# Development of spectral pseudo-static method for dynamic clayey slope stability analysis

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# ABSTRACT

The pseudo-static method is considered the simplest approach to evaluate the stability of earth-slopes against shakes. It replaces the action of the earthquake by a constant inertial force proportional or not to the peak ground acceleration of the excitation and applies it to the potential unstable mass of the ground. However, it involves some deficiencies: it ignores the earthquake effect on shear strength and, in addition, masks the dynamic aspect of the problem (site effect, degradation of the modulus, synchronization of the movement, etc.) This study aims to examine the limitations of the pseudo-static method in order to examine the possibility of developing a more effective analysis method that allows keeping the benefits of pseudo-static method. This paper will also present the results of a case among large number of numerical simulations that were used to develop a new method of pseudo-static analysis.

# RÉSUMÉ

La méthode pseudo-statique est considérée comme la méthode la plus simple pour évaluer la stabilité des pentes contre les tremblements de terre. Elle remplace l'action du tremblement de terre par une force d'inertie constante proportionnelle ou non à l'accélération de pic du sol et l'applique à la masse possiblement instable. Toutefois, cela implique certaines lacunes: elle ignore l'effet du tremblement de terre sur la résistance au cisaillement et, en plus, masque l'aspect dynamique du problème (effet de site, dégradation du module, synchronisation du mouvement, etc.) Cette étude vise à examiner les limites de la méthode pseudo-statique afin d'examiner la possibilité de développer une méthode d'analyse plus efficace qui permet de garder les avantages de la méthode pseudo-statique. Ce document présentera également les résultats d'un cas parmi un grand nombre de simulations numériques qui ont été utilisées pour développer une nouvelle méthode d'analyse pseudo-statique.

# 1 INTRODUCTION

Since 1920s, the pseudo-static method is the approach used to evaluate the seismic stability of earth structures, but the first time to be explicitly used to analyze slope stability under seismic loading is attributed to Terzaghi in 1950 (Kramer 1996). The philosophy of the method can be summarized by applying static seismic vertical and horizontal forces in the center of gravity of the entire sliding soil mass. These static forces simulate the inertial forces due to seismic movement. They are assumed to be proportional to the weight of the mass in question of stability i.e. multiply the weight of the mass by a vertical and horizontal force, respectively.

Commonly, the vertical seismic force is assumed to be zero ( $k_v = 0$ ) and only the horizontal force is considered in the analysis. As for the critical slip surface, static long term stability analysis is done using conventional methods to determine the most critical surface (since it is the most stressed area) then the analysis is redone using the seismic forces. However, several trial failure surfaces can be investigated to determine the minimum safety factor.

From the above, the method is fairly simple and straightforward. Yet the difficulty of this method comes from the selection of congruent seismic coefficient.

## 2 EVALUATION OF THE PSEUDO-STATIC COEFFICIENT

Terzaghi (1950) proposed approximate values for  $k_h$ : 0.1 for severe earthquake, Rossi-Forel scale IX; 0.25 for violent, destructive earthquake, Rossi-Forel scale X; and 0.5 for catastrophic earthquakes. In 1970, the US army corps of engineers published a map arranging in the USA and Puerto Rico into five zones according to the damage probability: the coefficient ranges between 0 and 0.15 (USACE 1970). Seed (1979) mentioned that in Japan the values have been less than 0.2. He also provided the design seismic coefficient used in many earth dams across the world; the coefficient varies between 0.1 and 0.15 except in Chili where its maximum value reached 0.2. Marcuson and Franklin (1983) proposed that  $k_h$ ranges from one third to one half the peak ground accelerations to ensure a factor of safety greater than1. Hynes-Griffin and Franklin (1984) proposed to take the horizontal coefficient equals to 0.5PGA providing a factor of safety greater than 1 and a strength reduction of 20%.

In the state of California, the values vary between 0.05 and 0.15 (Abramson et al. 2002). According to the US army corps of engineers, the coefficient varies between 0 and 0.15 depending on the seismic zone and the expected damage. Roughly, from the aforementioned, the most used values range between 0.1 and 0.15.



