Measuring the small-strain elastic modulus of gap-graded soils using an effective-medium model and the resonant column apparatus

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Challenges from North to South

Des défis du Nord au Sud

Jesús González-Hurtado, Tim Newson
Geotechnical Research Centre, Department of Civil Engineering, Western University, London, Ontario, Canada.

ABSTRACT
Much of our soil mechanics knowledge relates to testing results from uniformly graded materials, but many natural soils are composed of a wide range of different sized particles. Many soils such as sandy silts, sandy clays, (with the fine particles acting as the soil matrix and sand being the floating particles acting as inclusions) can be considered to be a combination of two poorly graded soils with different grain size distributions. These types of soils are usually known as "gap-graded soils". Although there is a lack of mechanical knowledge of gap-graded soil mechanics in the literature, some work has been conducted on binary mixtures, which provide a reasonable representation of this type of soil. The main focus of this work was to investigate the elastic properties of binary mixtures through the application of an effective-medium model used in rock mechanics. Results from resonant column test experiments on sand-glass bead mixtures were used in this study. Amongst the findings, it was possible to determine a threshold value where the mixture properties abruptly changes and how the elastic moduli of the binary mixtures is affected by this sudden change.

RÉSUMÉ
Une grande partie de notre connaissance sur la mécanique des sols est liée aux résultats des essais sur des matériaux à granulométrie uniforme, mais plusieurs sols naturels sont composés d'une large gamme de particules de différentes tailles. De nombreux sols, tels que les silts sablonneux, les argiles sableuses (avec les fines particules agissant comme la matrice du sol et les particules flottantes du sable, agissant en tant qu'inclusions) peuvent être considérés comme une combinaison de deux sols mal étalés, avec différentes distributions de tailles des grains. Ces types de sol sont généralement connus comme des « sols à granulométrie discontinue ». Bien qu'il y ait un manque de connaissance de la mécanique des sols à granulométrie discontinue dans la littérature, certains travaux ont été réalisés sur des mélanges binaires qui fournissent une représentation raisonnable de ce type de sol. L'objectif principal de ce travail était d'étudier les propriétés élastiques des mélanges binaires par l'application d'un modèle effectif moyen utilisé dans la mécanique des roches. Les résultats des essais à la colonne résonnante sur des mélanges sable-perles de verre ont été utilisés dans cette étude. Parmi les conclusions, il a été possible de déterminer un seuil où les propriétés du mélange changent brutalement et comment le module d'élasticité des mélanges binaires est affecté par ce changement soudain.

1 INTRODUCTION

The term "gap-graded soils" can be used to describe a soil composed of two different grain size distributions. Figure 1 shows typical particle size gradation curves for well-graded (1), uniform (2), and gap-graded soils (3). Soils described as gap-graded have missing particles in a certain size range. Gap-graded soils are sometimes considered to be a form of poorly-graded soil (Coduto et al., 2011). Figure 2 shows a typical probability density function (PDF) for particle size for a gap-graded soil. Two peaks are observed for the PDF curve, which indicates that the particle-size distribution is concentrated around two different mean particle sizes. As stated by Durner (1994), from a mathematical perspective, a gap-graded soil can be conceived as the combination of two or more individual soils.

Much of our soil mechanics knowledge relates to testing results from poorly graded materials, but many natural soils are composed of a wide range of different particle sizes. For example, materials forming glacial tills, residual soils, engineered fills, debris flows, mudflows, and colluvial soil deposits all have a well-defined structure, consisting of a soil matrix that could be either clay, sand, or silt, or a combination of these soils. A number of these materials may also include larger dispersed particles of gravel and/or

