Anisotropy in Granular Geomaterials

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ABSTRACT
The primary objective of this study is to demonstrate that a significant degree of anisotropy may occur in particulate materials that have nearly spherical aggregates (i.e. are typically considered as isotropic) provided there is a bias in the distribution of pore space due to the initial densification process. The study includes both an experimental and theoretical component. An experimental program is conducted on dynamically compacted samples of Ottawa standard sand (C109), and incorporates a series of tests performed at different orientation of the specimens. Following the experimental part, a plasticity formulation based on the critical plane approach is presented for transversely isotropic granular media.

RÉSUMÉ
Cette étude a pour objectif principal de démontrer qu'un degré significatif d'anisotropie peut exister dans des matériaux pulvérulents composés d'agrégats quasi sphériques (généralement réputés isotropes), pourvu qu'il existe une distorsion dans la distribution de la porosité attribuable au processus de densification initiale. L'étude comporte un volet expérimental et un volet théorique. Le programme expérimental, ayant été mené sur des échantillons dynamiquement compactés de sable d'Ottawa de type C109, comprend des essais réalisés à diverses orientations des échantillons. À la suite de la partie expérimentale, une formulation de la plasticité fondée sur l'approche du plan critique est présentée pour les matériaux granulaires transversalement isotropes. Nous avons étudié les effets de l'anisotropie tant inhérente qu'induite.

1 INTRODUCTION

Many geomaterials such as sedimentary rocks and soils exhibit significant inherent anisotropy. In this case, both the deformation response and the failure mode are strongly dependent on the loading orientation with respect to their microstructure. Such anisotropy may occur in sands that comprise flat, elongated grains (Azami et al. 2010), but may also be induced by densification process in sands that have nearly spherical particles (Haruyama, 1981). In either case, mechanical characteristics at the macro-scale display a pronounced directional dependence.

The existing evidence indicates that a parallel alignment of particles is often observed in river, beach and dune sand (e.g. Oda and Koishikawa 1977) as well as artificially deposited sand (Azami et al. 2010, Symes et al. 1984). Anisotropy is clearly reflected the soil fabric composition in relation to the spatial arrangement of soil particles and voids (inherent anisotropy) and it may undergo an evolution induced by applied loads (induced anisotropy). Anisotropy in granular materials affects both the deformation response and the conditions at failure (Lam and Tatsuoka 1988, Yamada and Ishihara 1979). Consequently, it has a significant impact on the behavior of geotechnical structures (such as tunnels and foundations, slopes, retaining walls, etc.) and should be adequately accounted for in the context of design and/or stability analysis.

The objective of this study is to introduce a methodology, with both experimental and theoretical components for characterizing the anisotropic response of granular materials that comprises nearly spherical particles. In particular, the anisotropic response of Standard Ottawa sand C109 has been examined.

The content of this paper is organized as follows. In section 2, an experimental program is described that it designed to investigate the mechanical properties of Ottawa standard sand (C109), in which the structural anisotropy of the soil skeleton is generated by the initial densification process. The program involves a series of direct shear as well as triaxial tests. Following the experimental part, a plasticity formulation based on the critical plane approach is presented for describing the directional dependency in the mechanical behavior. Section 3 concludes and summarizes the results of this work.

2 ANISOTROPY IN STANDARD OTTAWA SAND (C109)

2.1. Experimental Program

The inherent anisotropy in sands is known to occur if the grains are flat or elongated. The primary objective of the experimental program carried out here is to demonstrate that anisotropy may also occur in sands that have nearly spherical particles (i.e. are typically considered as isotropic) provided the distribution of pore space has a preferred orientation due to the initial densification process.

In this investigation, a series of direct shear as well as conventional triaxial tests were conducted on samples prepared at different orientations relative to the direction of deposition. The standard soil used throughout the test program was Ottawa sand C109. This is commonly used quartz sand composed of rounded to sub-rounded particles passing the #16 sieve (1.18 mm) and retained on the # 200 sieve (0.075 mm). Figure 1 and 2 provide the particle size distribution curve and the image of particles shape of Ottawa sand.

