Effects of varying modulus of subgrade reaction on structural design of mat foundation

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ABSTRACT
Geotechnical engineers are normally requested by structural engineers to provide "modulus of subgrade reaction - k_s" for the design of mat foundation or slab-on-grade. The geotechnical engineers will provide the values of k_s which are deemed applicable to the planned sizes of mat foundation to the structural engineers, who will subsequently use the k_s as elastic springs underneath the mat foundation in typical analytical models, e.g., slab on elastic foundation, to determine contact pressure, settlement, bending moment, etc. From the previous paper on the k_s by the same authors, the values of k_s will not normally lead to the same foundation settlements when comparing the analyses by geotechnical engineers (using elastic modulus) and structural engineers (using k_s). Such discrepancy can be resolved by repetitive analyses to be performed between the two engineers to satisfy a common criterion (e.g., similar foundation settlements). This paper presents the effects of varying k_s values on the structural design of mat foundation, if used without the benefit of repetitive analyses between geotechnical and structural engineers to match the same foundation settlements.

RÉSUMÉ
Les ingénieurs géotechniciens sont normalement appelés à fournir un «coefficient de réaction du sol de fondation - ks » à la demande des ingénieurs en structure pour la conception d’un radier ou d’une dalle sur sol. Les ingénieurs géotechniciens fourniront aux ingénieurs en structure les valeurs du coefficient ks qui sont considérées applicables pour les dimensions du radier. Ils utiliseront ensuite les coefficients ks comme des ressorts en-dessous du radier dans des modèles d’analyses typiques, comme par exemple, une dalle sur fondation élastique, pour déterminer la pression de contact, les tassements, le moment de flexion, etc. Dans un article précédent, préparé par les mêmes auteurs, les valeurs du coefficient ks ne mènent généralement pas aux mêmes tassements de la fondation, lorsqu’on compare les résultats des analyses préparés par les ingénieurs géotechniciens (en utilisant le module d'élasticité) à ceux préparés par les ingénieurs en structure (en utilisant le coefficient ks). Une telle différence peut être résolue par des analyses répétitives exécutées entre les deux ingénieurs pour satisfaire un critère commun (par exemple, des valeurs de tassement de fondation similaires). Ce papier présente les effets de la variation du coefficient ks sur la conception structural du radier, sans utiliser l'aide des analyses répétitives entre les ingénieurs en géotechniques et les ingénieurs en structure pour faire correspondre les mêmes tassements de fondation.

1 INTRODUCTION

In a typical design of a mat foundation, the values of modulus of subgrade reaction for the soils supporting the mat foundation are normally required by a structural engineer. The modulus of subgrade reaction “k_s” is defined as the ratio of the applied pressure ‘p’ to the settlement ‘y’ produced by the pressure applied at the same point, i.e., $k_s = p/y$. The value of k_s is normally provided by a geotechnical engineer based on the results of geotechnical investigation, field tests (e.g., plate loading test) and/or laboratory tests. However, the value of k_s depends on the interaction between the structure and the supporting soil, such that reciprocal and repetitive analyses should be conducted between the structural engineer and the geotechnical engineer in order to arrive at a reasonable value of k_s. Using the criterion that the mat foundation settlement calculated by k_s and the soil modulus of elasticity ("E") should be similar, the values of k_s that satisfy such criterion for different soil stiffness have been presented by Boonsinsuk et al (2013).

For a small project, the value of k_s provided by a geotechnical engineer is normally used for the design of mat foundation without the benefit of reciprocal and repetitive analyses between a structural engineer and a geotechnical engineer. As a result, the effects of such k_s value on the structural design of the mat foundation should be known. This paper addresses the effects of k_s on the structural design of mat foundation with respect to contact pressure, displacement and moment.

2 METHOD OF ANALYSIS FOR MAT FOUNDATION

In order to determine the effects of k_s on the structural design of mat foundation, a commercial computer program using the finite element method (FEM) for analysis was used. A square spread footing (i.e., small mat foundation) and a mat foundation, both to be constructed with reinforced concrete, were analyzed. The soil stiffness supporting the mat foundation was considered as Winkler-type springs. The contact pressure, p, under each FEM node was directly proportional to the nodal deflection, y, through the use of soil spring, k_s, i.e., $p = k_s y$.

