Probabilistic analysis of two mechanisms of failure in geosynthetic reinforced slopes using Monte Carlo simulation

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ABSTRACT
Reinforced slopes with horizontal layers of geosynthetic reinforcement can have different mechanisms of failure. In this paper two major mechanisms of failure of reinforced slopes are investigated. An external mechanism occurs when the critical slip surface passes beyond the reinforced zone. Internal mechanisms are characterized by failure surfaces that intersect all of the reinforcement layers. For a target value of the factor of safety, and a specific value of the reinforcement length, there is a minimum value of the reinforcement tensile strength to generate only external mechanism types. On the other hand, increasing the reinforcement length generates an internal mechanism type. In this study, probabilistic slope stability analysis of these two mechanisms is carried out using Monte Carlo simulation of slopes with different purely frictional soils and slope angles. For a target value of the factor of safety, two sets of charts for external and internal failure mechanism types are presented that can be used to calculate probability of failure for simple slopes with purely frictional soils.

RESUMÉ
Les pentes avec des couches horizontales de renforcement géosynthétique peuvent avoir différents mécanismes de défaillance. Dans cet article, deux grands mécanismes de défaillance des pentes renforcées sont étudiés. Un mécanisme externe se produit lorsque la surface de glissement critique passe au-delà de la zone renforcée. Les mécanismes internes sont caractérisés par des surfaces de rupture qui croisent toutes les couches de renforcement. Pour une valeur cible du facteur de sécurité, et une valeur déterminée de la longueur du renforcement, il existe une valeur minimale de la résistance à la traction du renforcement pour générer uniquement des types de mécanismes externes. D'autre part, l'accroissement de la longueur d'armature génère un type de mécanisme interne. Dans cette étude, l'analyse probabiliste de la stabilité des pentes de ces deux mécanismes est réalisée en utilisant une simulation de Monte Carlo de pentes avec différents sols purement frictionnels et différents angles de pente. Pour une valeur cible du facteur de sécurité, il est possible de générer un ensemble de graphiques pour les types de mécanisme de rupture internes et externes qui peuvent être utilisés pour calculer la probabilité de défaillance des pentes simples dans des sols purement frictionnels.

1 INTRODUCTION

Deterministic reinforced slope stability design charts are available in the literature to estimate the tensile strength, number of layers and reinforcement length to satisfy a minimum factor of safety against the external base sliding and internal modes of failure.

For the case of a simple slope with uniform soil, a given friction angle and slope angle, the key design parameter to ensure a satisfactory factor of safety against external failure modes is the length of the reinforcement. For internal stability modes of failure the spacing, allowable reinforcement tensile strength and anchorage length are the key design parameters to ensure a satisfactory factor of safety against these limit states (CFEM 2006).

Schneider and Holtz (1986), Leshchinsky and Boedeker (1989), Jewell (1991) and Bathurst and Jones (2001) used limit equilibrium methods to produce stability design charts for simple reinforced slopes. These deterministic design charts provide the minimum number, length and spacing of reinforcement layers required to achieve a target minimum factor of safety against collapse.

Limit-equilibrium methods for reinforced slopes include circular slip (Kitch 1994), log-spiral (Leshchinsky and Boedeker 1989) and two-part wedge methods (Bathurst and Jones 2001). These methods have been modified to include the stabilizing contribution of the reinforcement layers. While these methods can give quantitative differences in some cases, the differences are typically not large in practical terms. These limit-equilibrium methods typically provide factors of safety against reinforcement rupture and pullout modes of failure. The utility of these methods is that they can be extended to account for any slope geometry, a wide range of geotechnical soil properties and stratigraphy, pore water pressure and surcharge loading (Woods and Jewell 1990; Jewell 1996).

For more complicated slope problems, commercially available computer programs are now used routinely to perform these calculations (e.g. Slope/W - Geo-Slope Ltd. (2012) and SVSlope - Fredlund and Thode (2011)).