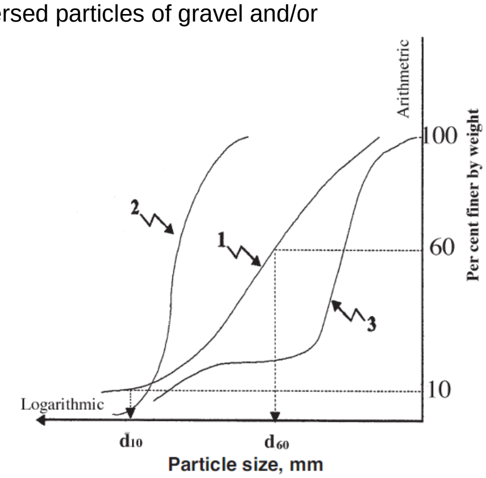

Figure 1. Characteristic particle size gradation curves (Senyur, 1998).

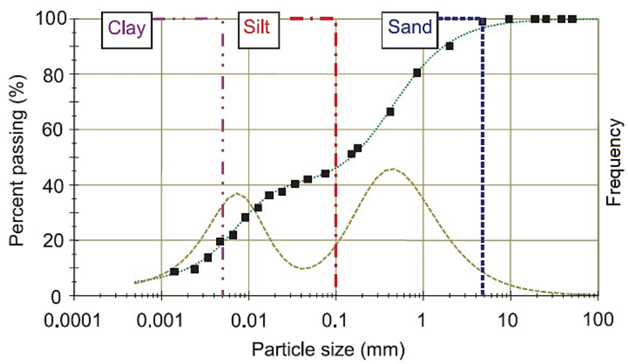

Figure 2. Arithmetic probability density function of a gap graded soil (Fredlund et al., 2000).

pebbles. Moreover, many soils such as sandy silts, sandy clays, (with the fine particles acting as the soil matrix and sand being the floating particles acting as inclusions) can also be considered to be a combination of soils with dispersed oversized particles, if the difference in size between the sand, clay and silt particles is taken into account (Vallejo and Lobo-Guerrero, 2012). Hence, natural gap-graded soils are not common and are occasionally found in engineered or waste soils (e.g. sand-gravel, sand-silt or sand-clay mixtures for: man-made fills, earth embankments, seepage control of dams and mine tailing facilities (Peters and Berney, 2010)).

The particle-size distribution curves shown in Fig. 1 provide a quantitative representation of the relative proportions of the different sizes of particle within a soil mass. The shape of the grain size distribution curve can be described through two simple parameters, which are the coefficient of uniformity (C_u) and the coefficient of curvature (C_c):

$$C_u = \frac{d_{60}}{d_{10}} \quad [1]$$

$$C_c = \frac{(d_{30})^2}{d_{10} \cdot d_{60}} \quad [2]$$

where d_{60} is the grain size for 60% passing, d_{10} is the grain size for 10% passing, and d_{30} is the grain size for 30% passing.

According to the Unified Soil Classification System (USCS), poorly-graded soils have low values of C_u (< 4), while flat curves (well-graded soils) C_u > 4 (gravels) or C_u ≥ 6 (sands). Soils with smooth curves have C_c values between 1 and 3. Most gap-graded soils have C_c values outside of this range (Coduto et al., 2011).

Whilst there is a lack of mechanical knowledge of gap-graded soil mechanics in the literature, some work has been conducted on binary mixtures (the combination of two poorly graded soils with different grain size distributions), which provide a reasonable representation of this type of soil. These studies include the stress-strain response at small strains of silty sands by Salgado et al. (2000), the effect of confining stress on the undrained shear strength of silty sands by Thevanayagam (1998), the determination of the transitional fines content of mixtures of sand-fines by Yang et al. (2006), and the study of the percolation threshold of sand-clay mixtures by Peters and Berney (2010).

The main purpose of the work in this paper is to investigate the elastic properties of binary mixtures through the application of an effective-medium model proposed by Dvorkin and Gutierrez (2002). This is achieved through comparison with an experimental study conducted on binary mixtures and existing predictive methods for small strain elastic moduli in the literature.

