Numerical Investigation of the Bearing Capacity of Ring Foundations on Inhomogeneous Clay

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ABSTRACT
Although the bearing capacity of ring foundations has been addressed in the literature, this is currently limited to drained soil behaviour only. Ring foundations are commonly used for supporting cylindrical structures such as oil tanks, bins, hoppers, bridge piers and silos. With more construction of large cylindrical structures occurring, ring foundations have the potential to become a very economical solution. The time and construction cost reduction of ring foundations compared to circular footings has motivated engineers to look for more guidance for the determination of the bearing capacity of ring foundations. This paper presents the results of a numerical investigation of the performance of ring footings on soft clay soils with increasing shear strength with depth. The study includes investigation of the effect of the footing diameter ratios and soil strength increase on the undrained bearing capacity factor N_c. This research has been conducted using finite element analysis with the software PLAXIS. The variation of Nc is shown to vary with both the geometry and soil properties. The research has both practical application to the design of circular foundations and academic interest related to the interaction of other closely spaced foundation systems on clay soils.

RÉSUMÉ
Bien que l'étude de la capacité portante des fondations annulaires ait été abordée dans la littérature, elle est actuellement limitée à l'analyse du comportement drainé du sol. Les fondations annulaires sont couramment utilisées pour soutenir les structures cylindriques, telles que les réservoirs d'huile, les trémies, les piliers de ponts et les silos. En raison de l'accroissement du nombre de constructions de grandes structures cylindriques, les fondations annulaires ont le potentiel de devenir une solution très économique. La réduction des coûts et de la durée de la construction des fondations annulaires par rapport aux fondations sur semelles circulaires a incité les ingénieurs à définir des orientations pour la détermination de la capacité portante des fondations annulaires. Cet article présente les résultats d'une analyse numérique de la performance de telles fondations sur des argiles molles dont la résistance au cisaillement augmente avec la profondeur. L'étude comporte une évaluation de l'effet du rapport du diamètre de la fondation et de l'accroissement de la résistance non drainée du sol sur le facteur de capacité portante Nc. Cette recherche a été menée en effectuant une analyse par éléments finis avec le logiciel PLAXIS. On montre que le facteur Nc varie en fonction de la géométrie du système et des propriétés du sol. Cette recherche a des applications pratiques pour la conception des fondations circulaires et présente aussi un intérêt académique lié à l'interaction avec d'autres types de fondations peu espacées sur sols argileux.

1 INTRODUCTION

Ring foundations are used for supporting cylindrical structures such as tanks, bins, hoppers, bridge piers and other similar structures. Silos and oil tanks in particular are a common example of the use of ring footings as a foundation for the overlaying cylindrical shaped structures. Today with increased construction of huge cylindrical structures, the use of ring foundations has become a very economical solution to minimize the time and the cost of foundation construction, as an alternative to circular or mat footings.

The bearing capacity of ring foundations on granular soils has been addressed to some extent in the literature but very little research has been done to date for clay soils (Al Massri and Newson, 2015). In particular, the behaviour of ring footings on clays with inhomogeneous strength with depth has not been reported at all. This paper presents the results of numerical investigations of the effect of shear strength increasing with depth on the bearing capacity factor (N_c) for ring footings on clay.

2 BACKGROUND

In general, the undrained vertical bearing capacity for shallow foundations has been studied comprehensively for both strip and circular footings on homogenous clay with uniform soil shear strength with depth (eg. Prandtl, 1921; Shield, 1955; Eason & Shield 1960). In addition, research has been conducted for non-homogenous soil with linear shear strength increase with depth (eg. Davis and Booker,1973; Houlsby & Wroth, 1983).

The bearing capacity of shallow foundations for a general-shear failure mechanism (Figure 1) is found from the well-known equation [1] below:

$$q_{ult} = c\, N_c + q\, N_q + 0.50\, \gamma\, B\, N_\gamma \qquad [1]$$

Where (q_{ult}) is the ultimate bearing capacity, (γ) is the soil unit weight, (c) is soil cohesion, (B) is the footing width, and (N_c, N_q and N_γ) are bearing capacity factors, which are functions of the material properties and geometry.