Kitch et al. (2011) investigated two example reinforced slopes designed using deterministic methods based on design guidelines published by Tensar (1988). These design guidelines are based on a two-part wedge failure surface satisfying only force equilibrium. Kitch et al. (2011) showed that for a specific uniform length of the reinforcement layers, there is a minimum reinforcement tensile strength to ensure that the failure mechanism is always external. For tensile strength values greater than this minimum value, there is no change in the value of the deterministic factor of safety. They investigated 18 mechanisms of failure for the two example slopes. Two main failure mechanism types were identified by Kitch et al. (2011): 1) external stability corresponding to cases where critical slip surfaces pass beyond the reinforced zone, and 2) internal stability corresponding to the case where critical slip surfaces intersect all reinforcement layers.

An important limitation of deterministic methods for conventional slope stability analyses is that nominal similar slopes may have the same factor of safety but different probabilities of failure. This is attributed to random variability of soil properties. An estimate of the coefficient of variation of friction angle of soil is $COV_{\phi} = 0.2$ (Phoon and Kulhawy 1999). Soil properties may also have spatial variability, which is not considered in this study. Soil random variability is also expected for reinforced soil slopes. However, for these slopes there is also the potential influence of random variability of reinforcement strength and pullout capacity on probability of failure. For example, the variability in the available tensile strength of geogrid reinforcement after installation may be as high as COV = 0.2 (Bathurst et al. 2011). Variability in pullout capacity based on project-specific laboratory testing can be as high as COV = 0.2, and using current default models in the absence of project-specific testing, as high as COV = 0.55 (Huang and Bathurst 2009). In this study $COV_T = 0.15$ is used for the variation of the tensile strength of the geosynthetic reinforcement.

In this study, external and internal failure mechanisms for three example slopes with four, eight and nine layers of geosynthetic reinforcement, and a range of soil properties and slope geometry are investigated. For each failure mechanism type, deterministic and probabilistic analyses were carried out and results were discussed.

2 DETERMINISTIC SLOPE STABILITY ANALYSIS

2.1 General

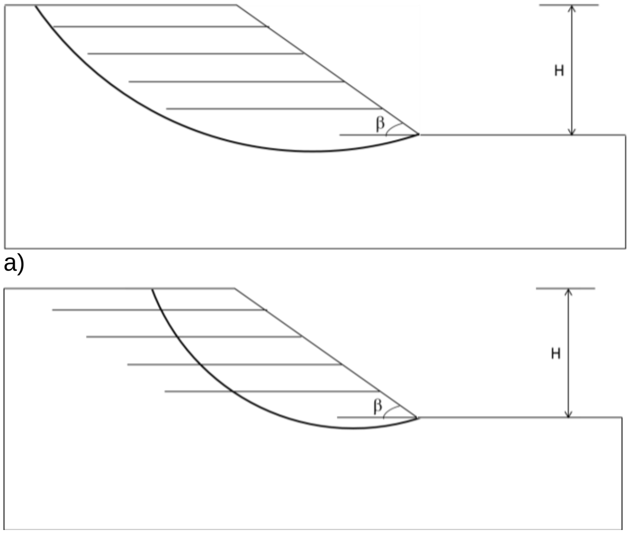
The two reinforced slope failure mechanisms introduced above are shown in Figure 1a (external failure mechanism) and Figure 1b (internal failure mechanism). If the reinforcement is strong enough, the critical slip surfaces are forced by the reinforcement layers to pass beyond the reinforced zone. For a target value of the factor of safety and specific slope angle and number of reinforcement layers, it is possible to find a minimum value of reinforcement tensile strength and reinforcement length which will always generate an external failure mechanism (Figure 1a). If the minimum tensile strength of the reinforcement required to generate only external failure mechanisms is kept constant, increasing the length of the reinforcement will generate an internal failure mechanism (Figure 1b).

Figure 2 shows the example slope with n = 4 reinforcement layers. For n = 4 and n = 8 layer cases, the height of the slope is H = 5 m and $\gamma = 20$ kN/m^3, and for n = 9 the height is H = 10 m and $\gamma = 16$ kN/m^3. For all cases the cohesive shear strength component is c = 0.

In this study, a Visual Basic code was written by the authors for the analysis of reinforced slopes. The code couples Monte Carlo simulation together with the Simplified Bishop's Method to carry out probabilistic slope stability analysis. Hereafter, the code is called PRSS which stands for Probabilistic Reinforced Slope Stability code.

