Poisson's ratio effect of slope stability calculations

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ABSTRACT
This paper presents the results of a study on the effect of varying Poisson's ratios on limit equilibrium slope stability analyses performed using the dynamic programming search technique. An example model is completed to show the impact of Poisson's ratio on the strain vectors, the critical slip surfaces and factor of safety. The results showed that decreasing Poisson's ratio causes the location of the critical slip surface, selected by the dynamic programming method, to move deeper into the slope, leading to a higher factor of safety. The critical slip surface is selected based upon kinematic admissibility criteria. The results of the method were also shown to become numerically unstable as cohesion approaches zero if adjustments to the kinematic admissibility criteria are not made. Therefore, care needs to be taken when analyzing cohesionless soils with low values of Poisson's ratio.

RÉSUMÉ
Cet article présente les résultats d'une étude sur les effets de la variation du coefficient de Poisson sur les analyses de stabilité de la pente d'équilibre limite, en utilisant la programmation dynamique. Un exemple de modèle est complété pour montrer les impacts du coefficient de Poisson sur les vecteurs de contrainte, les surfaces de glissement et le facteur de sécurité. Les résultats démontrent que la diminution du coefficient de Poisson cause l'emplacement d'une surface de glissement critique, sélectionnée par la méthode de programmation dynamique, pour progresser plus bas dans la pente, ce qui conduit à un facteur de sécurité plus élevé. La surface de glissement critique est choisie sur la base de critères d'admissibilité cinématiques. Les résultats de la méthode peuvent devenir instables numériquement, lorsque la cohésion tend vers zéro, si les ajustements du critère d'admissibilité cinématique ne sont pas faits. Par conséquent, une attention particulière doit être portée lors de l'analyse de sols non-cohésifs à faible coefficient de Poisson.

1 INTRODUCTION

The results presented in this paper are intended to clarify the role of Poisson's ratio when using the dynamic programming technique under certain modeling conditions. The influence of the Poisson's ratio appears to be magnified under certain conditions as illustrated in Figure 1.

This figure raised the following issues with regards to a slope stability analysis:

- The effect of the Poisson's ratio appears to have an adversely large effect on the location of the critical slip surface.
- The effect of the Poisson's ratio appears to have an adversely large effect on the calculated factor of safety (F_s).
- In the example problem used, the classic slope stability methods predict a factor of safety close to 1.0 and a critical slip surface near to the ground surface. However, this behavior is not observed in this figure when using a low Poisson's ratio's?

The Poisson's ratio effect was noted when undertaking a slope stability analysis on waste rock material. It is important to determine the reason for the variation of the computed factor of safety, F_s, because of its potential influence on the calculation of the F_s when dealing with cohesionless materials. There also appears to be a tendency in geotechnical engineering practice to ignore the cohesion component of the soil and analyze situations with cohesion set to zero. This study directly comments on the use of such practices.

2 ANALYSIS

The use of the dynamic programming method can be viewed as comprising two separate processes, namely:

- A searching algorithm (i.e., the dynamic programming method)
- The algorithm calculating the factor of safety (i.e., the limit equilibrium analysis)

Any critical review of the dynamic programming procedure should be studied by separating the algorithm into the two components and asking the following questions:

- Is the LOCATION of the critical slip surface reasonable and correct?
- Is the CALCULATION of the factor of safety for that location reasonable and correct?

It must be noted that the definition of the factor of safety used for the dynamic programming method is the same as the definition used for classic limit equilibrium methods. It should also be noted that a finite element stress analysis is used as the basis for determining the shear stress and the normal stress on the slip surface.