Two cases of loading were considered:

- one column load on a square spread footing (Figure 1);
- four column loads on a square mat foundation with each column at 8 m spacing (Figure 2).

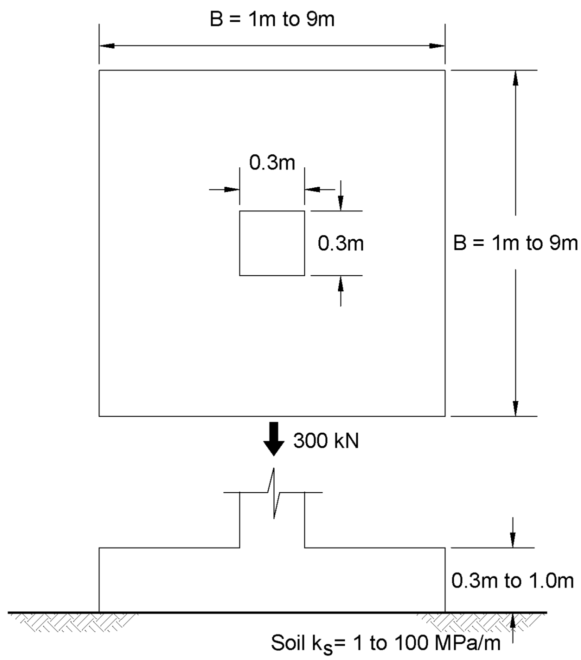


Figure 1 - Plan showing a spread footing with one column

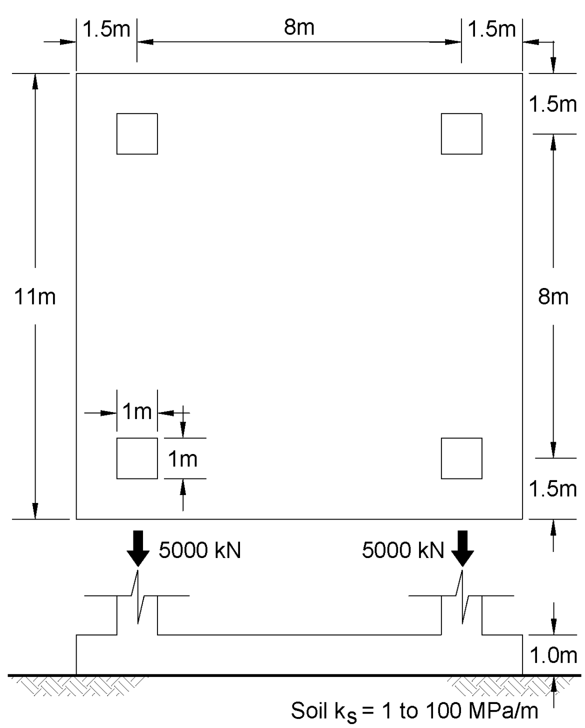


Figure 2 - Plan showing a mat foundation with four columns

The width of the spread footing was varied from 1 m to 9 m, while that of the mat foundation was fixed at 11 m. The thickness of the spread footing was maintained at 0.3 m, while that of the mat foundation was constant at 1.0 m. The column load for the spread footing was kept constant at 300 kN and the column was square with 0.3 m in width. The column load for each of the four columns of the mat foundation was 5000 kN, with each column size of 1 m by 1 m in width. The weight of the foundation was included in the analysis. The values of k_s were varied from 1 MPa/m to 100 MPa/m.