To verify the PRSS code, deterministic results were compared with results from the commercially available program SVSlope (Figure 3).

The value of the minimum reinforcement tensile strength is plotted against the minimum value of the reinforcement length, to generate external failure mechanisms for a slope



a)
b)

Figure 1. a) External failure mechanism, b) Internal failure mechanism

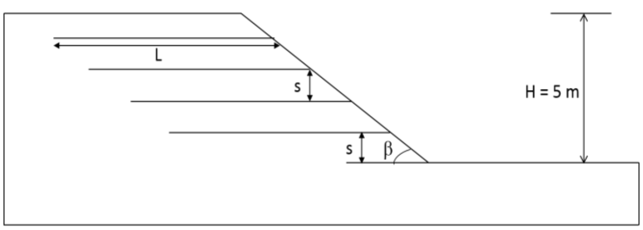


Figure 2. Example slope with four layers of geosynthetic reinforcement (n = 4) and H = 5 m

with slope angle β = 45 degrees and for a range of factor of safety from F_s = 1 to F_s = 1.7. It can be seen that using program SVSlope gives slightly higher values of tensile strength to generate external failure mechanisms. This is likely due to differences in numerical computational details in the two programs. From a practical point of view the differences are negligible

2.2 Minimum reinforcement length and tensile strength

Results of deterministic analysis showed that for each value of the slope angle and for constant minimum length, F_s is constant for values of reinforcement tensile strength greater than the minimum value to generate only external failure mechanisms. Also, for a constant or lower values of the minimum tensile strength, F_s is constant for the values of reinforcement length greater than the minimum value to generate only internal failure mechanisms. Therefore, for each value of slope angle, it is possible to generate a contour plot for a factor of safety for a combination of external and internal failure mechanism (Kitch et al. 2011). Figure 4 shows this contour plot for the factor of safety corresponding to the example slope with β = 45 degrees and n = 4. The green region corresponds to external failure mechanisms and the yellow region corresponds to internal

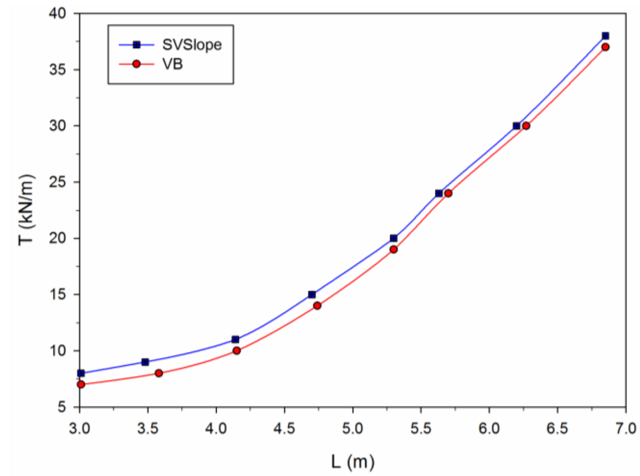


Figure 3. Comparison of results using PRSS code and SVSlope software

failure mechanisms. The locus of points dividing the two regions in Figure 4 corresponds to the minimum values of reinforcement tensile strength and reinforcement length to

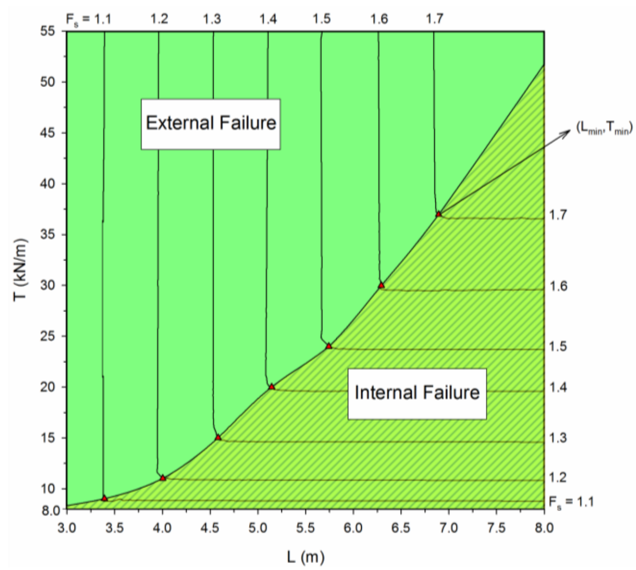


Figure 4. Contour plot for F_s values corresponding to internal and external failure mechanisms for β = 45 degrees and n = 4 reinforcement layers