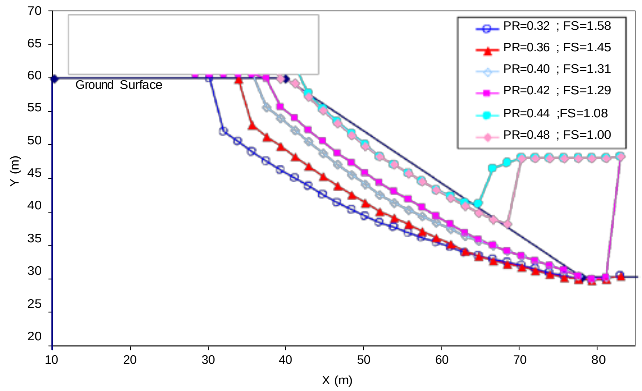

Figure 1 Effect of varying the Poisson's ratio on the location of the slip surface when cohesion = 0.0 kPa

2.1 Model Description

The examined model in this situation is defined by the following properties.

Cohesion	= 0 kPa
Friction angle	= 40 degrees
Slope angle	= 38 degrees
Poisson's ratio	= between 0.32 and 0.48

It is significant to note that the frictional properties of the defined material (i.e., zero cohesion) are representative of a coarse and relatively dense sandy material. As the Poisson's ratio approaches 0.5, the material exhibits no overall volume change upon deformation. As the Poisson's ratio decreases, the lateral expansion becomes less than the vertical deformation. The deformation behavior (and Poisson's Ratio effect) for the material can be seen in Figure 2.

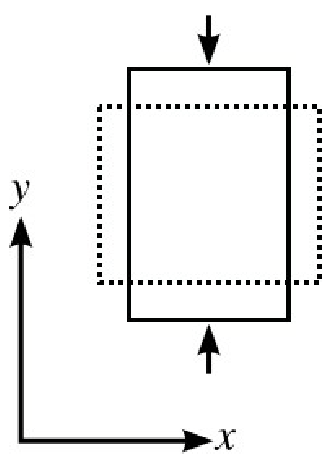

Figure 2 Illustration of the behavior of a material with a Poisson's ratio equal to 0.5

The definition of Poisson's ratio is as follows.

$$\nu = -\frac{\varepsilon_{trans}}{\varepsilon_{axial}} = -\frac{\varepsilon_x}{\varepsilon_y} \qquad [1]$$

Where υ=Poisson's Ratio, ε_x=strain in the x direction and ε_y=strain in the y direction. Values of the Poisson's ratio for typical materials are shown in Table 1.

Table 1 Typical values of Poisson's ratio

Material	***Poisson's Ratio***
rubber	~ 0.50
saturated clay	0.40-0.50
magnesium	0.35
copper	0.33
aluminum-alloy	0.33
clay	0.30-0.45
sand	0.20-0.45
concrete	0.2
glass	0.18-0.3
foam	0.10 to 0.40
cork	~ 0.00

2.2 What Effect Does Poisson's Ratio Have?

Let us consider the following question, "What changes in the slope stability analysis when the Poisson's Ratio is changed?" The primary effect of a change in Poisson's ratio is the ratio between the vertical and horizontal strains. If a linear elastic analysis is performed, then the strain vectors change as shown in Figures 3, 4, and 5 for varying values of Poisson's Ratio.