The pseudo-static method has points of weakness: The selection of the seismic coefficient depends on judgment and experience which is not considered a rational basis (Marcuson III et al. 2007) – It cannot be applied to materials that may lose strength during the seismic event (United States Society on Dams 2007). Seed (1979) concluded that loose sand above the water table, dense sand, and clays of low degree of sensitivity, in a plastic state have great resistance to sliding during earthquake. During the earthquake of Saguenay 1988, nine slope failures were recorded: seven of which are in granular materials and two slope failures occurred in sensitive clayey soils (Lefebvre et al. 1992).

From this brief review of the pseudo-static method, the failure surface is first determined from static stability analysis and it is presumed that the same failure surface would be developed under seismic condition. However, the use of the pseudo-static method in the finite element or finite difference codes, in which the failure surface is automatically determined, may lead to erroneous failure surfaces as it will be demonstrated hereinafter.

## 3 NUMERICAL MODEL

The main objective of this study is to develop a simple analytical method which can be used numerically and that allows the engineers to consider both the effect of earthquake characteristics on the stability of clayey slopes and the geometry of the slope. Thus, an extensive parametric study was made on a clayey slope as the one shown in Figure 1. The parametric study considered three different slopes: 1.75H:1V - 3H:1V - 6H:1V. For each case, three slope heights were studied: 5 m - 10 m - 15 m and seven deposit thicknesses were studied ranging from 5 m to 50 m. However, only one example of the parametric study will be presented in this paper, being the case of a slope of 10m high and 30m deep underlying soil with an inclination of 3H:1V. The slope was analysed statically, dynamically and pseudo-statically. Figure 1 depicts a sketch of the model where  $H_S$  is the height of the slope;  $H_D$  is the thickness of the soil deposit and  $H_t$  is the total height.

The boundaries of the model are situated at 100m from the toe and the head of the slope. The boundary conditions used in the initial state for all the analyses (except the dynamic state) are: the vertical limits are fixed horizontally and the horizontal boundary at the base of the model is fixed both horizontally and vertically. For the dynamic analysis, the free field is used.

The slope and the underlying deposit are divided into 5m thick sub-layers. The properties of each sub-layer are constant. The parameters of a Mohr-Coulomb model are: the density,  $\rho$ , the cohesion, *c*, the friction angle,  $\varphi = 0$ , the elastic modulus, *E*, and the Poisson's ratio, v. The latter two can be replaced by the bulk modulus, *K*, and the shear modulus, *G*. The cohesion of the first sub-layer is 25 kPa and is increased by 5 kPa in the underlying sub-layers. This increase of the undrained shear resistance ensures that the deposit becomes normally consolidated, if water table is considered on surface, at a depth ranging between 30 m and 50 m below the top of slope according

to the proposed formula by Jamiolkowski et al. (1985). Despite this increase can be somehow considered low, it helps to have deeper failure surfaces. From the cohesion, values of shear wave velocity,  $V_s$ , and the shear modulus were determined by the equations 1 and 2, respectively. These correlations were determined by Locat and Beauséjour (1987). They analysed 28 samples coming from 20 different sites in the lowland of Saint-Lawrence and from Saguenay. These correlations were established between the undrained shear strength determined by the unconfined compression test and the shear wave velocity and the maximum shear modulus. The density is therefore determined by the elastic relationship between the maximum shear modulus and the shear wave velocity  $G_o = \rho V_s^2$ . Finally, the bulk modulus was calculated so that the Poisson's ratio is close to 0.5. Table 1 summarizes the properties of each layer in the slope and underneath. As the shear wave velocity is an important factor in dynamic response, two other ratios of  $V_s$  were considered in the parametric study, but not presented in this paper, were studied.