2 BACKGROUND

2.1 Intergrain State Concept / Transitional Fines Content

The intergrain state concept was introduced by Thevanayagam (1998). Research in this area suggested that the behaviour of binary mixtures can be characterized into two forms, where one is dominated by the fine particles, and the other is dominated by the coarse particles. Figure 3 depicts the concepts underpinning the intergrain state concept; it is possible to observe the two different domains, one being coarse-dominated when the fines content (f_c) is less than a certain value termed the "transitional fines content (TFC)" or "threshold fine content (f_{cth})", and the other one fines-dominated when f_c > TFC.

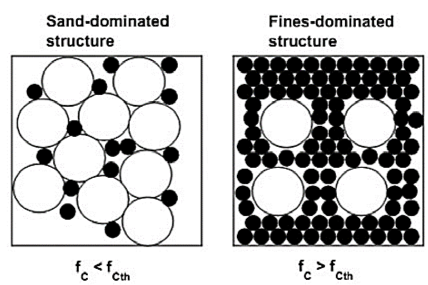

Figure 3. Configuration of Sand with Fines (Benahmed, 2014).

According to the intergrain state concept, the volume of fines is considered to be voids and the fines are assumed not to participate in the force chains of the skeleton formed by the coarse grains and do not significantly influence the mechanical properties (e.g., G_{max}). Once the fines content increases and is higher than the coarse content, the coarse particles are regarded as voids and in this case the coarse grains are assumed to not contribute to the shearing resistance (Yang et al., 2006).

2.2 Percolation Theory

From the spread of epidemics and forest fires, to electrical conductivity, "*Percolation Theory*" has been widely used in several branches of science to describe critical phenomena (Efros 1986). As mentioned in the previous section, binary mixtures exhibit a volume-change response at a threshold value called the *Transitional Fines Content* (TFC), which can be described by percolation theory.

Percolation theory attempts to define the reaction of a binary system to variations in the relative fractions of its two components. The distinctive feature of a percolation phenomenon is the abrupt change in the behaviour of a system due to a small change in the ratio of its internal components (Peters and Berney, 2010). Some studies have shown, in binary mixtures, that the percolation threshold depends on the volume fraction of each material and also the relative concentration (Consiglio et al., 2003).

For a binary mixture such as a soil, it is important to know the volume fraction where the transition (percolation threshold) between coarse to fine dominated mixtures occurs. Being a percolation phenomenon, the range of coarse/fine fractions where the transition occurs can be relatively narrow, causing a sudden change in behaviour with apparently small differences in the composition.

2.3 Topology of Binary Mixtures: The Dispersed Mode approach

Recent investigations of dispersed sand/shale structures in rock mechanics have become popular for studies of the elastic properties for different material mixtures. Dvorkin and Gutierrez (2002) presented an effective-medium model for calculating the porosity and elastic modulus of binary mixtures. This model can be used for theoretical mixing of sand (coarse particles) and shale/clay (fine particles) in the dispersed mode (shale coating the grains, or pore filling). The dispersed mode approach is assumed to be the most compact way of mixing particles of different sizes (Dvorkin and Gutierrez, 2002). When using this method, the fine grains pack can fit within the pore space of the coarse grain pack and still retain their local porosity of ϕ_{SH}.

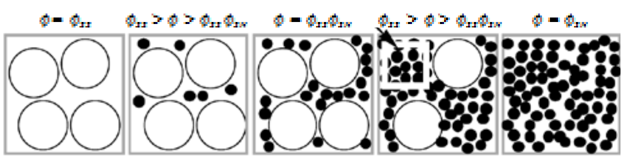

Figure 4. Dispersed mixing mode of coarse and fine grains (Dvorkin and Gutierrez, 2002).