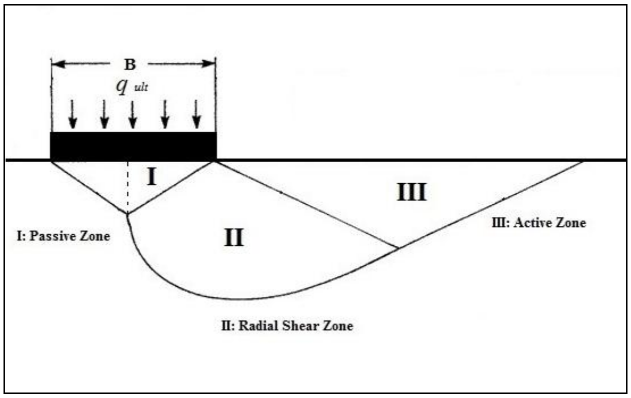

Figure 1. General shear failure mechanism of shallow foundation with no embedment depth (Terzaghi, 1943)

For a surface strip foundation on a clay soil the bearing capacity can be calculated as shown in equation [2].

$$q_{ult}= C_u \,.N_c \qquad [2]$$

The bearing capacity factor (N_c) takes the value ($2+\pi$) based on the Prandtl (1921) solution for this case.

The plasticity solution of the bearing capacity of a continuous strip footing on inhomogeneous strength soil is shown in equation [3] (Davis and Booker, 1973):

$$q_{ult}= F\,[\,(2+\pi\,).\,C_{uo} + kB/4] \qquad [3]$$

Where F is a dimensionless factor depending on the ratio kB/C_{uo}, k is the gradient of increase of shear strength with depth (z) and C_{uo} the undrained shear strength at the surface. The shear strength (C_u) in this study was assumed to vary linearly with depth according to the relationship shown in (Figure 2).

A range of solutions for (N_c) have been presented for strip & circular foundations for inhomogeneous strength clays by Gourvenec and Randolph (2003). In common with Houlsby and Wroth (1983), they found that the ratio of the bearing capacity for strip and circular foundations decreases as (kD/C_{uo}) increases. This was found to be due to the contraction of the central wedge or core of the failure mechanism (Davis and Booker, 1973). The reported values for (kD/C_{uo}) of 0, 1 and 6 give ranges of (N_c) from 5.91-6.05, 6.77-6.95 and 9.56-9.69 respectively (Gourvenec and Randolph, 2003; Cox et al., 1961; Houlsby and Wroth, 1983; Martin, 2001).

The interaction of closely spaced strip footings on clay has been investigated by a number of researchers. For example, theoretically by Stuart (1962) who studied the interference effect of two strip footings on their ultimate bearing capacity. Experimental studies have also been conducted using small-scale model tests (eg. Das and Larbi-Cherif, 1983; Kumar and Saran, 2003). Upper bound limit analysis combined with finite elements has been investigated (eg. Kumar and Kouzar, 2008).

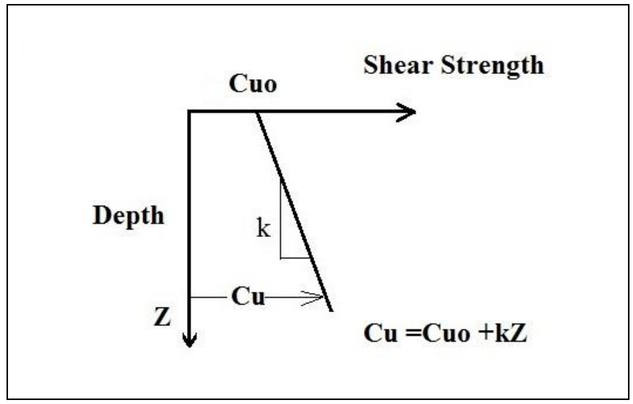

Figure 2. Definition of shear strength increase with depth.

Further research was completed by Martin and Hazell (2005) who investigated the bearing capacity of parallel strip footings on clay using the method of characteristics.

Recently the bearing capacity of ring foundations has received further attention and research has been completed to determine the bearing capacity of ring foundations for drained soil conditions (eg. Zhu and Fanyu, 1998; Laman and Yildiz, 2003; Boushehrian and Hataf, 2009), but no analysis for undrained ring foundations has been conducted to date (Al Massri and Newson, 2015).

3 METHODOLOGY

The methodology used in this study was split into two main approaches: i) modeling two parallel continuous strip footings with the same width (B), and a specified space between the two footings (S) as shown in (Fig.3a) and ii) a ring footing having the same diameter (D) as the strip footing width (B) and with the same spacing as the strip footing space (S) as shown in (Fig. 3b).