Figure 1. Sketch of the model.

$$V_{s} = 17,2c_{u}^{0,487}$$
 [1]

$$G_o = 0.379 c_u^{1.05}$$
 [2]

#### 4 STATIC, DYNAMIC AND PSEUDO-STATIC ANALYSES

Three types of numerical stability analysis were initially carried out: static, dynamic and conventional pseudostatic. FLAC is equipped with an integrated module to calculate the factor of safety for slope stability analysis. It uses the shear strength reduction method to determine the factor of safety. Static analysis can be accomplished using the integrated module in FLAC. However, since this module cannot be used to estimate the factor of safety in other types of analysis, the factor of safety was (manually) estimated using the shear strength reduction method by plotting the relative displacement between a point in the middle of the slope and a point at the bottom of the foundation soil as a function of the reduction factor. Then, the integrated module was used to verify if the shear strength reduction method is properly applied. Static stability analysis was carried out using the undrained shear parameters for the purpose of, on one hand, comparing its results with the subsequent analyses and on the other hand to evaluate the method used in estimating the factor of safety. The curve of the static analysis is shown in Figure 2 along with the curves of dynamic and conventional pseudo-static analysis. The factor of safety is found to be about 1.42 which was validated by the integrated module.

Table 1. Characteristics of clay layers used in the model.

Clay	Density	Bulk Modulus	Shear Modulus	Shear Wave Velocity
	ρ	K	G	$V_s$
	(kg/m³)	(MPa)	(MPa)	(m/s)
Clay 1	1636.16	552.77	11.13	82.48
Clay 2	1658.99	669.39	13.48	90.13
Clay 3	1678.54	787.00	15.85	97.16
Clay 4	1695.66	905.46	18.23	103.69
Clay 5	1710.91	1024.65	20.63	109.81
Clay 6	1724.66	1144.52	23.04	115.59
Clay 7	1737.20	1264.98	25.47	121.08
Clay 8	1748.73	1386.00	27.91	126.32

The dynamic analysis was performed using a seismic excitation compatible with the seismicity of the region of Quebec. The accelerogram shown in Figure 3 is generated using the conditioned earthquake ground motion simulator SIMQKE (Vanmarcke et al. 1976 and 1997). It was selected so that its response spectrum is compatible with that given by the NBC 2005 for the Quebec City area for site class A (rock). However, the accelerogram is reduced to 0.8 of the original so that the response spectrum is consistent with the spectrum of soil class A (Figure 4). The maximum acceleration - after the multiplication by the reduction factor - is approximately 0.18g. The relative displacement - reduction factor curve is as well plotted in Figure 2, from which the factor of safety was estimated 1.08. It can be noticed that for all the analyses except the dynamic analysis, there is an abrupt change and dramatic increase in the relative displacement indicating the occurrence of the failure. Whereas in the dynamic analysis, the change is more or less smooth. As the minimum reduction factor leading to the failure is not discernible in the dynamic analysis, the factor of safety is determined by constructing two tangents: one to the first segment of the curve and the other one to the lower segment of the curve. Both segments are almost straight lines. The bisector is then drawn intersecting the curve at a point. This point is considered where the failure occurs.

Moreover, the factor of safety is linked to the development of the failure surface, so the creation of the failure surface is examined throughout the shear strength reduction analysis. The aforementioned approach found to properly estimate the factor of safety.

A conventional pseudo-static analysis was also conducted using four different accelerations: 0.05g-0.1g-0.15g-0.2g. The purpose of examining more than value in the conventional pseudo-static analysis is: 1) to determine the value of the constant seismic coefficient corresponding to the accelerogram used in the dynamic analysis and gives the same factor of safety; 2) to study the effect of changing the constant seismic coefficient on the developed failure surface. The curves of relative displacement-reduction factor of the four accelerations are plotted in Figure 2. From this figure, the constant seismic coefficient giving the same dynamic factor of safety is about 0.06. This value is generally less than the most common used values.

The use of a constant seismic coefficient gives, as it will be discussed later, a discordant failure surface compared to the dynamic failure surface.



Figure 2. Relative displacement curves for static, dynamic and conventional pseudo-static analyses versus the reduction factor.



Figure 3. Accelerogram used in the dynamic analysis.