generate only external failure mechanisms. For the same soil properties and same number of reinforcement layers, similar charts can be generated for other slope angles.

Figure 5 presents minimum values of the normalized reinforcement tensile strength ($nT/(\gamma H\tan(\phi_f))$ versus normalized reinforcement length (L/H) for different slope angles and n = 4 where ϕ_f is the factored soil friction angle ($\phi_f = \tan^{-1}(\tan(\phi)/F_s)$). This figure shows that for a fixed value of F_s and increasing slope angle the minimum reinforcement tensile strength to generate only external failure

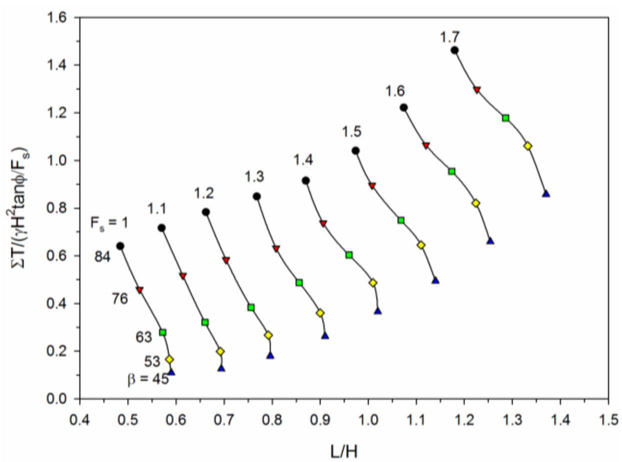


Figure 5. Minimum normalized reinforcement tensile strength versus minimum normalized reinforcement length to generate only external failure mechanisms for reinforced slopes with n = 4 reinforcement layers

mechanisms increases while the minimum length to height ratio of the reinforcement layers also decreases.

3 PROBABILISTIC SLOPE STABILITY ANALYSIS

3.1 General

In this study, probabilistic slope stability analysis of reinforced slopes was carried out using the PRSS code that couples Monte Carlo simulation and the circular slip Simplified Bishop's Method. The advantage of using the PRSS code is that the reinforcement tensile strength is treated as a random variable. Also, the software has the option to select the Fixed and Floating search method to find the critical factor of safety in the probabilistic analysis (Javankhoshdel and Bathurst 2014). In this study, the Fixed method is used (i.e., find the critical failure first and then carry out Monte Carlo simulation), provided the location of the critical slip surface is known for both failure mechanism cases. However, if random variability of the reinforcement tensile strength is not considered, Fixed and Floating methods for the case of purely frictional soil slopes give the same results.

In the probabilistic analysis, the two random variables are soil friction angle, ϕ and reinforcement tensile strength, T. They are assigned $COV_\phi = 0.2$ and $COV_T = 0.15$, respectively, as noted earlier.

Numerical results showed that the random variability of the reinforcement tensile strength using the Fixed method for external failure mechanism cases did not change the value of the probability of failure. However, for internal failure mechanism cases where the critical slip surface intersects all the reinforcement layers, random variability of the reinforcement strength had a very small effect on probability of failure. This outcome is demonstrated further in the next section.

In agreement with lessons learned by Javankhoshdel and Bathurst (2014), 5000 Monte Carlo simulations were shown to be sufficient to compute P_f for the case of two random variables in the probabilistic analysis to follow.

