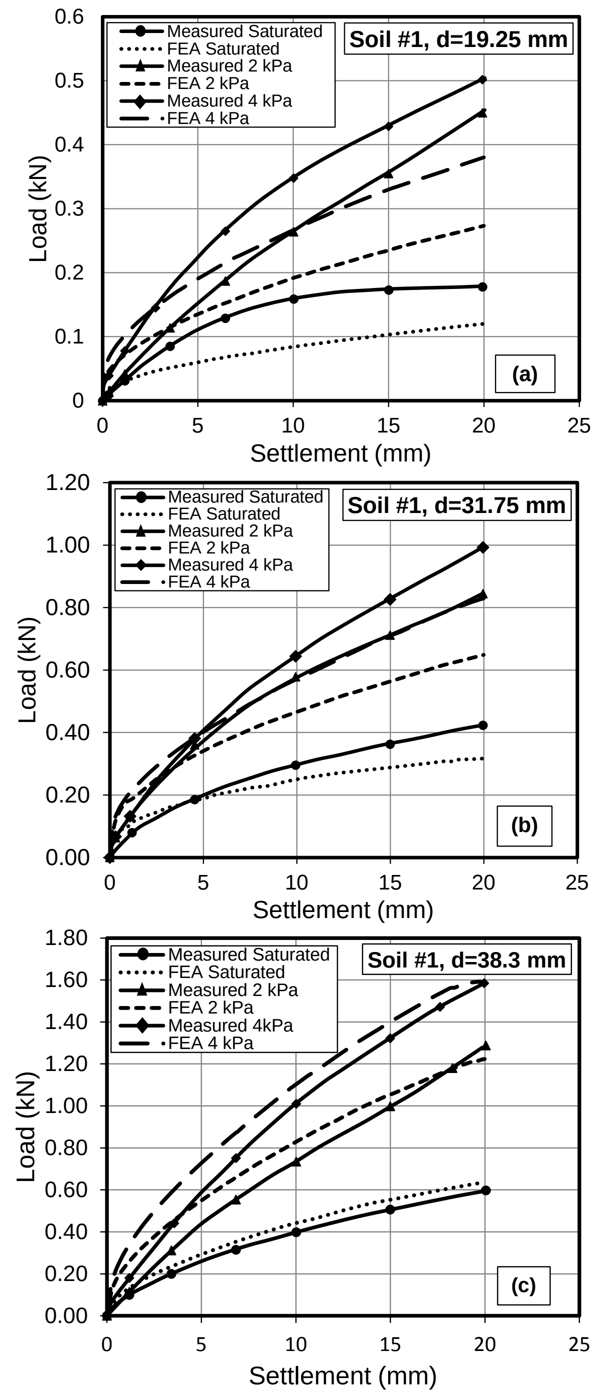
Figure 6 ((a) through (f)) shows the comparison between the measured load-settlement behavior and those estimated using the FEA for the three model piles for three different suction values (i.e. 0, 2, and 4 kPa). For both soils, high values of coefficient of determination, R^2 (Table 4) are achieved using Mohr-Coulomb model with a limited number of input parameters. The results show that the model pile capacity under unsaturated condition is much higher in comparison to the saturated condition. As the capillary suction increased from 0 kPa to 4 kPa, the pile capacity increased approximately 2 to 2.5 times in comparison to the saturated condition.

Table 4. Coefficient of determination, R^2 values for measured and predicted behavior.

$(u_a - u_w)$ (kPa)	Model pile diameters (mm) in Soil #1			Model pile diameters (mm) in Soil #2		
	19.25	31.75	38.3	19.25	31.75	38.3
0	0.956	0.969	0.998	0.988	0.994	0.991
2	0.966	0.983	0.981	0.969	0.987	0.994
4	0.987	0.989	0.988	0.989	0.986	0.993

Figure 7 shows the comparison of the variation of ultimate pile capacity with respect to matric suction for both the soils. It can be seen that the ultimate capacities increase with matric suction; however, the rate of increase is at higher rate up to the air-entry value for both sands. Once the sand starts desaturation, the rate of increase reduces and reaches a constant value at the residual suction. In Figure 8, the ratio of the measured/predicted capacity is related to the ratio of the pile diameter, d to the mean particle diameter, D_{50}. The ratio of the measured/predicted capacity of the small model piles (i.e., d=19.25 mm, and 31.75 mm) oscillates and shows no consistency for both the sands. The capacity of the larger diameter model pile (i.e., d=38.3 mm), however, is more consistent with the measured one. It can be seen that R^2 values increase as the model pile diameters increase (Figure 9). The reason for such a behavior can be attributed to the smaller diameter model piles that have access only to a limited number of contacts points with the sand grains in comparison to the larger diameter model pile. Any change in the stress state or applied load can disturb the sand under the small pile tip, which will mobilize extra settlement and reduce the bearing capacity. These observations are consistent with Gui and Bolton (1998) and Oh et al. (2009) findings. Gui and Bolton (1998) stated that particles sizes, angularity and roughness cause differences in pile tip resistance. On the other hand, Oh et al. (2009) concluded that load from small model footings is mostly carried by individual soil particles while soil under larger model footings, typically a well-defined failure plane develops. Consequently, the predicted capacity becomes more compatible with the experimental results as the pile tip surface area Increases.