Figure 4 illustrates the grain size variations from pure sand/coarse grains (on the far left) with grain radius *R* and porosity ϕ_{SS}, to pure fine/shale grains (on the far right) with grain radius r and porosity ϕ_{SH}, where R>r. The critical concentration point is in the middle. Here the fine grains completely fill the pore space of the coarse grain pack, while the coarse grains are still in contact with each other, the critical point also represents the inflection point between two different domains. The domain on the right is where the coarse grains are floating among the fine particle structure, which now is considered to be load-bearing. The domain on the left is where the external load is supported by the coarse grain framework. The total porosity is given above each frame. The fourth frame from the left shows a sub-volume of the fine particles that retains the porosity of the pure fine grain specimen.

2.4 Elastic bounds of binary mixtures

When only the elastic modulus of the constituents and their volume fractions are specified (without geometric details of their arrangements) it is possible to predict only the upper and lower bounds of the elastic moduli of the granular material (Avseth et al. 2008).

The best bounds for an isotropic linear elastic composite, giving the narrowest possible range without specifying aspects of the geometries of the constituents, are the Hashin-Shtrikman bounds (Mavko et al., 2009). These bounds can be used to compute the estimated range of average mineral modulus for a mixture of mineral grains, as well as to compute the upper and lower bounds of mineral and pore fluid phases. The HS bounds can only be applied if each constituent is assumed to be isotropic, linear, and elastic. The resulting expressions for a mixture's elastic modulus are:
when $C \geq \phi_{SS}$:

$$K_{MIX} = \left[\frac{C}{K_{SH} + \left(\frac{4.G_{SH}}{3}\right)} + \frac{1-C}{K_1 + \left(\frac{4.G_{SH}}{3}\right)}\right]^{-1} - \left(\frac{4.G_{SH}}{3}\right) \quad [3]$$

$$G_{MIX} = \left[\frac{C}{G_{SH} + Z_{SH}} + \frac{1-C}{G_1 + Z_{SH}}\right]^{-1} - Z_{SH} \quad [4]$$

$$Z_{SH} = \frac{G_{SH}}{6}\frac{9K_{SH} + 8G_{SH}}{K_{SH} + 2G_{SH}} \quad [5]$$

where K_{SH} and G_{SH} are the effective bulk and shear modulus of the pure fine grain specimen. K_1 and G_1 is the grain material bulk and elastic modulus for the coarse grains. *C* is the volume of fines content given by the equation:

$$C = \frac{1}{1 + \frac{1-\phi_{SS}}{\beta}} \text{ ; and } \beta = \frac{r^3.n_s/1-\phi_{SH}}{R^3.n_L/1-\phi_{SS}} \quad [6]$$

where β is the volume fraction of fines in the entire mass, and n_S and n_L are the number of fine and coarse grains in the mixture respectively.

On the other hand, the simplest bounds, but not necessarily the best, are the Voigt upper bound simple arithmetic average (1910), and the Reuss lower bound harmonic average (1929). According to Avseth et al. (2008), it is not possible, in a natural state, that a mixture of materials can exist that is elastically stiffer than the constituent modulus given by the Voigt bound, or elastically softer than the modulus given by the Reuss bound. The Voigt and Reuss bounds are sometimes called the *isostrain* average and the *isostress* average, respectively. These methods can be used to compute the estimated range of the average mineral modulus for a mixture of grains, and a mixture of grains and pore fluid. It is also important to note that when the Voigt and Reuss bounds are calculated for two similar materials, they tend to give a narrow envelope almost approaching a single line having the HS bounds in between.

The lower (Reuss) bound proposed in the study by Dvorkin and Gutierrez (2001) gives the following:

when $C \geq \phi_{SS}$:

$$K_{MIX} = \left[\frac{C}{K_{SH}} + \frac{1-C}{K_1}\right]^{-1} \text{ and } G_{MIX} = \left[\frac{C}{G_{SH}} + \frac{1-C}{G_1}\right]^{-1} \quad [7]$$

3 EXPERIMENTAL STUDY OF ELASTIC MODULI OF BINARY MIXTURES

A study of the elastic moduli of binary mixtures was conducted at Western University by Reipas (2012) using the resonant column device. Sand-glass bead mixtures were used in this study and were created by mixing both components quantified by mass percentage. Silica sand (Ottawa silica sand, Barco 49) was chosen for the fine particles, since it has less variable grain size compared to natural sands, and it is commonly used as benchmark material in laboratory experiments. Solid glass beads were chosen to represent the coarse diameter particles. The beads were chosen to represent the coarse diameter particles (or inclusions) due to their constant shape and roughness, their uniformity, and similar mineralogical composition to Ottawa silica sand. The engineering properties of these materials are summarized in Table 1. The ratio of d_{50} for these materials is approximately 1:40.