3.2 Probabilistic analysis results

3.2.1. External Failure mechanisms

Figure 6 shows the results of the probabilistic slope stability analysis of reinforced slopes with external failure mechanisms. Different number of reinforcement layers, different slope angles and heights of slope and different values of unit weight were investigated. The soil in each analysis was assigned a mean value of ϕ = 30 degrees. Since the value of F_s changes from 1 to 1.7, the factored friction angle (ϕ_f) also changes. For this range of F_s with different slope angles, the reinforcement tensile strength and reinforcement length must also be changed (Figure 5). However, as can be seen in Figure 6, changing the value of the slope angle, reinforcement tensile strength, reinforcement length and number of reinforcement layers does not change the probability of failure. For the same mean value of F_s and external failure mechanisms (same value of ϕ_f), P_f is the same. This is because the location of the critical slip surface passes beyond the reinforced zone, and hence the values of F_s and P_f depend only on the soil properties.

Figure 6 shows that for external failure mechanisms, the value of probability of failure depends only on the value of the factored friction angle, even with different reinforcement properties or different slope angles; therefore a general chart can be produced which gives the probability of failure for each mean value of the factor of safety and value of the factored friction angle. Figure 7 is the probabilistic slope stability chart for a range of factored friction angles between 20 and 45 degrees with 5 degree increment and mean values of factor of safety varying from 1 to 2. This chart covers a range of slope angles between 30 to 90 degrees

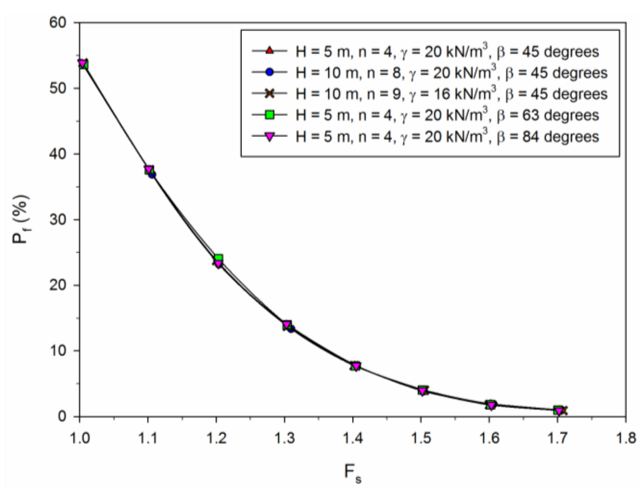


Figure 6. Probability of external failure mechanism versus factor of safety for different slope angles and n = 4

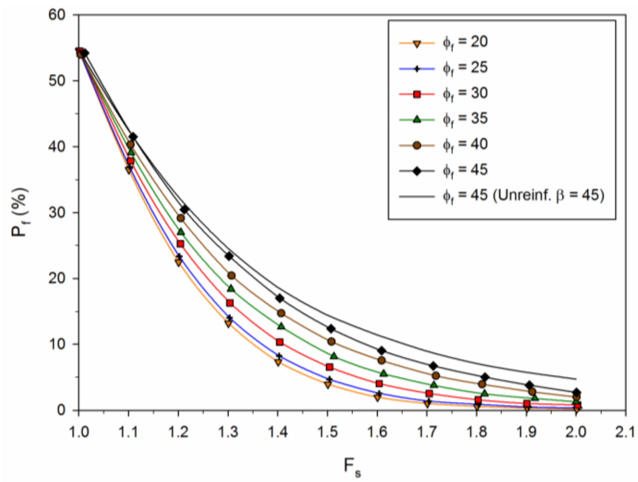


Figure 7. Probabilistic slope stability chart for different values of ϕ_f to generate external failure mechanism

and different reinforcement properties which generate external failure mechanisms. In this figure, a solid line is also presented which is for an unreinforced slope with ϕ_f = 45 degrees and β = 45 degrees. This curve falls above the curve with the highest values of P_f for ϕ_f = 45 degrees and reinforced slope cases. It can be seen in this figure that for the same mean value of F_s, the probability of failure of the unreinforced slope case is greater than for reinforced cases with the same value of ϕ_f.

It should be noted that for unreinforced slopes with purely frictional soil, $F_s = \tan(\phi)/\tan(\beta)$. Therefore, for a target value of F_s for unreinforced slopes for each value of ϕ, there is only one slope with slope angle $\beta = \phi$. However, as mentioned earlier (i.e. Figure 5 and Figure 6), by changing the length and tensile strength of the reinforcement, it is possible to get the same F_s and P_f in reinforced slopes with external failure mechanisms and for different values of β.