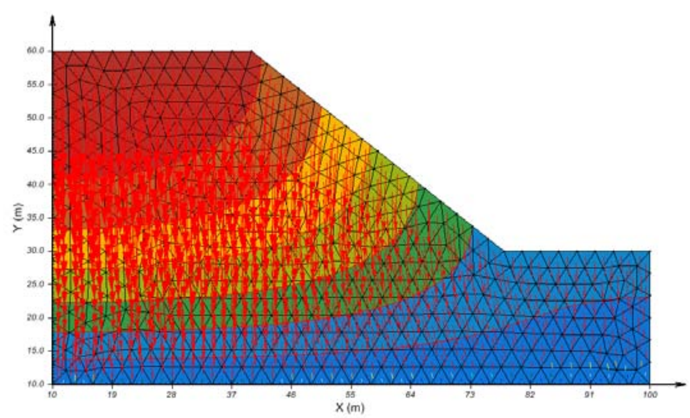

Figure 3 Strain vectors for Poisson's ratio equal to 0.20

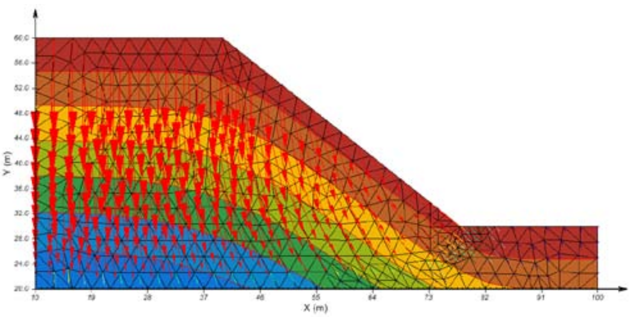

Figure 4 Strain vectors for Poisson’s ratio equal to 0.32

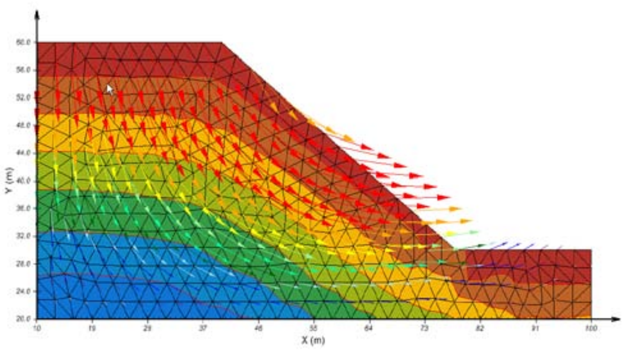

Figure 5 Strain vectors for Poisson’e ratio equal to 0.48

Figures 3, 4, and 5 show that the lateral strain vectors are significantly increased as Poisson’s ratio is increased toward 0.48. Stated another way, the lateral direction of strain introduced by the choice of Poisson’s Ratio is significantly diminished by using low values of Poisson’s ratio. It would appear, based on visual examination of the strain vectors that a lower Poisson’s ratio would lead towards a lesser tendency for slope failure. It should also be noted that the dynamic programming method only gives consideration to stresses and not strains as far as the calculation of the factor of safety is concerned.

2.3 Is the Calculated Factor of Safety Correct?

The first priority in addressing the slip surfaces in Figure 1 is to determine whether or not the calculated factors of safety arecorrect for each of the presented slip surfaces. Each of the slip surfaces was examined by stepping through the calculations as well as comparing each analysis to results obtained from a fully specified slip surfaces using the limit equilibrium method of slices. It was found that the factors of safety presented in Figure 1 are the same for each presented slip surface once its location was fixed.

An important question to ask regarding each of the slip surfaces would appear to be, “Is each of the slip surfaces a true minimum?” The examination should not center around whether the correct factor of safety is being calculated for each scenario but rather, “Is each slip surface truly the slip surface with the minimum factor of safety?”

2.4 When May There be a Problem?

Further studies of the issues associated with dynamic programming have been undertaken both by SoilVision Systems Ltd. (2008) and Stianson (2008). The results of these studies have indicated that the location of the critical slip surfaces is changed when Poisson’s ratio is changed but this only occurs when the slip surfaces are passed through cohesionless materials. The problem was illustrated by Stianson (2008) by plotting various analysis relative to the stability number, *N*, as defined below:

$$N = \frac{c'}{\gamma H \tan\phi'} \quad [2]$$

where *c’* is cohesion, γ is the unit weight, *H* is the slope height, and φ' is the angle of internal friction. It was noted from the equation that a high stability number indicates a soil with significant cohesion, such as clay, whereas a soil with a low stability number indicates a soil with little cohesion, such as sand. The differences between published factors of safety and the dynamic programming factor of safety are illustrated in Figure 6. These differences are generally less than 5% except for the case of zero cohesion.