The proposed model, however, has some limitations that can be seen in Figures 6(d) and 6(f). In Figure 6(d), the FEA results overestimated the model pile capacity (i.e., d = 19.25 mm.) The reason is that Soil #2 has larger grains in comparison to Soil #1. Coop et al. (2004) conducted a series of ring shear tests to investigate carbonate sand particle crushing under different shear strain levels. Their study showed that the initial and final grading are not the same after load application. Zhang et al. (2012) stated that conventional bearing capacity analysis predicts an increase in the soil resistance with an increase in the angle of internal friction of the crushable soil (i.e., coarse sand). Consequently, the expected resistance against pile penetration would be high. However, due to crushing



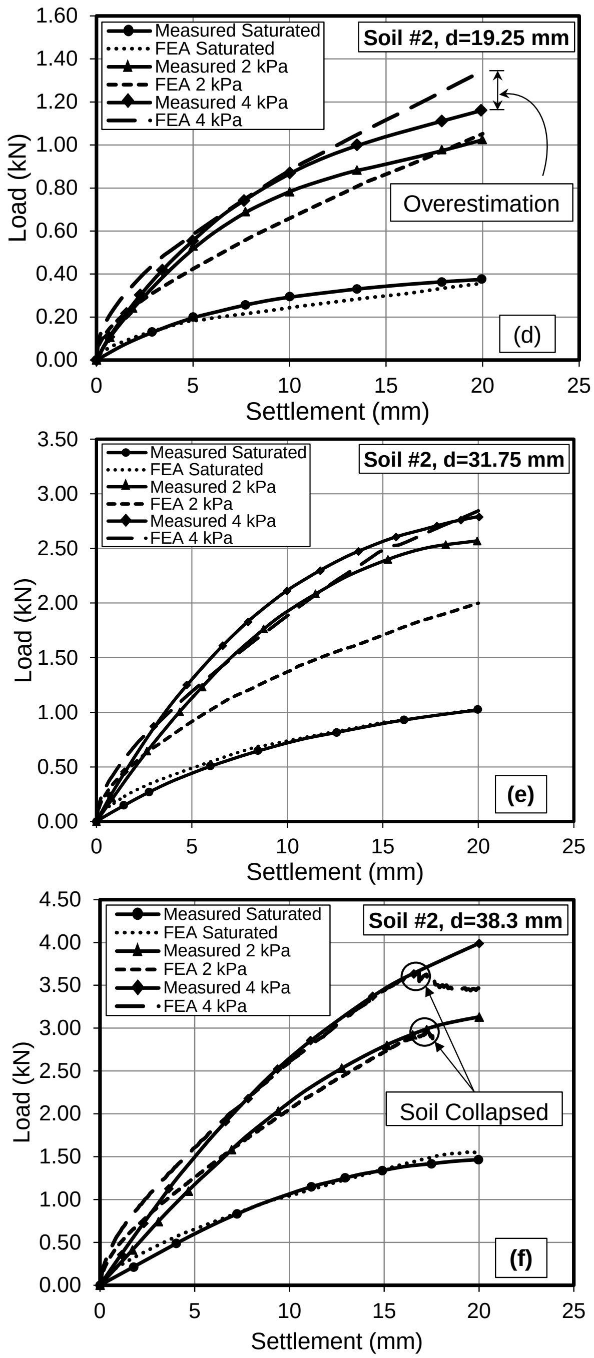


Figure 6. Results of the pile load tests and FEA using PLAXIS 2D under different saturation conditions (i.e., under matric suction of 0 kPa, 2 kPa, and 4 kPa.)