Figure 4. Response spectrum of the accelerogram compared to the response spectrum of Quebec City.

# 5 SPECTRAL PSEUDO-STATIC ANALYSIS

Spectral pseudo-static analysis has been made with the aim of finding a coefficient leading to the same factor of safety and almost the same slip surface of the dynamic analysis. This latter is the non-coinciding outcome of the use of a constant seismic coefficient which may reflect only one aspect of the seismic analysis characteristics. Hence, it is necessary to develop a formula for the seismic coefficient which takes into account the height of the slope or the thickness of the deposit as well as the dynamic properties of the seismic ground motion. It was found that the pseudo-spectral force varies with the thickness of the deposit in such a way that it is minimal at the bedrock and increases gradually to its maximum at the surface. It also depends on the maximum acceleration applied to the bedrock.

Different formulae were examined, but it was found that the variation of the coefficient/force takes more or less a hyperbolic form. The simplest and most practical form can be given by equation 3

$$\mathbf{k}_{h}(z) = \mathbf{k}_{ho} \left[ 1 + \mathbf{a} (z/\mathbf{H}_{t})^{b} \right]$$
[3]

Where  $k_{ho}$  is the seismic coefficient at the bedrock (initial value);  $H_t$  is the total height of the slope and the thickness of the deposit; *a* and *b* are two coefficients that were found for this example to be equal to 2; and *z* represents the variation of the height measured from the presumed bedrock.

This variable coefficient is then multiplied to the mass of each element in the model to obtain the seismic force to be applied in favor of the instability of the slope. The magnitude of  $k_h(z)$  influences the intensity of the force. Hence, different values of  $k_{ho}$  were examined in order to have the same factor of safety of the dynamic analysis as well as the same form of the slip surface. Figure 5 shows the comparison of the relative displacement curves versus the reduction factor of the spectral pseudo-static analyses to the dynamic analysis. Three values of  $k_{ho}$  were examined: 0.03-0.04-0.05. From Figure 5, the factor of safety in the spectral pseudo-static analysis decreases as the value of  $k_{ho}$  increases as expected. The relationship between the value of the factor of safety and  $k_{ho}$  is linear. Hence, the estimated dynamic factor of safety is situated between the factor of safety of both  $k_{ho}=0.03$  and 0.04, namely 0.035. This leads to a seismic coefficient equals to 0.105 on the surface. The same was made for soil deposit thickness of 5 m, 10 m and 20 m (results are not presented). The resulting  $k_{ho}$  values and  $k_h(z)$  on the surface of the presented example along with the cases of 5 m, 10 m and 20 m thick of the soil deposit are plotted in Figure 6. From this figure, the soil deposits of 5 m and 10 m deep have the same order of magnitude for both  $k_{ho}$ and  $k_h(z)$  on the surface. This may be attributed to that the slip surface in both deposits has the same trend (the surface passes by the bedrock as presented in Figure 9(a) and (b)) and a dynamic factor of safety of 1.28 and 1.25, respectively. Both soil deposits of 20 m and 30 m deep have almost the same seismic coefficients as they have a similar failure surface and a similar factor of safety.



Figure 5. Relative displacement curves for dynamic and spectral pseudo-static analyses versus the reduction factor.



Figure 6. Variation of seismic coefficient,  $k_h(z)$ , with the soil deposit thickness.

### 6 FAILURE SURFACE

Although the static analysis is of no interest, the failure surface of the slope 3H:1V is shown. Figure 7(a) and (b) show the slip surface for the static and dynamic cases, respectively. In both cases, the deepest point of the slip surface is at 10 m below the toe of the slope. However, a second failure surface arises below the first one where the lowest point is almost at 15 m deep. As for the extent of the slip surface, in the static case, it meet the earth surface at about 20 m backward the top of the slope. On the other hand, the extent of the failure surface in the dynamic analysis is located between 20 m and 40 m rearward the top. Generally speaking, there is a slight difference between the failure surface in the static and dynamic cases.