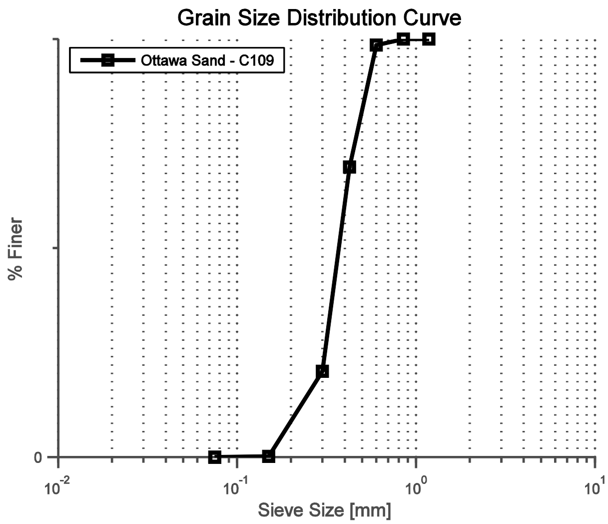

Figure 1. Grain Size distribution Curve for Ottawa Sand C109

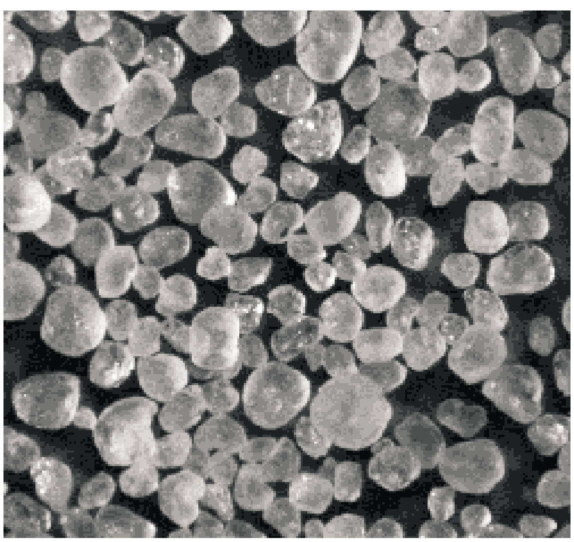

Figure 2. Image of ASTM graded Ottawa sand C109 (http://www.nrcresearchpress.com/doi/pdf/10.1139/t00-031)

Direct shear tests: In order to investigate the directional dependency of shear strength parameters, a series of direct shear tests was conducted on samples prepared at different orientations of material axis. The direct shear tests were performed on dry samples, with a constant displacement rate of $0.2\ mm/\min$ at three different levels of normal stress $\sigma_n = 10,\ 50,\ 75\ kPa$. The initial void ratio of all specimens was approx. 0.52.

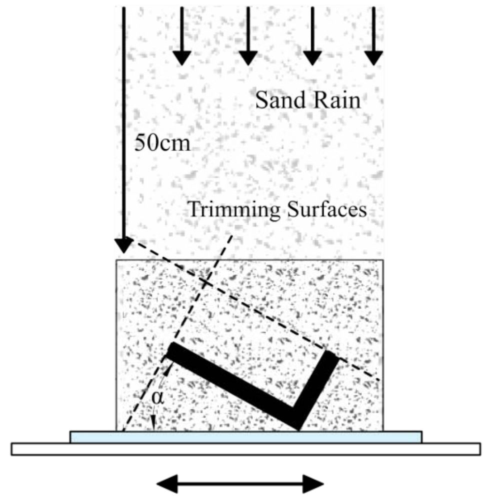

Figure 3. Schematic diagram of sample preparation and definition of α

The main stages of the sample preparation are summarized schematically in Figure. 3. In order to prepare samples with different orientation of material axes, a modified version of the shear box was used (Guo 2008). The modification included direct shear box is obtained by cutting off one side of the standard direct shear box and making this side removable. The first stage of sample preparation involved mounting a container on a shaking table and then placing the modified direct shear box at the desired orientation, inside this container. The samples were prepared using the technique of raining which involves pluviating the sand through a grid into the container. The height of fall of sand particles during pluviation process was 500 mm with respect to the top of the container. The sand grains were slowly poured until the container was full. It was then covered with a lid and a dead load of 2 kg was applied at the top. The set up was then vibrated to produce specimens of desired initial void ratio. The specimens were vibrated with the amplitude of 5.3mm (peak-to-peak) at 5 Hz for 40 minutes. After vibration, the extra material was carefully removed, the sample was trimmed and the removable side parts were reassembled. The deposition angle α , is defined here as the angle between the sand rain direction and the direction normal to the plane of isotropy. The deposition angles were taken as $\alpha = 0^{\circ}, 15^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 90^{\circ}$.