Table 1. Properties of Ottawa Sand and Glass Beads.

Property	Ottawa Sand (Barco 49)	Glass Beads
e_{min}	0.476	0.351
e_{max}	0.727	0.923
G_s	2.66	2.50
D_{10}	0.16	9.90
D_{50}	0.26	10.26
D_{60}	0.30	10.35

Six different samples were prepared by varying the sand content as follows: 0%, 36.3%, 54.5%, 76.4%, and 100% (percentages by mass of the sample). The size of all of the samples were 50 mm in diameter by 100 mm in height. Air pluviation from a constant drop height was selected to prepare repeatable and uniform samples. The estimated initial properties of each mixture are presented in Table 2.

Table 2. Initial properties of the sand-glass bead mixtures.

% Glass beads	% Sand	Void Ratio, e	Density (kg/m^3)
100	0	0.653	1513
63.7	36.3	0.188	2151
45.5	54.5	0.306	1978
23.6	76.4	0.430	1835
0	100	0.563	1702

To study the elastic properties of the mixtures, all samples were tested using the resonant column apparatus (RCA). Samples were tested from low to high confinement pressures (60 kPa, 120 kPa, and 240 kPa). At each confinement pressure, the samples were subjected to low and high levels of strains, and for each level of strain one torsional resonant frequency test was performed. A summary of the shear modulus (G_{max}) obtained from the RCA for each sample can be found in Table 3.

Table 3. G_{max} values measured with the RCA (at an average strain of 2.5 x10^{-5}).

% Glass beads	% Sand	G (MPa) at 60 kPa	G (MPa) at 120 kPa	G (MPa) at 240 kPa
100	0	99	126	159
63.7	36.3	220	380	550
45.5	54.5	115	176	252
23.6	76.4	93	135	198
0	100	81	120	175

Figure 5 shows the correlation between the variation of the void ratio with increasing sand content and the variation of density again with increasing sand content. There is an apparent inflection point between 30% and 40% sand content, where the mixtures are clearly divided into two domains, as would be expected from percolation theory.

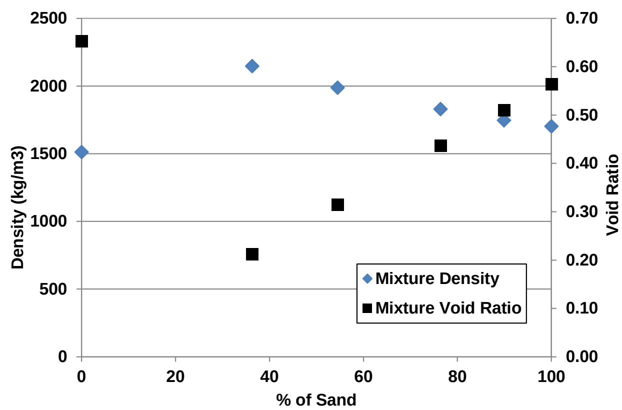

Figure 5. Mixture void ratios and densities with respect to sand content.

Figure 6 shows a graphical representation of the variation of minimum void ratio for the binary mixtures. This graph has been created with the theory developed by Lade et al. (1998).