The two models were investigated using the same soil parameters considering the following assumptions: inhomogeneous soil (with shear strength increasing linearly with depth), no embedment and undrained soil conditions. The footings were assumed to be rigidly connected and a rough surface is assumed between the soil and the edge of the footing.

For the strip footing case a parametric study was conducted to investigate the effect of spacing ratio (S/B) on the bearing capacity of the soft clay. The same models were adopted for the axisymmetric case, which is a ring footing and the numerical simulation was repeated for (S/D) keeping all dimensions and soil parameters the same. The degree of inhomogeneity was represented by the non-dimensional groups (kB/C_{uo}) or (kD/C_{uo}) for each case. The values of these dimensionless groups were taken as 0 (homogeneous), 1 and 5. The soil was modeled with undrained shear strength of C_{uo}=20 kPa, soil unit weight $\gamma=\gamma_{sat}$=20 kN/m^3, modulus of elasticity E was variable and Poisson's ratio υ=0.485.

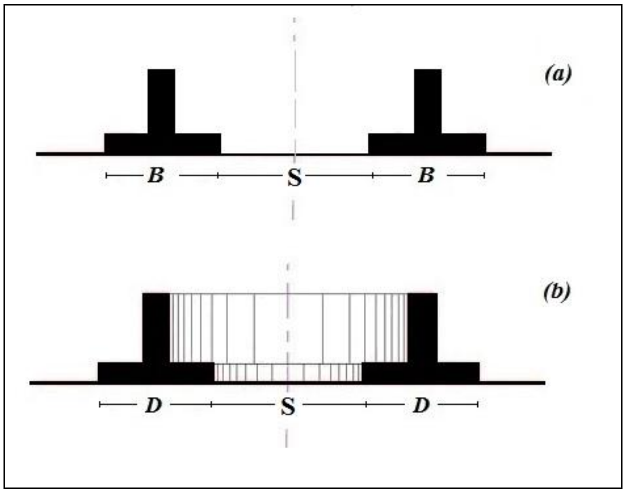

Figure 3. Ring and strip footing model cases investigated.

The soil undrained stiffness modulus (E_u) was varied with depth relatively to the undrained shear strength giving an undrained stiffness ratio (E_u/C_u = 150). The loading effect of the foundation was modelled with uniform vertical nodal displacements, simulating a rigid foundation of width (B, D).

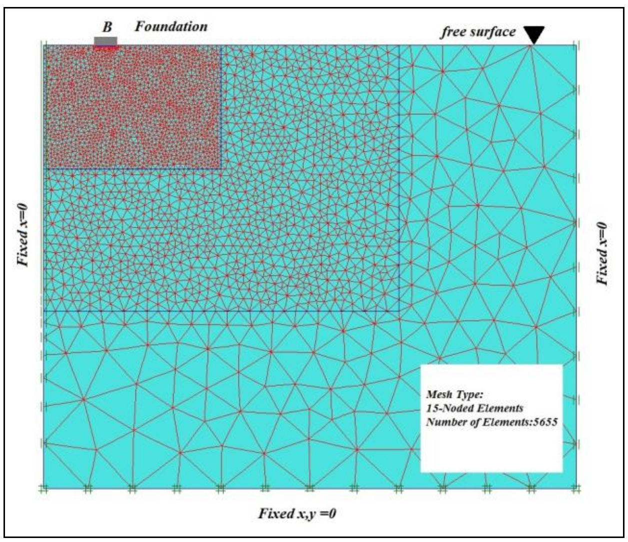

Figure 4. Typical finite element mesh showing elements, nodes and fixities.

A typical mesh is shown in (Figure. 4) with the boundary conditions and fixities. Analyses were repeated with increasing mesh refinement until converged results were found. A typical mesh had 5655, 4th order 15 node triangular elements. The soil model adopted for analysis was a Mohr-Coulomb perfectly plastic yield surface with isotropic linear elasticity.