In spite of nearly spherical shape of particles, the material exhibits a significant degree of anisotropy, as both the strength and deformation characteristics are affected by the angle of deposition. The variation of shear strength and the corresponding evolution of volume change for $\sigma_n = 10\ kPa$ are presented in Figure 4. It should be noted that the results of direct shear tests have been used to identify the material parameters/functions for the constitutive model described later.

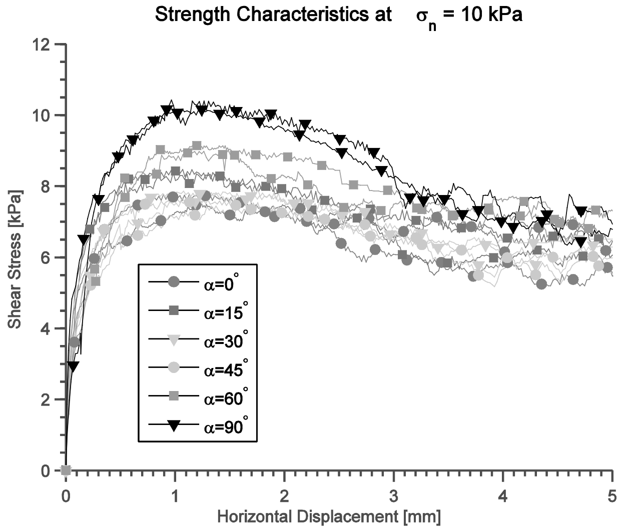

Figure 4. Mechanical response at normal stress of $\sigma_n = 10\ kPa$ for different values of α; Shear stress vs. horizontal displacement characteristics.

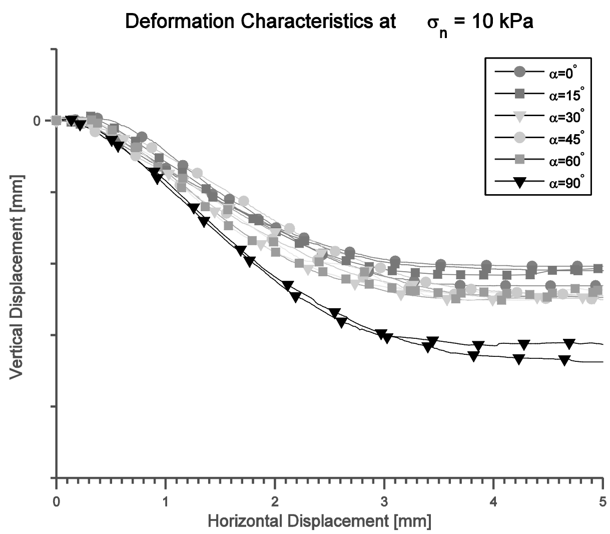

Figure 5. Mechanical response at normal stress of $\sigma_n = 10\ kPa$ for different values of α; Evolution of volume change.

Triaxial tests: A series of drained triaxial compression tests were conducted on sand samples prepared at two different orientations of material axes (i.e. $0^\circ, 90^\circ$). The samples were prepared in a $100 \times 50 \times 50\ mm$ rectangular prism mold. Analogous to the direct shear samples, the triaxial samples were prepared by the sand rain method using a constant falling height of 500 mm. It is noted that in triaxial test configuration, for intermediate values of alpha the sample has tendency to distort under the increasing axial load. Such distortion is constrained by the presence of loading platens. Consequently, the stress field is no longer uniform and the results do not reflect the behavior of the material. Therefore, the triaxial tests were conducted on horizontal and vertical samples only.

In order to preserve the composition of the fabric, the specimen with the mold was first submerged in water. Then, excess water was drained out under gravity to obtain a moist specimen with low moisture content. The entire setup was then frozen. The frozen specimen was subsequently transferred to the base of the triaxial cell and covered with rubber membrane. A small confining pressure was then applied while unfreezing the specimen. Finally the specimen was saturated and consolidated under a hydrostatic pressure prior to the shearing phase. To minimize the end friction effects two layers of membrane with silicon grease in between were placed between the sample and loading platens. Proper cuts were made on the membranes to allow both seepage through the sample and free lateral expansion of the sample.