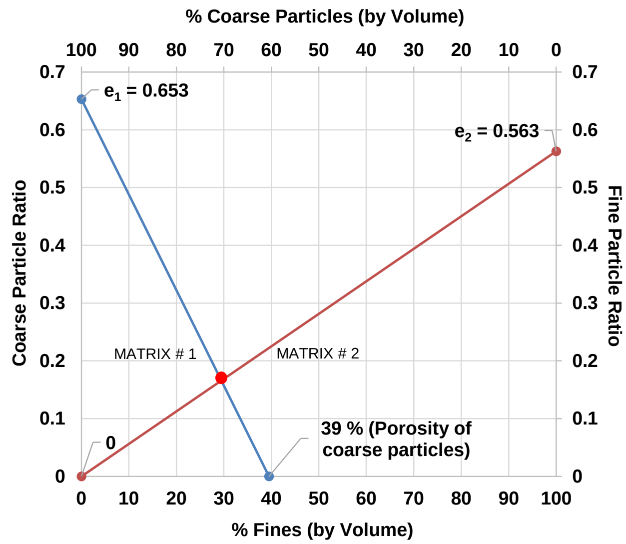

Figure 6. Theoretical variation of minimum void ratio in binary packing with % fines.

Their study suggested that, for binary packings, the minimum void ratio is reached when the voids of the large particles are completely filled with small particles. Using the theoretical approach presented in Fig. 6, it is possible to estimate a value for the inflection point. This estimation matches that of the experimental data shown in Figure 5.

4 RESULTS AND INTERPRETATION

To predict the values of small strain shear modulus G_{max} obtained for the binary mixtures in Section 3, using the RCA machine, three empirical equations were used. These equations are well-known and have proven to be very useful to estimate the shear modulus of sands. The first equation is the Hardin equation (1966), and G_{max} can be calculated with the following equation:

$$G_{max} = A\frac{(a-e)^2}{1+e}(\sigma')^n \quad [8]$$

where G_{max} is in MPa, σ′ is effective mean pressure in kPa, and e is void ratio. The constants A, a, n depend on particle shape (round or angular grains).

The second equation used was the Modified Hardin Equation, which was developed by Witchmann and Triantafyllidis (2009) to account for different grain size distributions.

$$G_{max} = A\frac{(a-e)^2}{1+e}(\sigma_{atm})^{1-n}(\sigma')^n \quad [9]$$

where G_{max}, σ_{atm}, and σ′, are in kPa. The constants *A*, *a*, and *n* are all modified as functions of the coefficient of uniformity (C_u).

The third equation used takes into account a correlation between G_{max} and the relative density, and was also developed by Witchmann and Triantafyllidis (2009).

$$G_{max} = A_D\frac{1+D_r/100}{(a_D-D_r/100)^2}(\sigma_{atm})^{1-n}(\sigma')^{n_D} \quad [10]$$

where the constants are A_D = 177000, a_D = 17.3, n_D = 0.48.

Rock-physics models provide a link between seismic properties (e.g. S-wave/P-wave velocity, elastic modulus, and bulk modulus) and geological parameters (e.g. sorting of particles, granular content, porosity, void ratio, saturation). These models are widely used in the hydrocarbon industry to estimate seismic response to assumed reservoir and overburden properties and conditions. Rock-physics models are adjusted to site-specific conditions to help deduce rock properties away from existing wells and to assist early exploration and evaluation.

To explore the abilities of these models for binary mixtures, the effective-medium model proposed by Dvorkin and Gutierrez (2001), already explained in Section 2.4, was also used to predict the elastic properties of the binary mixtures composed of Ottawa sand and glass beads, described in Section 3.

Along with equation 6, equations 11 and 12 were used to estimate the total porosity (Φ) and the mixture density (ρ_{dry}):

$$\phi = \phi_{SH}.C \quad [11]$$

$$\rho_{dry} = (1-C).\rho_{SS} + C.(1-\phi_{SH}).\rho_{SH} \quad [12]$$

where ρ_{SS} and ρ_{sh} are the glass beads (coarse particles) and Ottawa sand (fine particles) grain-material densities, respectively. The values of grain-material densities used in this study were 2579 kg/m^3 (glass) and 2643 kg/m^3 (quartz) (Mavko et al., 2009).