It can be seen in Figure 7 that for the same mean values of F_s, slopes with higher values of ϕ_f have a higher probability of failure. This is because slopes with stronger soil (higher ϕ_f) require reinforcement layers with lower value of tensile strength and shorter length to generate external failure mechanisms; therefore, the location of the critical slip surface changes and the failure surface becomes shallower, and shallower slip surfaces correspond to slopes with lower values of F_s and higher values of P_f.

3.2.2. Internal failure mechanisms

Figure 8 shows the influence of random variability (COV_T = 0 and 0.15) of the reinforcement tensile strength on probability of failure for the same mean values of F_s. In this figure, n = 9, β = 45 degrees and ϕ = 30 degrees. It can be seen that for the same mean value of F_s, some cases with COV_T = 0.15 have a small but detectable higher probability of failure. However, the difference between the probabilities

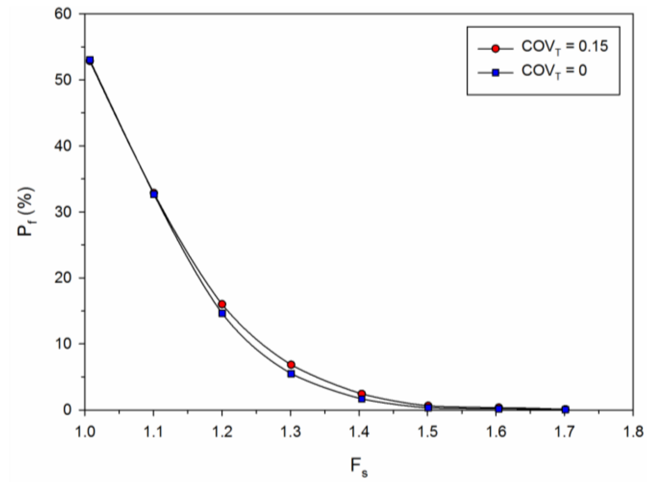


Figure 8. Effect of constant tensile strength (COV_T = 0) and random variability in reinforcement tensile strength (COV_T = 0.15) on the probability of internal failure mechanism

of failure with and without random variability in reinforcement tensile strength is negligible. To have more accurate results, random variability of soil tensile strength is considered in all the simulations.

Figure 9 presents the effect of number of reinforcement layers on probability of failure of an internal failure mechanism. In this figure, results are for cases with n = 4, 8 and 9, β = 45 degrees and friction angle ϕ = 30 degrees. It can be seen that for the same value of slope angle and friction angle, the number of reinforcement layers has negligible effect on the value of the probability of failure.

Figure 10 shows the effect of slope angle on the probability of an internal failure mechanism. The results are for cases with n = 9, slope angle $45 \le \beta \le 76$ degrees and ϕ = 30 degrees. The curves in Figure 10 show that increasing β, decreases the probability of failure for an internal failure

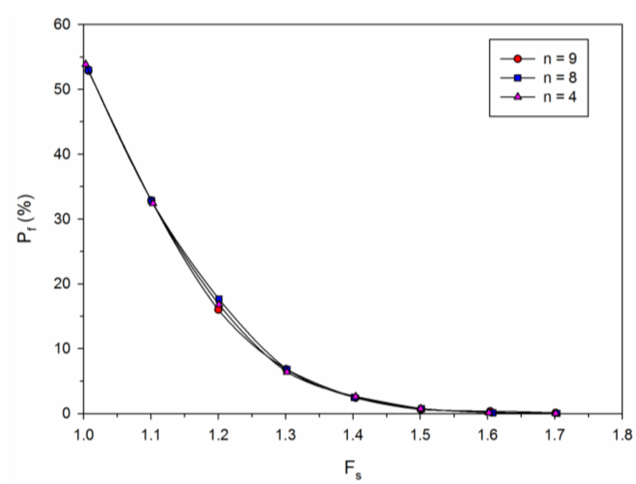


Figure 9. Effect of number of reinforcement layers on probability of internal failure mechanism