of large grains under the pile tip, the measured Mohr-Coulomb envelop is highly curved due to the friction angle, ϕ´ decreasing with an increase in normal effective stress (Yasufuku and Hyde 1995). In this case, increasing matric suction contributes to the soil stiffness increases. As a result, the stress state increases to a higher level causing larger grains crushing which is consistent with Gui et al. (1998), Bolton et al. (1999) and McDowell and Bolton (2000) findings. For this reason, the model pile experienced higher settlement and developed lower capacity than the predicted behavior. In other words, as grain crushing is not taken into account in the proposed numerical model, it contributes to an overestimation in the predicted capacity. In Figure 6(f), the FEA shows that the soil body collapses before reaching the final settlement (i.e., 20 mm). Such a behavior can be attributed to the fact that classical Mohr-Coulomb model can simulate soil as elastic perfectly-plastic material and cannot simulate the strain hardening of the sand (Potts and Zdravković 1999). For this reason, incorporating the dilatancy angle of the sand contributed to some difficulties in developing the full load-settlement curve. The numerical analysis will continue to dilate as long as the shear deformation occurs. This approach is unrealistic because once the soil reaches the critical state, no more dilation would occur in spite of increase in the shear deformation (Bolton 1986). Therefore, more sophisticated models such as the hardening soil model with small-strain stiffness is required to capture the real behavior of coarse grained soils. In hardening soil model with small-strain stiffness, dilatancy cut-off feature can be used to prevent sand dilation once the soil reaches the critical state (PLAXIS 2014).

In spite of the proposed model limitations, it provides a reasonable agreement with the measured behavior of the

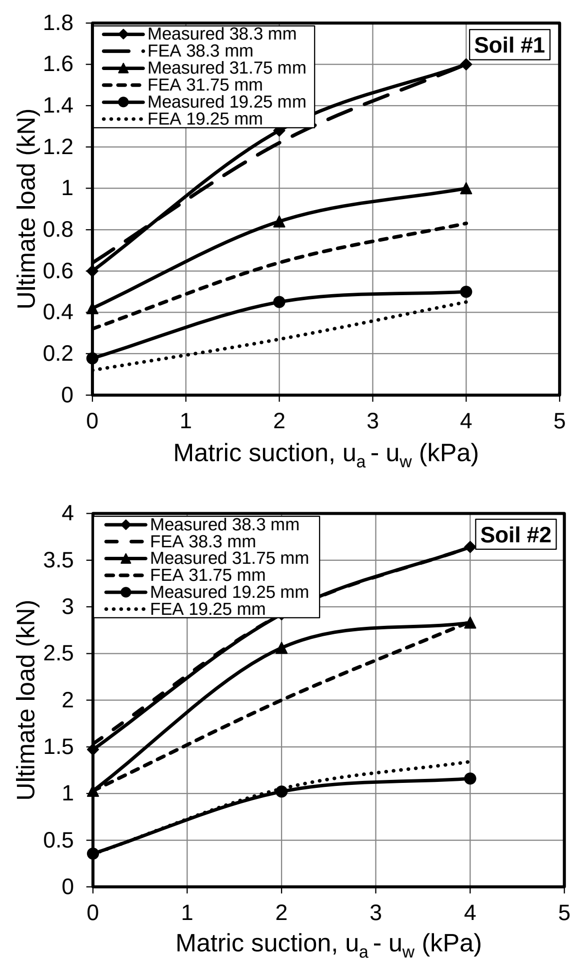


Figure 7. The variation of ultimate load with respect to matric suction, u_a - u_w for Soil # 1 and Soil # 2.

model piles under saturated and unsaturated conditions (Figure 9). Reasonably high coefficients of determination, R^2 are achieved for all model piles. In other words, the feasibility of using the Mohr-Coulomb model with limited number of input information (i.e., saturated shear strength parameters c´, and ϕ´, angle of dilation, ψ, Poisson's ratio, μ, and initial modulus of elasticity, E_i) is possible for investigating the behavior of single piles under different unsaturated conditions.

6 SUMMARY

In this study, FEA was undertaken using PLAXIS 2D to estimate the load-settlement behavior of three model piles tested with different capillary suction values of 0, 2, and 4 kPa for two coarse-grained soils. The Mohr-Coulomb model was used to simulate the behavior of the model piles using the information of the predicted stiffness, $E_{i(unsat)}$ and the apparent cohesion of unsaturated soils, c derived from the information of saturated soil properties and the SWCC.