Table 1 relates the k_s values to actual soil types and stiffness as shown in CFEM (2006):

Table 1 - Typical Ranges of Vertical Modulus of Subgrade Reaction (k_S) - CFEM (2006)

Soil Type	k_S (MPa/m)
Granular Soils (Moist or Dry)	
Loose	5 - 20
Compact	20 - 60
Dense	60 - 160
Very Dense	160 - 300
Cohesive Soils	
Soft	< 5
Firm	5 - 10
Stiff	10 - 30
Very Stiff	30 - 80
Hard	80 - 200

The results of the finite element analysis were considered mainly in terms of contact pressure, displacement (i.e., immediate settlement) and moment. Other design considerations, particularly shear and reinforcing bar requirement, were not included in the determination of the effects of k_s on the mat foundation design in this paper. Moment with zero value indicated that reinforcement for tensile resistance was not required.

3 SPREAD FOOTING WITH ONE COLUMN LOAD

The effects of k_s on the design of a reinforced-concrete square spread footing (i.e., small mat foundation) to support one column with 300 kN load were analyzed by varying the footing width. The variations of contact pressure, displacement and moment from the centre of the spread footing to the footing edges are shown in Figure 3 for k_s = 5 MPa/m, as an example. The smallest footing (1 m by 1 m) exerts the highest contact pressure of about 310 kPa (when compared with larger footing sizes supporting the same column load), resulting in the highest displacement of about 61 mm and the lowest moment of about 50 kN-m, at the centre of the footing. Larger footing sizes lead to much lower contact pressure and displacement, but higher moment at the centre of the spread footing. The contact pressure and displacement decrease from the centre of the footing (where the column is located) to the edges of the footing, displaying the flexibility of the spread footing analyzed.

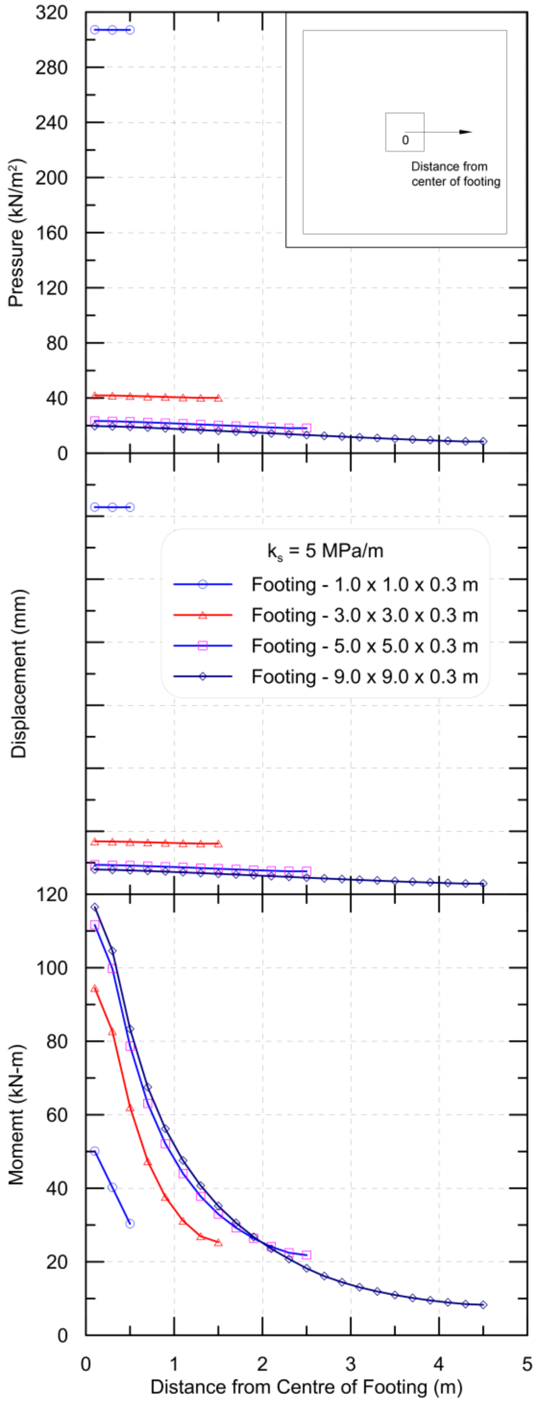


Figure 3 - Distribution of pressure, displacement and moment along the centre of spread footing