The results of drained triaxial tests, including the deviatoric and volumetric characteristics at initial confinement of $50\ kPa$, are presented in Figure 6. The results correspond to vertical and horizontal samples. The samples tested at $\alpha = 0^\circ$ display larger initial shear stiffness and a much higher strength.

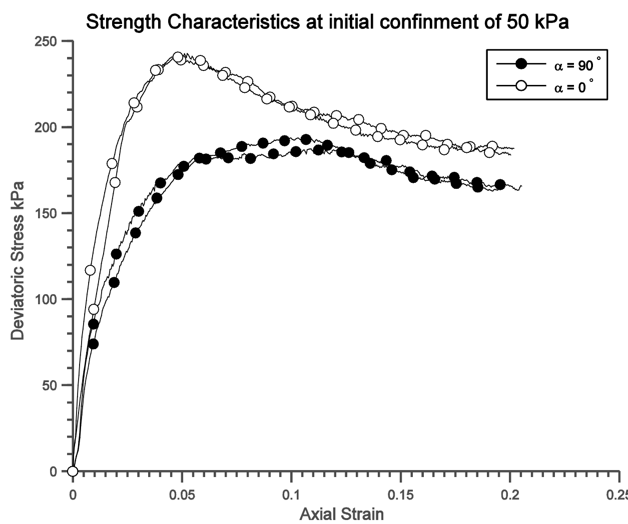

Figure 6. Results of triaxial tests at confining pressure $P = 50\ kPa$; Variation of deviatoric stress, $\sigma_1 - \sigma_3$ with axial strain

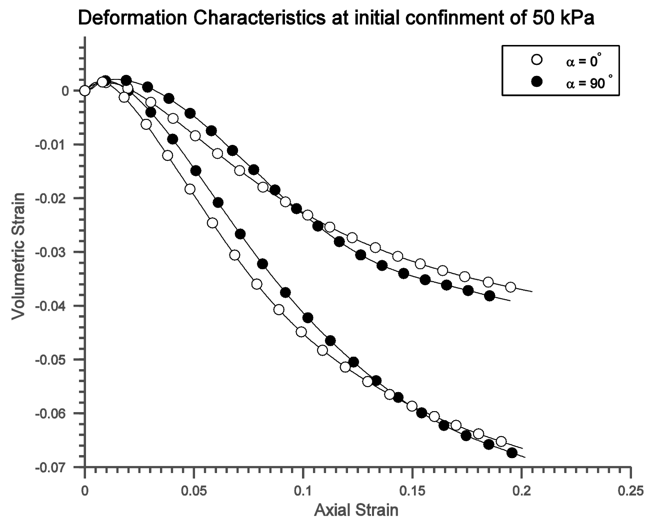

Figure 7. Results of triaxial tests at confining pressure $P = 50\ kPa$: volumetric strain with axial strain

Figure 7 presents the evolution of volumetric strain with increasing axial deformation. It is evident that the material undergoes initial compaction followed by dilation. Based on the experimental results obtained, it may be concluded that the stress strain behavior of dense sand strongly depends on the loading direction. Although the material tested here, i.e. Ottawa sand has nearly rounded particles, a marked inherent anisotropy is observed, which stems from a bias in the distribution of void space generated through the sample preparation procedure.

2.2. Mathematical Framework

Inherent anisotropy: In this study, the critical plane approach, outlined in the article by Pietruszczak and Mroz (2001), is employed. In the critical plane approach, the failure function is defined in terms of traction components acting on a plane with unit normal n_i . The simplest representation takes the form

$$F = f\ (\tau, \sigma_n) - c(n_i) \qquad [1]$$

where

$$\tau = \left| \sigma_{ij} n_i s_j \right|;\ \sigma_n = \sigma_{ij} n_i n_j \qquad [2]$$

and s_i is a unit vector orthogonal to n_i , such that $n_i \cdot s_i = 0$. The approach consists now of finding the orientation of a so-called critical or localization plane, for which the failure functions, reaches a maximum, i.e.

$$\underbrace{\max}_{n_i, s_i}(F) = 0 \qquad [3]$$

Consider a Coulomb's material,

$$F = \tau - \eta_f \sigma - c \qquad [4]$$

for which, $c = 0$, and

$$\eta_f = \eta_0 (1 + \Omega_{ij} n_i n_j + a_1 (\Omega_{ij} n_i n_j)^2 + ...) \qquad [5]$$

In this Equation, Ω_{ij} is a symmetric traceless tensor which describes the spatial variation of η_f .
Equation (5) represents a constrained optimization problem for specifying orientations. The problem can be solved by constructing a Lagrangian function.