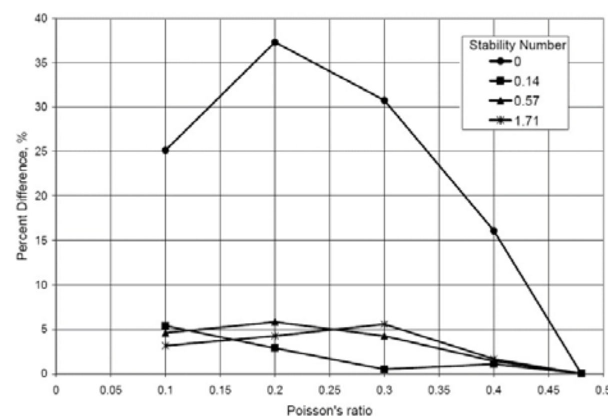

Figure 6 Relationship between Poisson's ratio and the factor of safety (Sianson, 2008)

A comparison of the dynamic programming, DP, method results and the Morgenstern-Price method of slices was studied for a typical homogenous slope (Figure 7). In this case, the stability number, *N*, was high. In this scenario there was a 4% difference between the Dynamic Programming method and limit equilibrium method as Poisson’s ratio reached lower values such as 0.10.

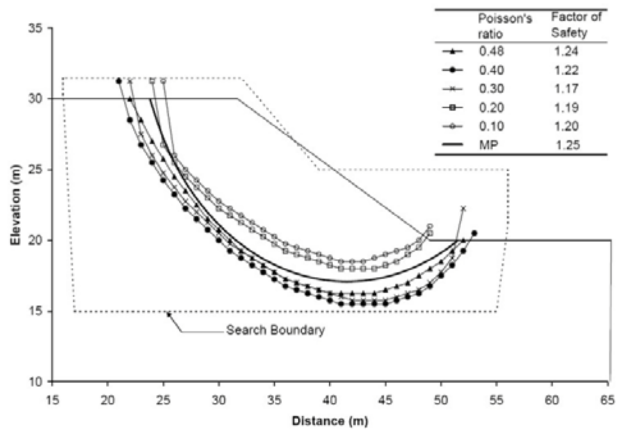

Figure 7 Family of slip surfaces where the stability number is equal to 1.71 (Stianson, 2008)

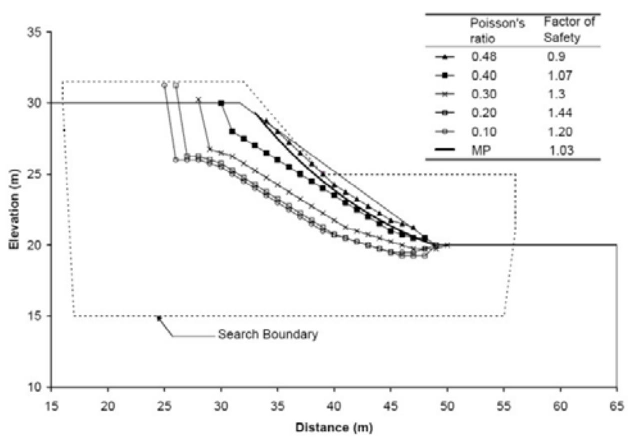

Figure 8 Family of slip surfaces for the case where the stability number is equal to zero (Stianson, 2008)

A similar comparison is presented in Figure 8 where the stability number is equal to zero (cohesionless material). This scenario is also compared to the limit equilibrium method of slices analysis. It can be seen that the computed slip surface location and the computed factors of safety converge to similar values for Poisson's ratios between 0.40 and 0.48. Differences between the two methodologies diverge up to a maximum of 17% when Poisson's ratio is lowered to 0.10.

It should therefore be noted that the current anomaly with the dynamic programming method only occurs in cohesionless soils with a selected Poisson's ratio below 0.40-0.45. It is also interesting to note that the traditional methods of slices have difficulty in analyzing this particular scenario and the analysis becomes numerically unstable. From a purely theoretical standpoint the critical slip surface rises to the ground surface. In the method of slices it is common in engineering practice to introduce a small amount of cohesion to the soil in order to bring numerical stability to the analysis.