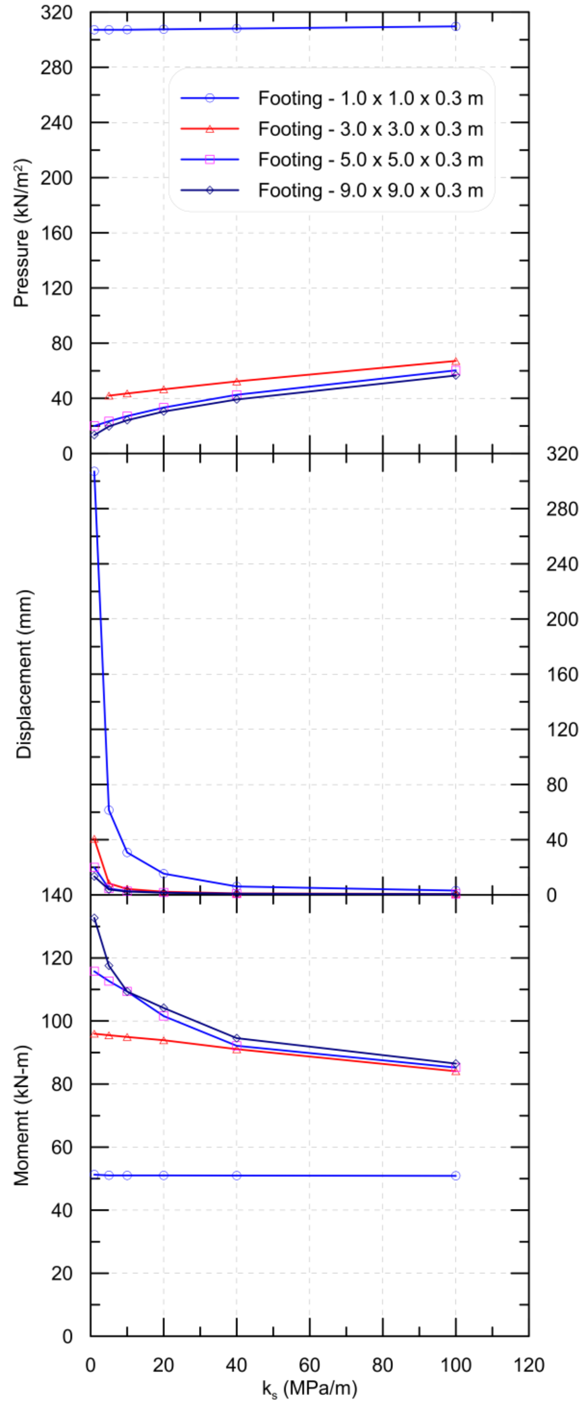


Figure 4 - Variation of pressure, displacement and moment at the footing centre with k_s

The results for the 0.3 m thick spread footing under a column load of 300 kN shown in Figure 3 and Figure 4 lead to the following observations:

a. Contact pressure, displacement and moment (Figure 3) are typically highest at the column location and decrease toward the edges of the footing, the values of which depend on the sizes of the footing and k_s (Figure 4).
b. The smallest footing size (1 m by 1 m) with a contact pressure of about 310 kPa settles the highest while larger footings settle less due to lower contact pressure under the same column load (Figure 3) and k_s (Figure 4). The difference in deflection becomes less when k_s is relatively high ($k_s \geq 60$ MPa/m - dense to very dense sand).
c. The moment in the smallest footing (1 m by 1 m) is the lowest and is the same for all k_s values considered (Figure 4). For larger footing sizes, the moments are approximately the same when k_s is relatively high ($k_s \geq 60$ MPa/m - dense to very dense sand).

The effects of k_s on the displacement (settlement) and moment to be used for the design of spread footing, in the case analyzed, are more pronounced in the lower range of k_s values ($k_s \leq 60$ MPa/m - loose to compact sand).