Using equations 4 to 6 (HS bound) and equation 7 (Reuss Bound), proposed by Dvorkin and Gutierrez (2001), the values of shear modulus were calculated. The values of K_{SH} and G_{SH} depend on the measurements done on the 100% Ottawa sand and 100% glass beads samples in the RCA for each confining pressure (see Table 3).

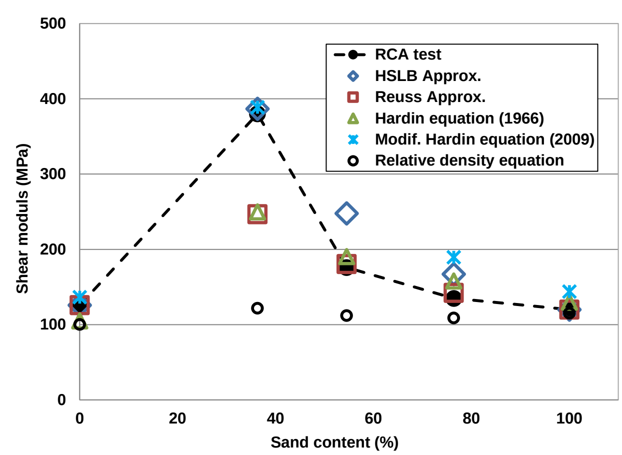

Figure 7. Measured and estimated values of G_{max} at 120 kPa.

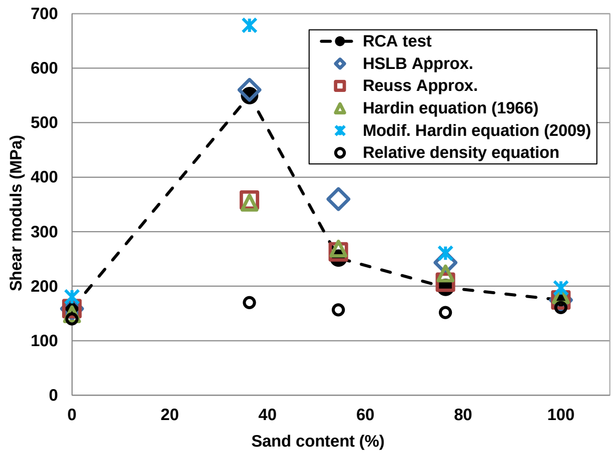

Figure 8. Measured and estimated values of G_{max} at 240 kPa.

The values of K_1 and G_1 used in this study were 45550 kPa and 25000 kPa, respectively. Comparison of the estimated and measured G_{max} values for two pressures (120 and 240 kPa) are shown in Fig. 7 and 8.

From both figures, at the inflection point between 30% and 40%, the HS lower bound provides a better approximation for the shear modulus of the experimental binary mixtures. For this concentration the Reuss bound and Hardin equation give approximately the same values of G_{max}. For sand contents higher than 40%, the best approximation of G_{max} is given by the Reuss bound.

5 DISCUSSION

Figure 9 shows a comparison between the estimated and measured values of small strain shear modulus (G_{max}). Both, HS and Reuss bounds, equations [4] and [7] proposed by Dvorkin and Gutierrez (2002) can be used to establish upper and lower limits to estimate G_{max} values for the binary mixtures. In all cases, traditional equations for G_{max} (equations 8 to 10) are less successful for estimating shear modulus for these types of materials. Since these equations have been developed for well-graded and poorly graded soils this is not wholly surprising. Also evident in Figure 9, is that the closeness of these limits depends on the fines content of the mixture. For the sand content of 36.6% the HS bound lies close to the measured value with the RCA, while the Reuss bound prediction slightly overestimates the shear modulus. According to Dvorkin et al. (1999), if there is a large difference in the elastic contrast between two elements, the HS lower bound can precisely predict experimental measurements, which may be why the HS bound better estimates the elastic properties of the binary mixtures.

