Estimation of Residual Friction Angle of Clay Soils Using Artificial Neural Networks Modelling

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ABSTRACT
Accurate estimation of site-specific soil strength parameters (e.g., the internal friction angle and cohesion) is challenging in geotechnical engineering due to the limitations and complexities associated with obtaining undisturbed soil samples and laboratory shear test analysis. The residual friction angle (ϕ'_r) of clay soils is particularly important parameter in slope stability analysis, especially in case of pre-existing slip surfaces and large deformations, and is commonly approximated from Atterberg limits and grain size distribution using traditional regression analysis. In this study, we tested the reliability of Artificial Neural Networks (ANNs) in predicting the residual friction angle degrees of different soil types based on their Atterberg Limits, clay size fraction and normal stress. The main objective was to find a satisfactory relationship between input and actual measured values using artificial neural network models. The effect of the network geometry on the performance of the models was also assessed. Strong correlation factors (e.g., 0.99) for training and testing data sets in model MLP741 demonstrate that ANNs are powerful tools for predicting soil strength parameters.

RÉSUMÉ
Dans cette étude, nous avons déterminé la fiabilité des réseaux de neurones artificiels (RNA), pour prédire la valeur de l'angle de frottement résiduel pour (ϕ'_r) les différents types des sols en fonction de leur limites d'Atterberg, la teneur en argile at la contrainte normale. L'objectif principal était de trouver une relation satisfaisante entre l'entrée et les valeurs mesurées en utilisant les modèles RNA. L'effet de la géométrie du RNA sur la performance des modèles est également évalué. Nous avons trouvé que, pour tous les modèles RNA testés, les facteurs de la corrélation sont supérieurs à 0,99. Cela montre que les RNA sont des outils puissants pour prédire des valeurs du ϕ'_r. Il a été également observé que parmi les modèles RNA, le MLP741 (perceptron multicouche) mène aux meilleurs résultats.

1 INTRODUCTION

The residual shear strength of cohesive soils is the foremost parameter in slope stability analysis where pre-existing slip surfaces or large deformations in earth works have been recognised. Under residual conditions, due to the large stain imposed to their sensitive grain orientation, clay soils show no cohesion and their angle of internal fiction exhibits substantial reduction, which is called residual friction angle (Skempton 1965, 1977; Stark et al. 2005).

Due to the limitations and complexities associated with undisturbed soil sampling and laboratory shear test analysis, the estimation of soil residual friction angle strength is a challenge in geotechnical engineering practice. Therefore, many empirical correlations for predicting ϕ'_r based on soil physical properties (e.g., clay-size fraction, liquid limit, plasticity index, and sand-fraction) have been proposed over the past 50 years (e.g., Skempton, 1964; Voight, 1973; Kanji, 1974; Peters and Lamb, 1979; Lupini et al., 1981; Skempton, 1985; Mesri and Cepeda-Diaz, 1986; Collotta et al., 1989; Stark and Eid, 1994; Stark and Eid, 1997; Mesri and Shahein, 2003; Wesley, 2003; Stark et al., 2005; Tiwari and Marui, 2005; Stark and Hussian, 2013).

One of the most commonly used correlation is ϕ'_r determination from liquid limit, clay size fraction (CF) and effective normal stress proposed by Stark and Eid (1994) and revised in 2005 (Stark et al., 2005). Stark and Hussain (2013) recently revised their model to better predict ϕ'_r, which is calculated from Liquid Limits (LL) and Clay Fraction for four different normal stresses; their results are presented in Figure 1.

Although the Stark and Hussain (2013) models are frequently used, the soils from which ϕ'_r was derived (measured or modelled) were primarily from California and Texas, which are unlike the typical soil conditions encountered in Canada and do not correlate well with shear test results for soil conditions in the Prairie Provinces. In this paper we propose that artificial neural networks (ANNs) can reliably predict ϕ'_r in the Prairie Provinces where the relationship between soil physical properties and ϕ'_r is not well known.

1.1 Artificial neural network Models

Artificial neural networks (ANNs) mimic the complicated organization and information processing of the central nervous system in animals (Goh, 1994) as compared to linear processing in conventional mathematical models and digital computers. The processing elements of ANNs are similar to nodes in the human brain and are arranged in an input layer, output layer and hidden layer as illustrated in Figure 1. Complex and highly non-linear real world problems cannot be solved by traditional regression analysis, but ANNs can overcome these limitations by changing the function of transfer and in the case of highly non-linear phenomena by changing the number of hidden layers and nodes (Gardner and Dorling, 1998).