4 NUMERICAL RESULTS ANALYSIS

Initially the strip footing results were plotted against the results of Martin & Hazell (2005) to compare the findings of the bearing capacity for parallel strip footings on inhomogeneous clay for a rough footing as shown in (Figure. 5a and 5b). The efficiency term as presented in Martin & Hazell (2005) refers to the ratio of the overall (group of footings) bearing capacity to the sum of the individual (isolated footings) bearing capacities. The numerical results for the two parallel strip footings show a good agreement between the Martin & Hazell (2005) findings for the two values of kB/C_{uo} (0 and 5) as shown in (Figure 5a and 5b). For the homogenous strength case, the efficiency is seen to peak at S/B=0.15, with a 7% increase. For the inhomogeneous case the efficiency reduced from a 30% increase at S/B=0. In both cases the efficiency returns to 1.0 within S/B =0.5. More detailed investigations of the changes in N_c for the strip and ring footings are shown in Figure 6. These results show greater variations in N_c for the 2D cases compared to the axisymmetric cases. The values of N_c for large spacing are comparable with those reported in Section 2 for circular foundations. As was found by Houlsby and Wroth (1983), as the inhomogeneity increases, the ratio of N_c for the strip and circular foundation reduces. This occurs for both low and high values of S/B and S/D.

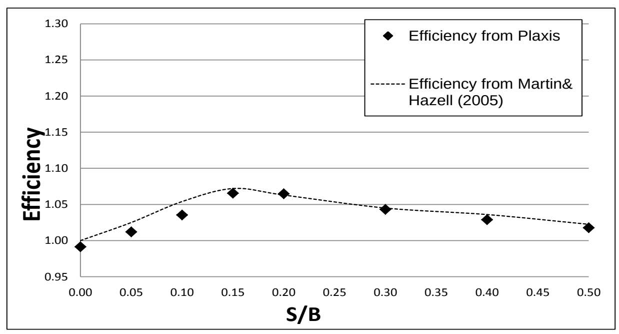

Figure 5 a. Efficiency for strip footing with kB/C_{uo}=0

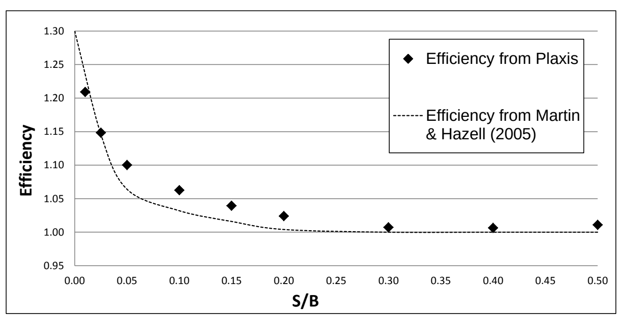

Figure 5b. Efficiency for strip footing with kB/C_{uo}=5

The failure mechanisms for two parallel strip footings are shown in (Figure. 7). These illustrate the effect of varying S/B (0 and 0.4) on the failure surfaces. Increasing spacing of the footing (S/B) shows a transition from a Prandtl mechanism with a single wedge beneath the foundation (Figure.7(a)) to a Pseudo-Hill type mechanism where a secondary wedge and radial fans extend beyond the edge of the footing towards the line of symmetry (Figure.7(b)). These mechanisms eventually show the reducing interaction between the two foundations and when S/B>0.8 the footings behave independently. Figure

8 shows the failure mechanism with zero spacing (S/B) for two parallel strip footings with kB/C_{uo}=5, with a typical Hill type mechanism, such as would be found for a rough strip footing (Tani and Craig, 1995). The failure mechanisms for ring footings are presented in (Figure.9) for kD/C_{uo}=0, for S/D=0 and 0.4. Comparing the mechanism with that of the strip footing shows a similar shape, but with a relatively larger radial fan and a smaller passive wedge. With a spacing of S/D=0.4 the mechanism is similar to that in Figure 7 (b), but has less lateral extent and a larger radial fan with smaller wedges.

Figure 10 shows the mechanisms for inhomogeneous ring footing case. The mechanism for zero spacing with (S/D=0) is a similar Hill type mechanism to Figure 8, but is shallower than that for the strip footings. With a spacing of S/D=0.4 it has a similar form to the homogenous case in Figure 9, but with a smaller wedge beneath the footing and shallower, more evenly sized radial fans. The transitions between the different mechanisms show how the values of N_c are affected by the depth and lateral extent of the deforming soils. Generally, greater spacing and stronger soils with depth decreases and increases N_c respectively.