Within the critical plane framework, the irreversible deformation is attributed to sliding/separation along an infinite set of randomly orientated planes. The inelastic deformation is then accounted for by invoking an appropriate plasticity framework; this approach is conceptually similar to the so-called multi-laminate framework (Pande and Sharma 1983).

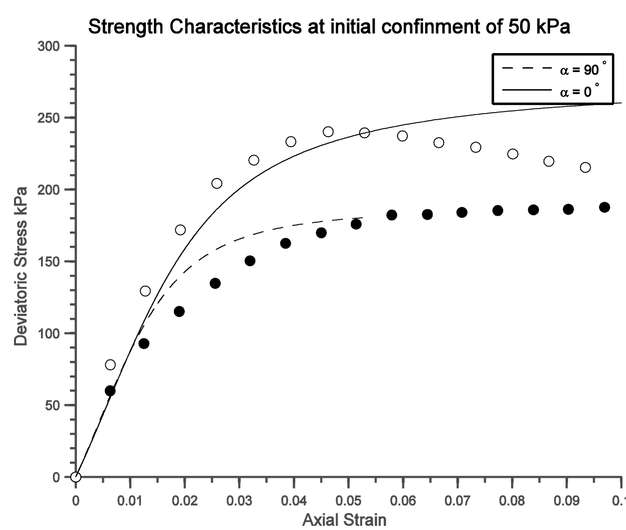

Figure 8 Results of triaxial compression tests at $P = 50\ kPa$: stress-strain characteristics

In this study, deviatoric hardening framework has been employed. Note that the results of direct shear tests have been used to identify the material parameters for the constitutive model just described.

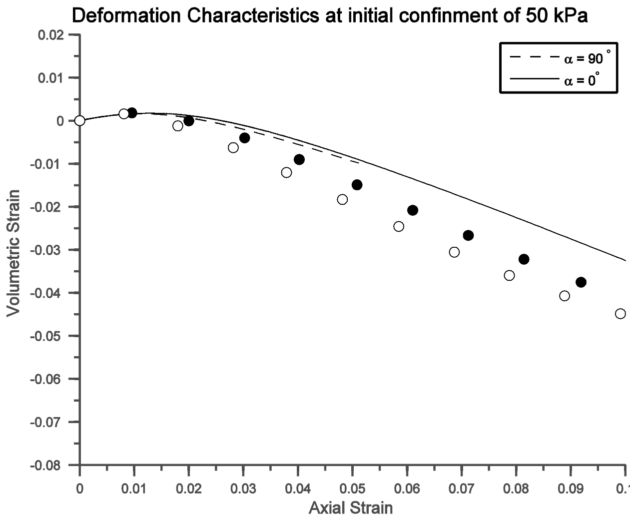

Figure 9. Results of triaxial compression tests at $P = 50\ kPa$; evolution of volume change

Figures 8 and 9 show the results of simulations of tests conducted at initial confining pressures of $P = 50\ kPa$ for the deposition angles of $\alpha = 0^{\circ}, 90^{\circ}$. The results include the deviatoric stress-strain characteristics, as well as the evolution of volume change. The simulations do not involve the unstable strain softening response, as the latter is associated with the onset of localized deformation and should be analyzed at the level of a boundary-value problem. It is evident that the obtained results, in terms of strength and volumetric characteristics, are fairly consistent with the experimental evidence. In particular, the predicted maximum level of the deviatoric stress intensities is close to the observed values. Also, the evolution of volume change shows a transition from a compaction to dilation, while the quantitative response is not very sensitive to the orientation of the sample, which is also consistent with the experimental observation.

Induced anisotropy: In order to capture the effects of induced anisotropy, an evolution law, which imposes coaxiality between the microstructure and the total strain tensors, is incorporated into the mathematical framework. In this case, the influence of induced anisotropy on the strength characteristics is described by the variation of the anisotropy parameter η_f , as defined by Equation (5).

$$\eta_f = \eta_0 (1 + \Lambda_{ij} n_i n_j + a_1 (\Lambda_{ij} n_i n_j)^2 + ...) \qquad [6]$$

The tensor Λ_{ij} which governs the bias in spatial distribution of strength parameter η_f is defined as

$$\Lambda_{ij} = \beta e_{ij} \qquad [7]$$

where e_{ij} denotes the deviatoric strain tensor which evolves during deformation and β is a constant.