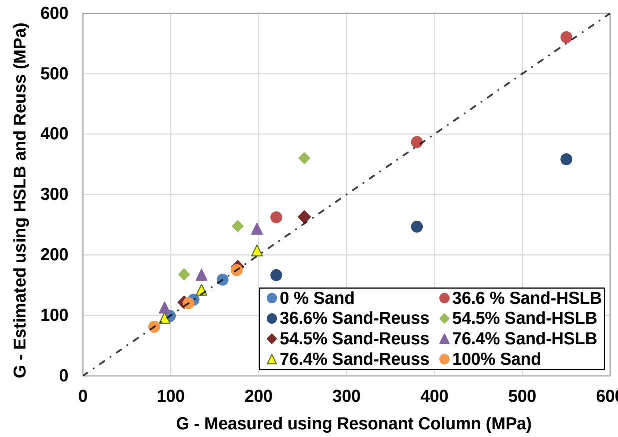

Figure 9. Comparison of estimated vs. measured values of G_{max}.

After the application of the effective-medium model and the analysis of the tested binary mixtures, it was possible to determine the threshold value where the mixture bifurcates into the two domains. This threshold value, also known as the percolation threshold or transitional fines content, was shown to be between 30% and 40% sand content. These findings agree with the work done by Lade et al. (1998), Yang et al. (2006), Polito and Martin (2001), and Thevanayagam et al. (2002), where they reported a transition value from fine-domains to coarse-domains close to 35%, 30%, 35%, and 40%, respectively.

Another important soil mechanics concept that ties into the percolation theory is that of force chains. Force chains are interconnected groups of particles that carry most of the interparticle forces within a soil matrix; those particles that do not actively participate in the force-chains are believed to act as stabilizers of the main force chains (Peters and Berney, 2010).

For the packing data shown in Figure 5, with the addition of sand up to the inflection point, the void ratio decreases making the density of the mixtures increase. This physical phenomena can be explained by using the TFC concept. The fine particles start to fill the voids between the large particles up to the critical point. Just before reaching the critical point, the mechanical behaviour of the mixtures is governed by the properties of the glass beads/coarse particles (for a sand content from 0% to 36% approx.). Right after the critical point, the amount of fines starts to be sufficient to break the connection between the large particles, which are then responsible for carrying the interparticle forces within the soil matrix.

Previous work on binary mixtures has clearly shown the effects of the relative particle sizes on the achievable packing densities and the threshold of fines content (Bouvard and Lange, 1991). Although the links between sample packing, coordination number and elastic moduli have been demonstrated previously, the complexity of the behaviour for gap-graded and binary mixtures requires further investigation. Bideau et al (1985) explained the form of the volumetric fraction v. void ratio curves for binary mixtures (e.g. Figure 5) in the following way. The left hand side of the curve (low fines content) shows void ratio decreases because of global particle segregation; fine grains preferentially fill the pore space of the coarse grain packing. The right hand side of the curve shows the response of what is known as the 'wall effect' due to the large particles. Changes in the number of particles (and hence coordination number) actively involved in force chains due to these structural transitions appear to be controlling the elastic response of the materials.

6 CONCLUSIONS

An investigation of the elastic properties of binary mixtures through the application of an effective-medium model used in rocks mechanics was carried out. The comparison with an experimental study conducted on binary mixtures and existing predictive methods for small strain elastic moduli in the literature has been performed. The links between percolation threshold and the change in the elastic behavior of the binary mixtures were investigated and the findings suggest that is possible to correlate the small strain shear modulus (G_{max}) variation with the change of fine content in the mixtures. By completing these types of analyses this may lead to better engineering practice and improved soil mechanics knowledge of the behavior of gap-graded soils. Further work needs to be conducted on binary mixtures with different particle size ratios to expand and confirm these findings.

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