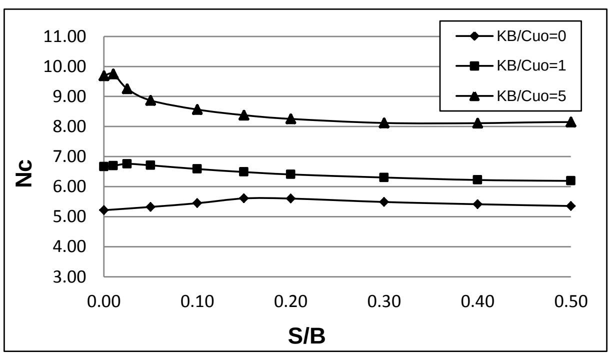

Figure 6a. Results for two parallel strip footings showing the effect of varying (S/B) on bearing capacity factor (N_c).

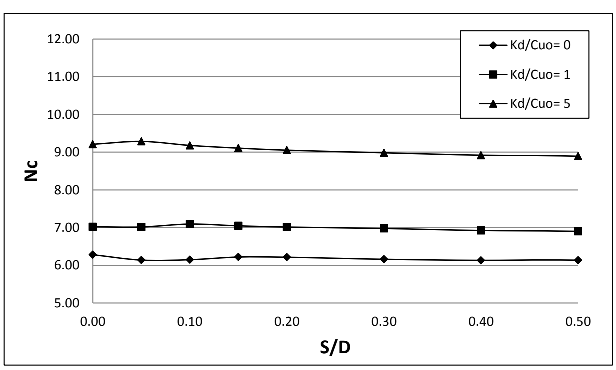

Fig. 6b: Results for a ring footing showing the effect of varying (S/D) on bearing capacity factor (N_c).

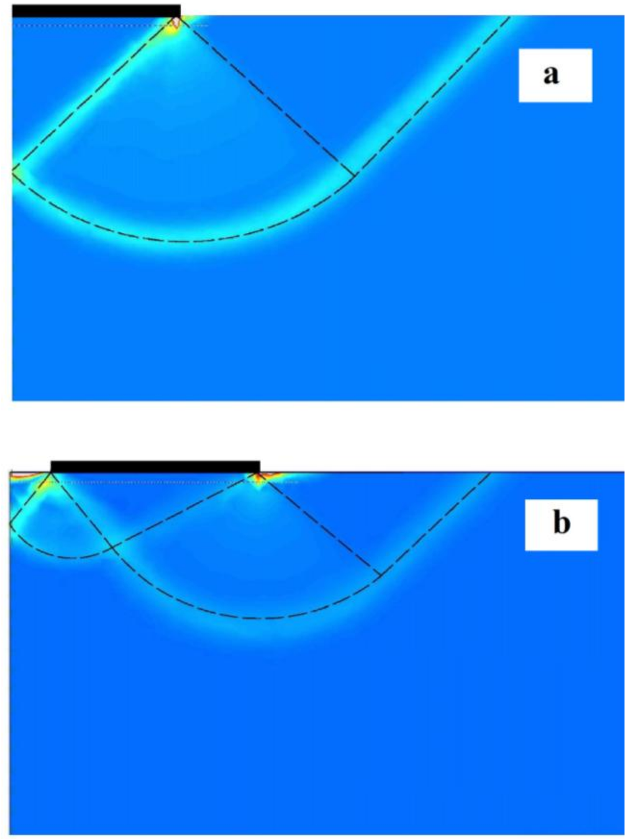

Figure 7. Incremental strain contours and assumed failure mechanisms for two parallel strip footings for kB/C_{uo}=0 showing the effect of variation of S/B (a) 0 and (b) 0.4.

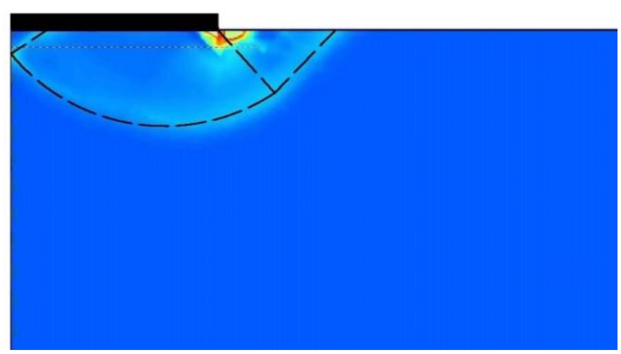

Figure 8. Incremental strain contours and assumed failure mechanism for two parallel strip footing for kB/C_{uo}=5 and of S/B (zero).