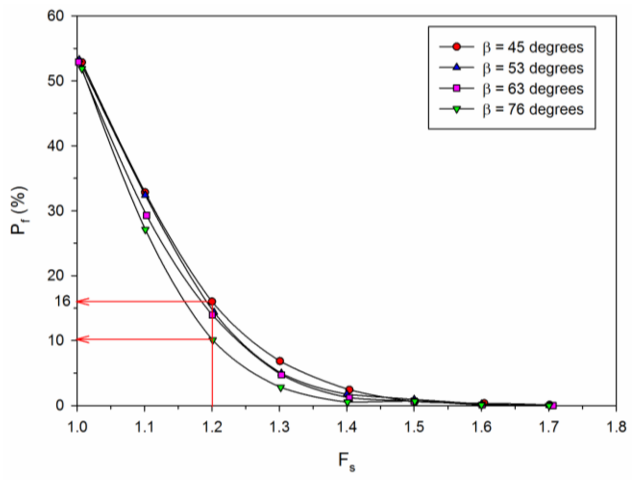


Figure 10. Effect of slope angle on probability of internal failure mechanism

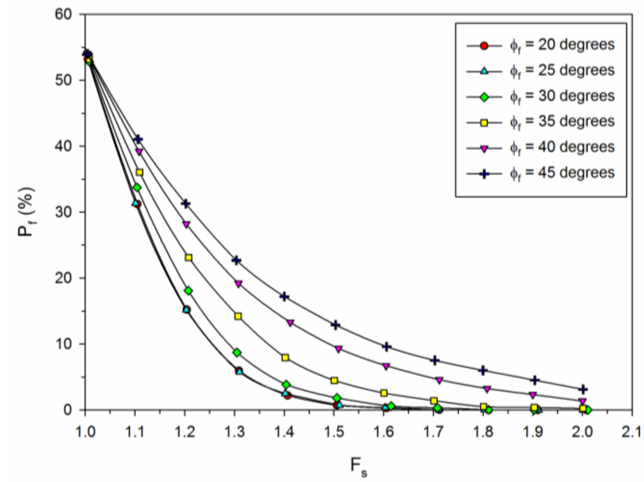


Figure 11. Probabilistic slope stability chart for reinforced soil slopes with internal failure mechanisms and different values of ϕ_f (β = 45 degrees)

mechanism. Also, it should be noted that for the same value of friction angle, steeper slopes require stronger reinforcement layers to achieve the same factor of safety; therefore the probability of failure decreases. However, it can also be seen in Figure 10 that, the difference in the probability of failure for different values of slope angle is small (e.g. in the worst case scenario for F_s = 1.2, P_f = 10% for β = 76 degrees, while P_f = 16% for β = 45 degrees); therefore the case with β = 45 degrees is likely the most conservative case for reinforced slopes with internal failure mechanisms.

Results obtained in Figures 9 and 10 demonstrate that it is possible to have a general conservative (β = 45 degrees) probabilistic slope stability chart similar to Figure 7 for reinforced slopes with internal failure mechanisms. Figure 11 presents such a chart for β = 45 degrees, different values of ϕ_f and $1 \le F_s \le 2$. It can be seen in this figure that for the same mean value of F_s, curves with higher ϕ_f values have a higher probability of failure. The reason is that for slopes with stronger soil (higher ϕ_f), tensile strength of the reinforcement layers should be lower to meet the same target values of F_s. For internal failure mechanisms, probability of failure depends mostly on the value of the tensile strength of the reinforcement; therefore, increasing ϕ_f increases P_f.

It should be noted that in Figure 11, the slope angle for the curve with the value of ϕ_f = 45 degrees is β = 50 degrees, provided that, as mentioned before, factor of safety for unreinforced slopes with purely frictional soils is $F_s = \tan(\phi)/\tan(\beta)$ and for ϕ_f = 45 degrees the unreinforced slope provides the target values of F_s without requiring any reinforcement. Therefore, β = 50 degrees is chosen for ϕ_f = 45 degrees in Figure 11. This problem does not arise for probabilistic charts for external failure mechanisms, provided that the probability of failure is the same for different values of slope angle with the same mean value of F_s.