Two examples are presented to show the noneffectiveness of the conventional pseudo-static method (i.e. the use of constant seismic coefficient) to be used in finite element or finite difference codes to determine the slip surface. Figures 7 (c) to (f) show the resulting slip surface from the conventional pseudo-static analysis. As cited previously, four different accelerations were examined: 0.05g, 0.1g, 0.15g and 0.2g. From latter figures, it can be noticed that by applying a greater constant seismic coefficient, the failure surface deepens and widens. Over the same figures, the dynamic failure surface was drawn. It can also be noticed that only applying a constant seismic coefficient of 0.05g gives almost a matching failure surface. Nevertheless, the failure surface is somewhat deeper and wider. In practice, as reviewed, this value is not common. The more common values, 0.1g and 0.15g, result in unrealistic slip surfaces.

Figure 8(a) shows the dynamic failure surface for the slope 1.75H:1V. The deepest point of the dynamic slip surface is approximately at 5 m below the toe of the slope. As in the first slope, a second failure surface arises below the first one where the lowest point is almost at 10 m deep. Figure 8 (b) shows the slip surface for the case of 1.75H:1V considering a seismic coefficient of 0.15g. The same unrealistic slip surface is anew obtained.

Figure 7(g) and Figure 8(c) show the slip surface obtained from spectral pseudo-static analysis over which the slip surface of the dynamic analysis is plotted in dashed line for the slope 3H:1V and 1.75H:1V, respectively. It is observed that both surfaces are nearly identical. The second deeper failure surface observed in the dynamic analysis emerges as well. However, in the case of 3H:1V, the spectral pseudo-static surface is a bit wider than the dynamic one.

Figure 9(a) and (b) show the dynamic slip surface for the cases of 5 m and 10 m, respectively. The failure surface passes in both cases by the bedrock. In the case of 10 m, the slip surface may intersect the surface at the top at a distance of 30 m, but in the case of 5 m, this distance becomes 20 m. Figure 9(c) shows the dynamic slip surface for a soil deposit of 20 m thick. As discussed previously, the slip surface is similar to the case of 30 m depth having the same factor of safety.

## 7 CONCLUSION

In this study, a numerical model of a clay slope is used to primarily compare the dynamic analysis with the pseudostatic analysis. The comparison between both analyses indicates that the concept of replacing the seismic dynamic force by a seismic static force cannot be efficient if the slip surface is to be automatically determined. From the examples presented in this study and in conventional pseudo-static analysis, the greater is the seismic coefficient, the deeper and wider is the slip surface. Furthermore, the typical values of constant seismic coefficient (0.1g and 0.15g) give unrealistic slip surfaces compared to the slip surface resulting from the dynamic analysis. This emphasises the concept on which the pseudo-static approach was established: the determination of the slip surface from a steady static stability analysis such as the case of steady seepage in dams. Based on this result, a spectral pseudo-static approach which is in the process of development is introduced. In this approach a variable seismic coefficient is used in lieu of the conventional constant seismic coefficient. The results of this approach conform to the dynamic analysis regarding the form and the extents of the arisen failure surface. According to the ongoing extensive parametric study, the spectral pseudo-static approach is function of the geometry of the slope and the thickness of the soil deposit as well as the seismic characteristics and the soil properties.

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Figure 7. Slip surface developed in the slope 3H:1V for: a) Static analysis; b) Dynamic analysis; c) Pseudo-static analysis with 0.05g; d) Pseudo-static analysis with 0.1g; e) Pseudo-static analysis with 0.15g; f) Pseudo-static analysis with 0.2g; g) Spectral pseudo-static analysis ( $k_{ho}$ =0.035).



Figure 8. Slip surface developed in the slope 3H:1V for: a) Dynamic analysis; b) Pseudo-static analysis with 0.15g; c) Spectral pseudo-static analysis ( $k_{ho}$ =0.044).



Figure 9. Slip surface of dynamic stability analysis of the slope 3H:1V: a) Thickness of soil deposit=5m; b) Thickness of soil deposit=20m.

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