The results of the analysis conducted for parameter $\beta = 0.1$ are shown in Figures 10 and 11. The tests were simulated for values of $\Delta\Psi = 0^{\circ}, 45^{\circ}, 90^{\circ}$. It is noted that $\Delta\Psi$ denotes the magnitude of rotation of principal stress directions with respect to the orientation of the previous principal stress. The results clearly demonstrate that the proposed formulation can account, at least in a qualitative manner, for various manifestations of this form of anisotropy in granular materials.

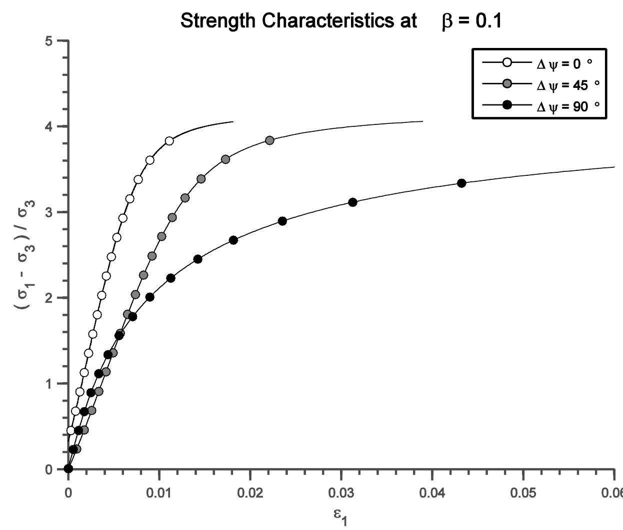

Figure 10 Effects of previous loading on induced anisotropy: $\Delta\psi = 0^{\circ}$, $\Delta\psi = 45^{\circ}$ and $\Delta\psi = 90^{\circ}$ with constant $\beta = 0.1$: strength characteristics

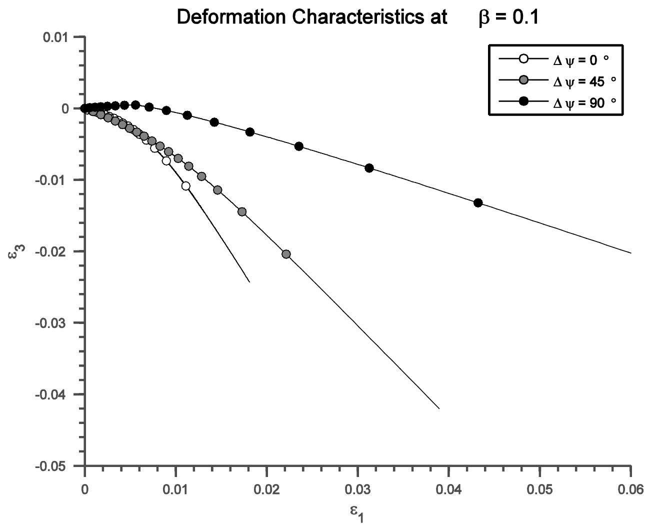

Figure 11 Effects of previous loading on induced anisotropy; $\Delta\psi = 0^{\circ}$, $\Delta\psi = 45^{\circ}$ and $\Delta\psi = 90^{\circ}$ with constant $\beta = 0.1$: deformation characteristics

3 CONCLUSIONS

The experimental investigations were conducted on Ottawa standard sand C109 that comprises nearly rounded particles. To induce a distinct microstructure, a sand rain method followed by a densification process was used for sample preparation which enabled uniform anisotropic specimens to be produced at the selected void ratio. The material tests involved direct shear and triaxial experiments and were aimed at examining the effects of inherent anisotropy on both the deformation and strength characteristics.

A general constitutive framework was outlined which is capable to model the effects of inherent anisotropy in particulate media. The incorporation of the proposed evolution law into this framework results in a constitutive law that can also describe, at least in a qualitative manner, manifestation of induced anisotropy in granular materials. The methodology presented here, can be incorporated into finite element framework for solving complex boundary value problems that involve granular materials with inherent/induced anisotropy. It is well known that, in practical geotechnical problems such as, tunnel excavation, slopes, foundations, etc., continuous rotation of principal stress directions occurs and has a considerable influence on deformation and strength behavior of soil mass.

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