It should also be noted that it is possible that values of Poisson's ratio between 0.40 and 0.48 will produce similar answers to the calculated location for the critical slip surface and the computed factors of safety. Further study is warranted on this issue.

2.5 The Kulhawy Link

The Kulhawy method (1969) provides a critical link between classic "method of slices" techniques and the more recent stress-based methods. Stresses from a linear elastic stress analysis are used as the basis for calculating the shear and normal stresses at the base of each slice in the Kulhawy method. The method makes use of all other aspects of the method of slices and is therefore valuable with respect to the examination of the transition from the "method of slices" procedure to the stress-based analysis.

A number of slopes were studied using the SVSLOPE (SoilVision Systems Ltd., 2008) slope stability software program. In each scenario the cohesion component was slowly removed from the material being studied. In each case, a number of different Poisson's ratios were considered. The results can be seen in Figures 9, 10 and 11. A circular slip surface searching method was utilized in each case to determine the critical slip surface.

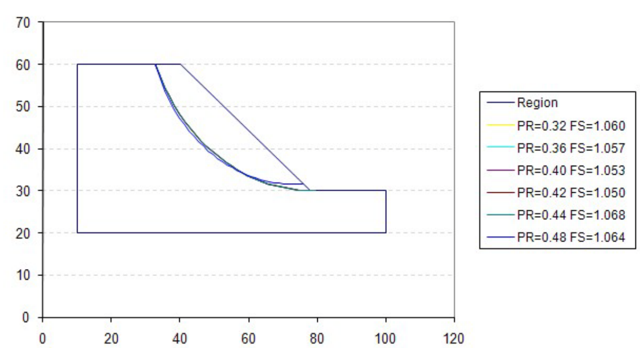

Figure 9 Results with the Kulhawy (1969) method with a friction angle of 20 degrees and cohesion equal to 30 kPa

It can be noted that the "classic" understanding of the slip surface rising to the ground surface and the calculated factor of safety show quite small differences for different values of Poisson's ratio as long as there is some cohesion component. For the scenario with the cohesion component equal to zero, (Figure 11) it can be seen that the LOCATION of the critical slip surface is consistently at the surface; however, the computed factors of safety show significant variance (i.e., 0.201 to 0.528). The question which remains regarding the Dynamic Programming method is the issue related to the location of the critical slip surface. In other words, why does the location of the computed critical slip surface, (CSS), go deeper into the slope in Figure 1 when the Poisson's ratio falls below 0.42?

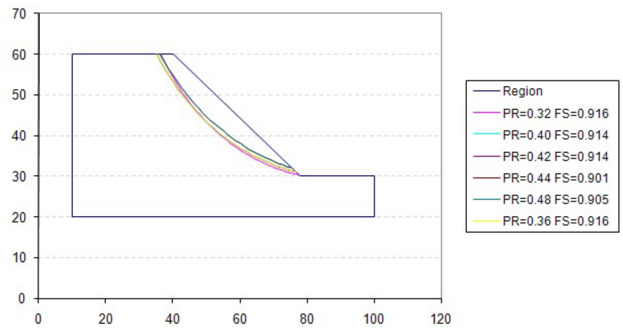

Figure 10 Kulhawy (1969) method with an angle of internal friction equal to 25 degrees and a cohesion component of 10 kPa

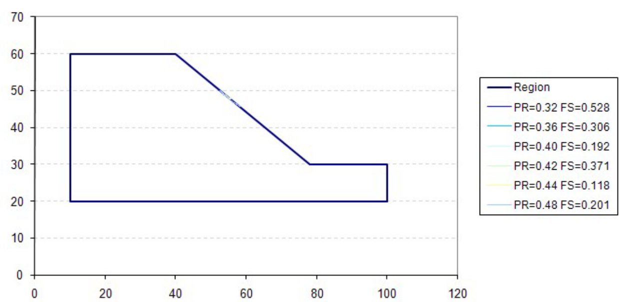

Figure 11 Kulhawy (1969) method with an angle of friction of 40 degrees and a cohesion component equal to 0 kPa

2.6 Kinematic Admissibility Criteria

Thousands of individual line segments are analyzed within a Dynamic Programming analysis in order to determine the most likely failure surface path. In order to reduce the total number of line segments analyzed, a number of kinematic admissibility criteria are applied to "throw out" line segments as being unreasonable. Insight into the performance of the Dynamic Programming method can be seen by plotting kinematically admissible line segments. Such line segments can be seen in the following plot where Poisson's ratio is equal to 0.48 and the stability number is equal to 1.71 (i.e., high cohesion).