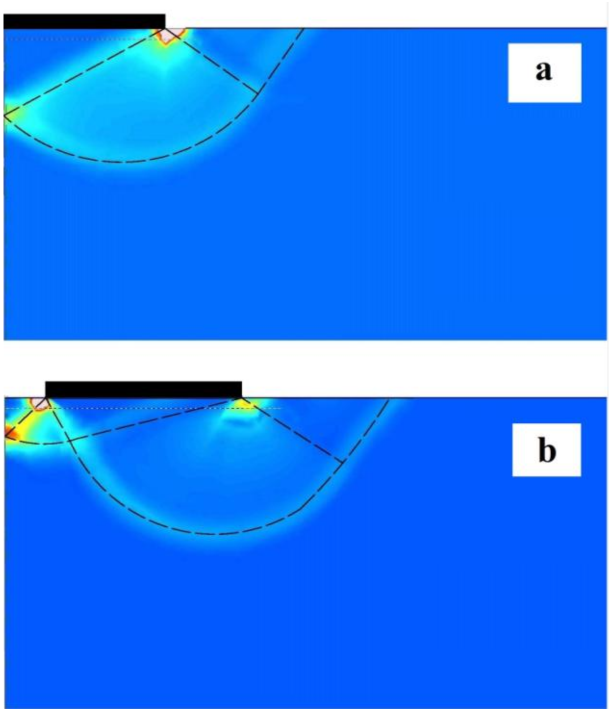

Figure 9. Incremental strain contours and assumed failure mechanisms for the axisymmetric case for $kD/C_{uo}=0$ showing the effect of variation of S/D (a) 0 and (b) 0.4

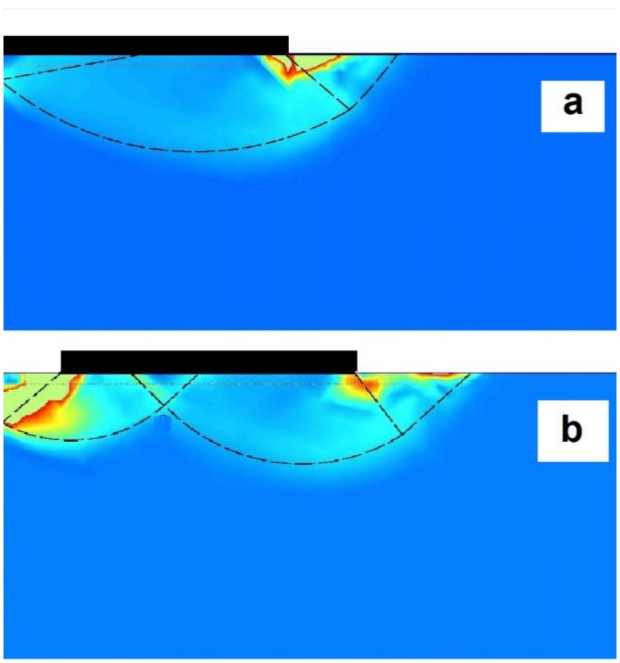

Figure 10. Incremental strain contours and assumed failure mechanisms for the axisymmetric case for $kD/C_{uo}=5$ showing the effect of variation (S/B=0 and 0.4) on the failure surface.

It should be noted that the final estimate of bearing capacity of these types of foundations must take into account two additional effects: the greater capacity of a circular footing on soil with a given (φ) and the reduced (φ) of the soil in an axisymmetric model compared to the plane strain case (Meyerhof, 1963).

CONCLUSIONS

This paper has presented a numerical investigation for the vertical bearing capacity of parallel strip footings and a ring footing on soft clay. Both footings were studied under undrained soil conditions. The results have been obtained using the finite element method and have been confirmed by comparing the results with the work of Martin and Hazell (2005), Tani and Craig (1995) and Eason and Shield (1960). The generally smaller extent of the axisymmetric soil failure mechanisms show less interaction and lower efficiencies than occur for comparable (S/B) ratios for the plane strain footing case. Increasing inhomogeneity of strength with depth gives greater N_c values but seems to be insensitive to ring gap size.

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