Figure 12 shows a comparison between probabilities of failure for the same mean values of F_s for external and

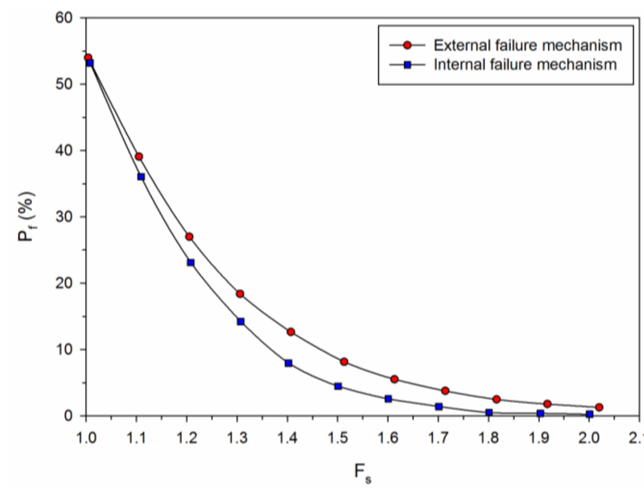


Figure 12. Difference between probabilities of failure in external failure mechanism and internal failure mechanism for the same mean value of F_s and for ϕ_f = 35 degrees

internal failure mechanism cases and slopes with n = 9, β = 45 degrees and ϕ_f = 35 degrees. Figure 12 shows that for the same mean value of F_s, probability of failure for external failure mechanisms is higher than for internal failure mechanisms. It can be concluded that increasing the reinforcement length changes the mechanisms of failure from external to internal failure type and probability of failure decreases. It should be noted that the probabilities of failure presented in Figure 11 for internal failure mechanisms are upper bound values for β = 45 degrees (Figure 10), and by increasing the slope angle the probability of failure decreases.

4 CONCLUSION

This study reports the results of the probabilistic slope stability analysis of reinforced slopes using coupled Monte Carlo simulation together with circular slip simplified Bishop's method and implemented within a program (PRSS) written by the authors. Two main mechanisms of failure of geosynthetic reinforced slopes were investigated; external failure mechanisms that comprise of critical slip surfaces that pass beyond the reinforced zone and internal mechanisms where critical slip surfaces intersect all the reinforcement layers.

The study demonstrates that for the simplifying assumptions adopted here, there are minimum values of reinforcement tensile strength and reinforcement length which will generate only external failure mechanisms. This study shows that for tensile strength greater than this minimum value, there is no change in the values of factor of safety and probability of failure. For constant value of the minimum tensile strength, increasing the minimum reinforcement length changes the failure mechanism from external failure to internal failure type. This change in the failure mechanism increases the value of F_s and decreases P_f. Contour plots can be generated for different values of slope angle that identify the combinations of reinforcement tensile strength and reinforcement length that will generate each type of failure mechanism (e.g. Figure 4).

Probabilistic analyses showed that the magnitude of the probability of an external failure mechanism is not changed by changing the reinforcement properties. Probability of failure in these cases depends only on the value of the factored friction angle; therefore it is possible to have a probabilistic slope stability chart for the case of purely frictional soil with external failure mechanisms and for different values of the factored friction angle. Comparison of probability of failure of reinforced slopes with an external failure mechanism type with unreinforced cases with the same mean values of factor of safety and same value of the factored friction angle shows that the probability of failure of the reinforced slopes is lower than for the unreinforced slopes.

For internal failure mechanism cases, the probability of failure decreases with increasing slope angle due to the change in the reinforcement tensile strength required to meet the same target value of factor of safety. However, reinforced slopes with β = 45 degrees have the highest probability of failure (except for cases with ϕ_f = 45 degrees and β = 50 degrees is used). Nevertheless, a probabilistic slope stability chart can be generated for this slope angle condition that gives conservative values for probability of failure.

Finally, this study shows that for the same mean value of the factor of safety, probability of failure is lower for internal failure mechanism cases compared to external failure mechanism cases when all other factors are the same.

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