The proposed FEA provided a reasonable comparison with the measured load-settlement behavior under different saturation conditions. The proposed numerical technique has several advantages and limitations:

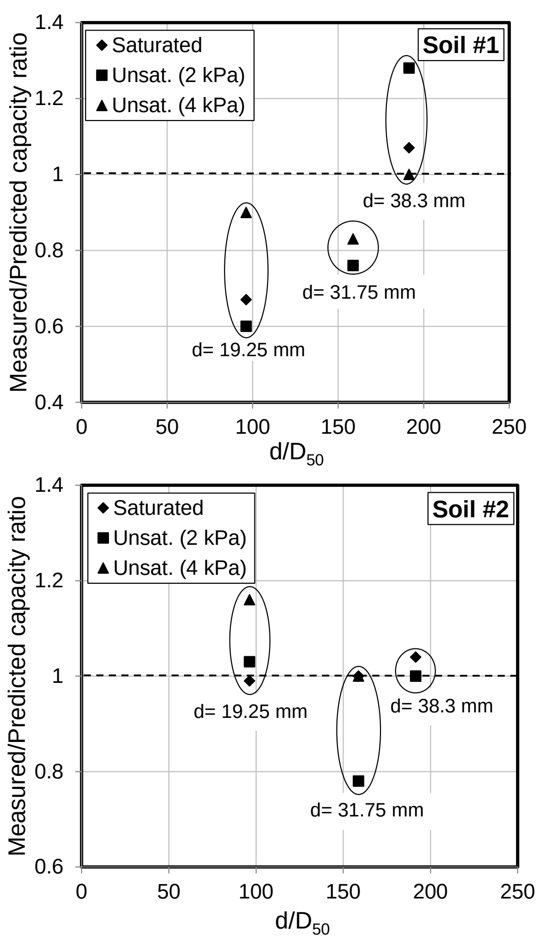


Figure 8. Scale effect of the model pile diameters and the normalized ratio of the measured to predicted capacity.

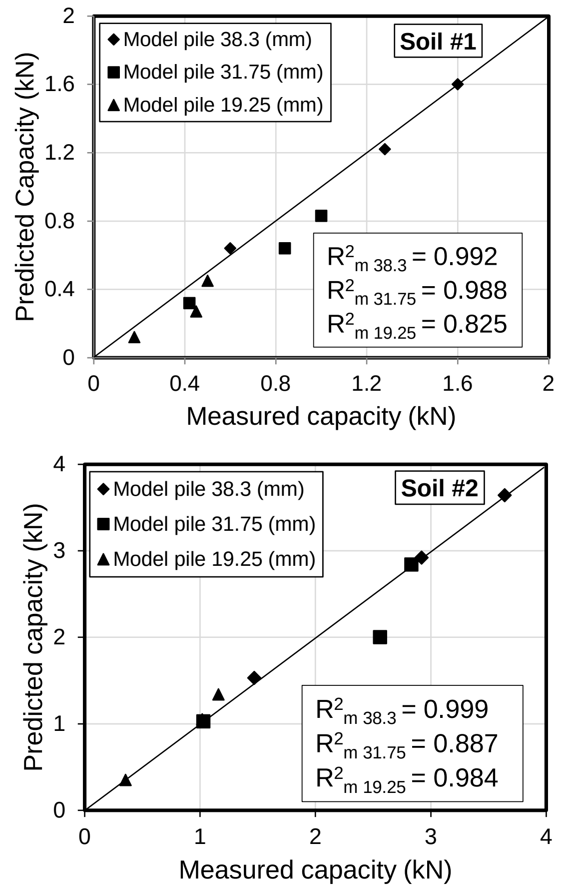


Figure 9. Comparison between the measured and predicted capacities.

- Advantages: Modelling the coarse-grained soils behavior without using complex stress-strain models is a challenge. In this study, a simple FE approach is proposed to simulate the behavior of single piles in unsaturated sand. The proposed model needs a limited number of soil parameters as input (i.e., c´, ϕ´, μ, ψ and $E_{i(sat.)}$), which can be obtained from conventional test results. The considerably long time period of testing and the need for complex experimental procedures to determine unsaturated soil shear strength and stiffness parameters can be alleviated. The predicted unsaturated stiffness and shear strength behavior from the previously introduced models can be easily incorporated into this model. In addition, there is no need for strain hardening/softening rule to define the behavior of coarse grained soil.
- Limitations: Neglecting the strain hardening/ softening rule can cause the numerical model to collapse under the applied load before reaching the ultimate capacity. Moreover, the FEA sometimes overestimates the load-settlement behavior of pile in coarse-grained soils which is attributed to grains crushing under significant loads.

Nevertheless, the proposed approach is capable to predict the load-settlement behavior of single model piles in unsaturated sandy soils. The results of the study presented in this paper are of interest to understand the contribution to matric suction towards load-settlement behavior of pile foundations in coarse-grained soils. In addition, the proposed numerical modeling methodology is of considerable promise for use in practice.

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