If foundation design criteria are set to be 300 kPa bearing value for Serviceability Limit State (SLS) at a settlement of 25 mm, the 1 m by 1 m footing will be subject to the same moment regardless of the soil stiffness (k_s). However, the deflection will be higher than 25 mm in loose sand (k_s less than 10 MPa/m - Table 1). A larger footing will then be required to reduce the footing settlement and the moment will be significantly increased as shown in Figure 4. In low k_s values (less than 10 MPa/m - loose sand), the effects of k_s on moment and displacement of a large footing could exceed 100 % of those for a small footing. A 'small' footing is defined as the minimum footing size that is designed with the SLS bearing value, but may exceed settlement criterion. A 'large' footing is defined as a footing that is designed with lower bearing value (than SLS) in order to satisfy settlement criterion.

Varying k_s values do not have significant effects on the displacement and moment of a spread footing supporting one column load when the foundation soil is dense to very dense sand ($k_s \geq 60$ MPa/m).

The high displacement of 61 mm under a contact pressure of about 310 kPa in loose sand (k_s = 5 MPa/m considered in Figure 3) seems to be rather low since the loose sand may not be capable of supporting a contact pressure of about 310 kPa without failure or higher displacement. The value of k_s to be used for the design of a small footing may have to be lowered than those in Table 1 to arrive at a reasonable footing settlement (Boonsinsuk et al, 2013).

4 MAT FOUNDATION WITH FOUR COLUMN LOADS

A square mat foundation supporting four (4) columns was analyzed as mentioned in Section 2. Each 1 m by 1 m square column carried 5000 kN at a spacing of 8 m. The dimensions of the mat foundation considered were 11 m wide by 11 m long by 1 m thick. The values of k_s were varied from 1 MPa/m to 100 MPa/m.

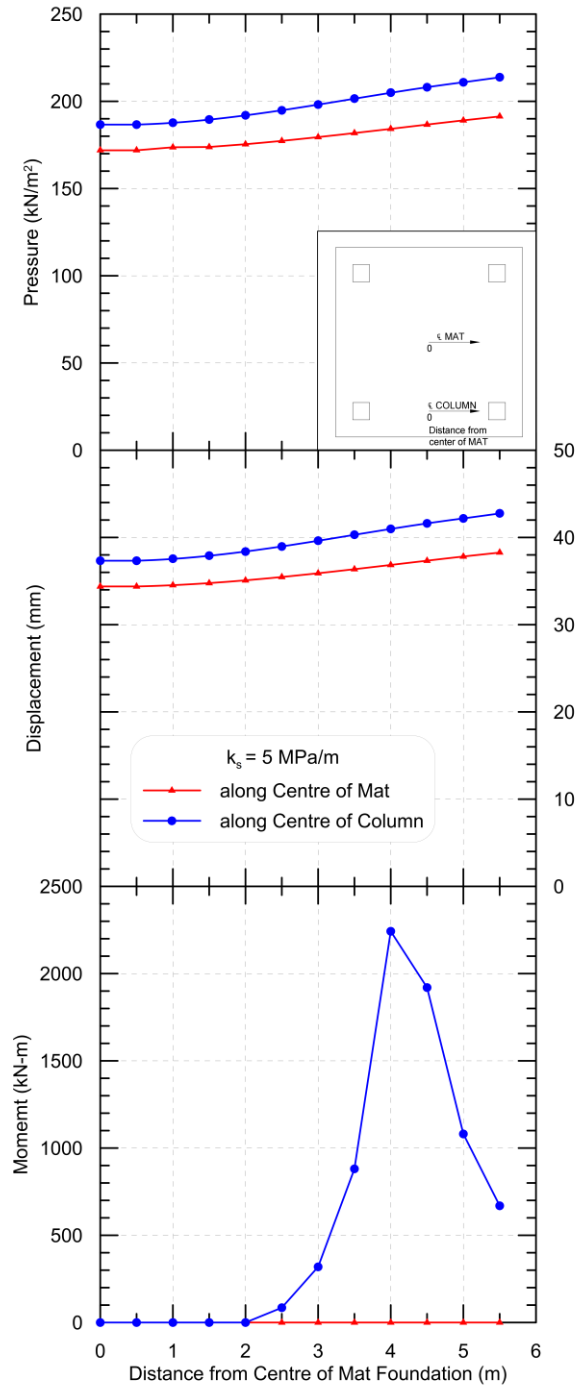


Figure 5 - Distribution of pressure, displacement and moment from the centre of mat foundation