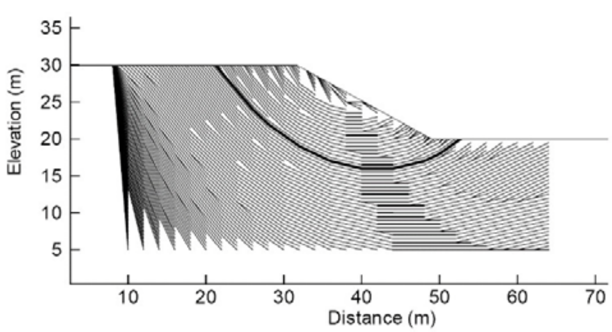

Figure 12 Admissibility plot for the case where Poisson's ratio is equal to 0.48 and the stability number is equal to 1.71 (Stianson, 2008)

From this plot it can be seen that in the location of the critical slip surface there are a number of connected and valid line segments from which to form a slip surface.

Alternatively the number of kinematically admissible line segments for the "zero cohesion" case can be plotted as shown in the following figure.

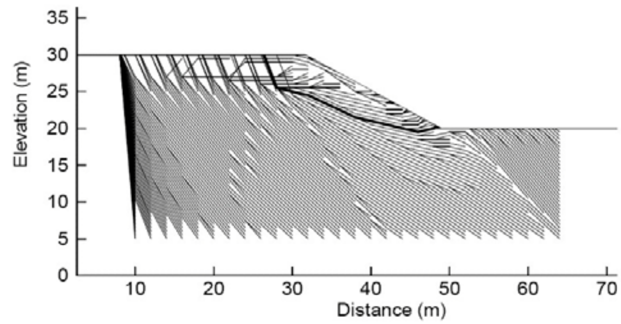

Figure 13 Admissibility plot for the case where Poisson's ratio is equal to 0.1 and the stability number is equal to 0 (Stianson, 2008)

From Figure 13 it can be seen that there is some uncertainty in the kinematically admissible line segments on the entering of the critical slip surface into the slope. This uncertainty can lead to the inability of the slip surface to remain shallow in certain situations. Therefore the location of the critical slip surface in specific scenarios when cohesion is equal to 0.0 and Poisson's ratio is unnaturally low can lead to an unrealistic location for the critical slip surface as computed by the Dynamic Programming method.

3 SUMMARY

From the analysis presented in this paper it is possible to draw a number of conclusions:

1. The Dynamic Programming method of analysis works well in materials with cohesion and there is less than a 5% difference in the factors of safety computation when compared to limit equilibrium results for various Poisson's ratios.
2. The introduction of a stress analysis utilizing various Poisson's ratio opens up new possibilities from an analysis standpoint, in comparison to the classic limit equilibrium methods of slices.
3. All current analytical methods become somewhat unstable when analyzing cohesionless materials.
4. The Dynamic Programming method has a tendency to suggest a critical slip surface which is too deep when analyzing cohesionless soils with a Poisson's ratio less than 0.48.
5. The reason for this deep critical slip surface appears to be linked to kinematic admissibility

assumptions in the Dynamic Programming method.

REFERENCES

Kulhawy, F. (1969). “Finite element analysis of the behaviour of embankments,” Ph.D. Thesis, University of California, Berkley, CA, USA.

Stianson, J.R., 2008, “A Three-Dimensional Slope Stability Method Based on Finite Element Stress Analysis and Dynamic Programming”, University of Alberta, Edmonton, AB, Canada

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