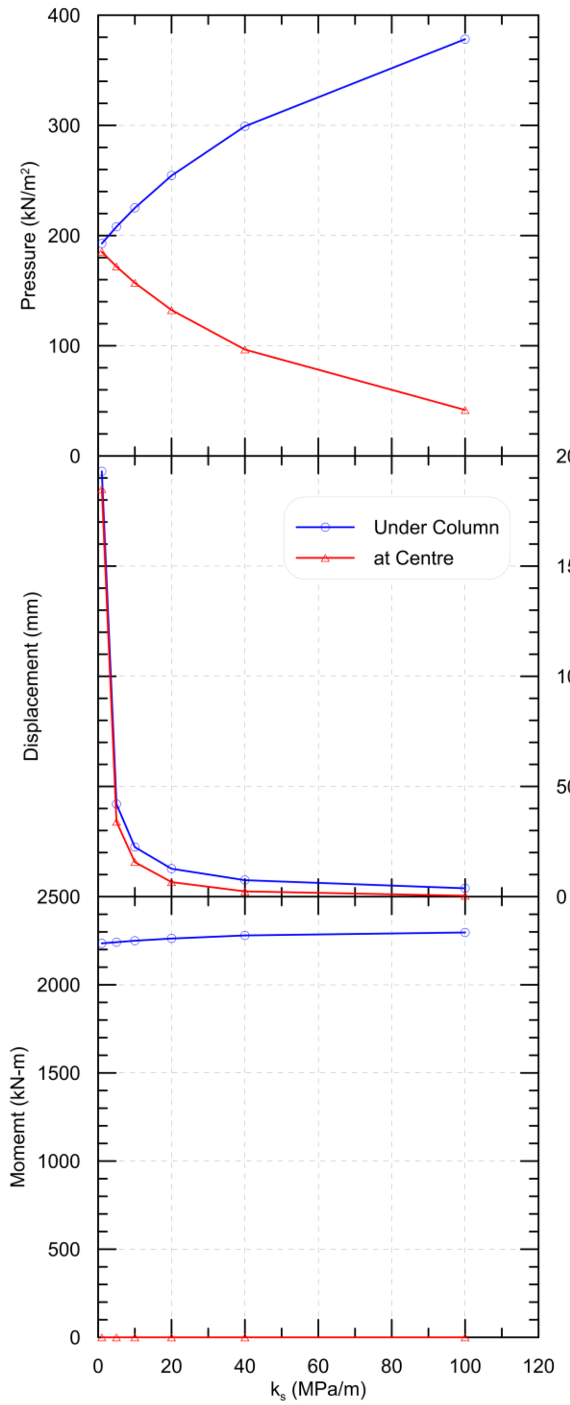


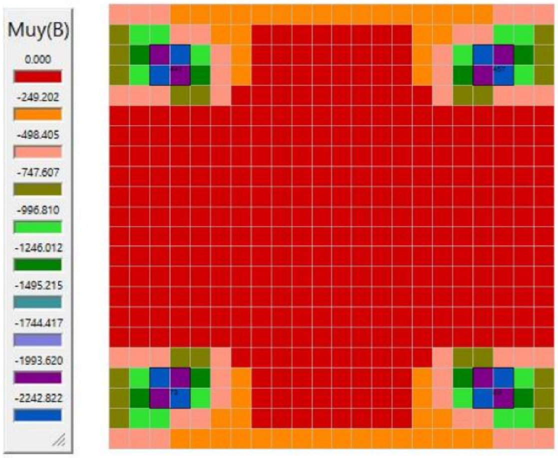
Figure 6 - Variation of pressure, displacement and moment with k_s

The results of the analysis are shown in terms of contact pressure, displacement and moment distributed within the mat foundation for k_s = 5 MPa/m (Figure 5) at two sections - one along the centre of the mat foundation and the other along the centre of column, to the edge of the mat foundation. The highest contact pressure, displacement and moment are located at the column locations. There is no moment at the central portion of the mat for the case analyzed, possibly due to relatively large mat foundation size with small difference in displacement under the column loads considered. Varying the k_s values leads to the variation of contact pressure, displacement and moment (Figure 6) as follows:

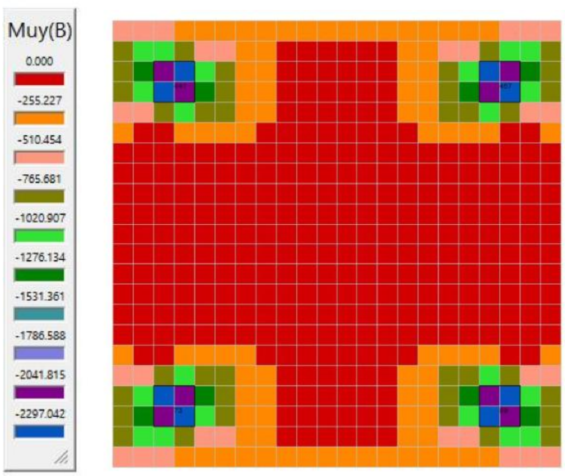
a. Contact pressures are approximately uniform across the mat foundation when k_s values are low (less than 10 MPa/m - loose sand), i.e., the contact pressures under the columns are slightly higher than those elsewhere under the mat foundation. When the k_s values are high ($k_s \geq 60$ MPa/m - dense to very dense sand), the contact pressures under the columns are significantly higher than elsewhere under the mat foundation, since the column loads can be supported locally by the soil in the close proximity underneath the columns and the column loads do not have to be distributed across the mat foundation.
b. Displacements (settlements) of the mat foundation are slightly different between those at the columns and those at the centre of the mat foundation, under the same k_s.
c. Moments are the highest at the column locations, while moments are practically zero in the central portion of mat foundation, i.e., no reinforcement for tensile resistance is required (except for the minimum requirement for reinforced-concrete).

The effects of k_s on the moment of the mat foundation in the case considered are minor when compared the moment at the column locations and at the centre of the mat foundation. It should however be noted that the area of high moment in the vicinity of column varies with different values of k_s, as compared in Figure 7 for k_s = 5 MPa/m and k_s = 100 MPa/m.

In practice, a mat foundation is normally used to support large column loads on weak soils (e.g., loose sand). In the case considered herein, the average contact pressure is about 190 kPa. When such mat foundation is founded on loose sand, a high settlement should occur. For the k_s value of 5 MPa/m (loose sand - Table 1), the calculated settlement is about 42 mm at the column locations and 34 mm at the centre of the mat foundation. Such calculated settlements seem to be low for a mat foundation with a bearing pressure of 190 kPa founded on loose sand. The k_s value used for the loose sand in this case should be lower if settlement is to be calculated directly by k_s (Boonsinsuk et al, 2013).



a) k_s = 5 MPa/m



b) k_s = 100 MPa/m

Figure 7 – Effects of k_s on the moment of the mat foundation

5 CONCLUSIONS

Based on the two cases of mat foundation analyzed to determine the effects of k_s on the structural design of mat foundation, the following conclusions can be made:

a. For a large mat foundation supporting a number of column loads, the values of modulus of subgrade reaction (k_s) for foundation soils may not significantly affect the moment used for the structural design of the mat foundation. However, reasonable values of k_s are still required for the structural design of the mat foundation in order to determine the area within the mat foundation with high moment.
b. For a mat foundation founded on loose sand (or similar weak soils), the size of the mat foundation may have to be large in order to reduce foundation settlement. As a result, the mat foundation may be subject to a high variation of moment with the values of modulus of subgrade reaction (k_s).
c. The values of the modulus of subgrade reaction (k_s) to be used for the structural design of a mat foundation should be determined from reciprocal and repetitive analyses between a structural engineer and a geotechnical engineer in order to arrive at an agreeable behaviour of the mat foundation when analyzed by using different soil parameters and analytical methods.

ACKNOWLEDGEMENTS

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