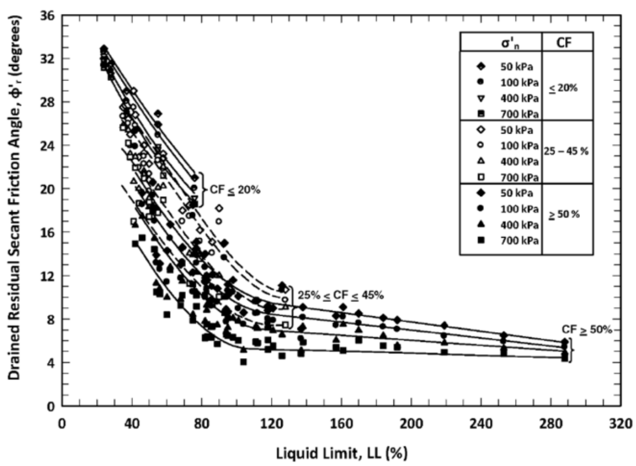

Figure1. Empirical correlation for ϕ'_r angle based on LL, CF, and normal stress (Stark and Hussain 2013)

1.2 Artificial neural network Models

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The main objective of using ANN models is to find a satisfactory relationship between input and output data. ANNs do this without any knowledge of the nature of the input and output variables by adjusting the weight of each variable to find the input-output mapping with the smallest error in a process called training. After the model is trained, the model performance is examined by using different sets of input-outputs to determine the reliability of the model (Maier and Dandy 2000; Shahin et al. 2001; Shahin and Indraratna 2006; Erzin 2007).

Artificial neural networks have been successfully applied in various fields of geotechnical engineering such as settlement of foundations (Sivakugan, 1998), and liquefaction (Ural and Saka, 1998; Najjar and Ali, 1998).

ANNs were used by Khanlari et al. (2011) to estimate the cohesion and effective friction angle (ϕ') from grain size, plasticity index, and density from soils collected in Iran. They obtained a correlation factor of 0.91 for cohesion and 0.89 for ϕ' for testing dataset. Rani et al. (2013) also used ANNs to predict cohesion, friction angle, permeability, and compressibility of soils in terms of their clay fraction, liquid limit, plasticity index, maximum dry density, and optimum moisture content. They obtained a correlation factor of 0.99 and 0.92 for the training and test dataset for cohesion, respectively; they also obtained a correlation factor of 0.99 and 0.94 for the training and test dataset for ϕ', respectively. Another example of ANN modeling of soil strength parameters was done by Tipza et al. (2014); they modeled coarse content, fine content, liquid limit, and soil bulk density. Although there have been several studies on the successful use of ANN in modeling of soil strength parameters, to the authors' best knowledge application of ANN in modeling of Canadian Prairie soils has not been previously reported.

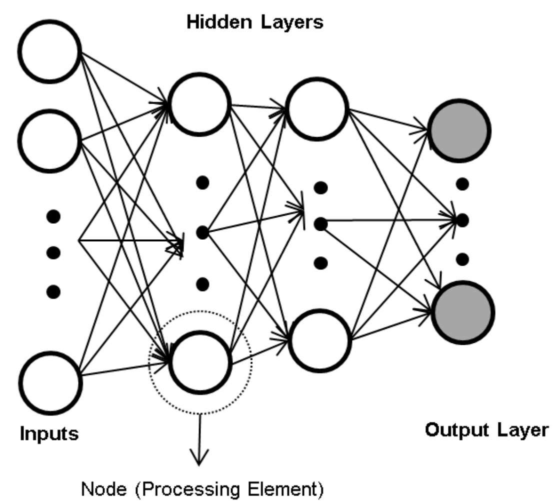

Figure 2. The architecture of ANNs.

2 MATERIAL AND METHODS

2.1 Database

The drained residual fiction angle dataset reported by Hussain (2010) and Stark and Hussain (2013) was used in this study. Laboratory measured ϕ'_r, Atterberg limits, clay size fraction, and normal stress of 98 clay and shale samples mainly from the Texas and California, Unites States are reported in this study (Stark and Hussain, 2013; Detwiler, 2104). In this dataset, measured ϕ'_r ranged from 4.5º to 31.3º; liquid limit ranged from 24% to 288%; soil samples CF ranged between 10% and 88%. For two of ANN models 90% of the Stark dataset was used for model training and the remained 10% was used for testing the model performance. For the other two models, 80% of dataset was used for training and 20% was used for testing the models.

2.1.1 Clifton Associates Database

A set of 24 laboratory measured ϕ'_r along with soil Atterberg limits, clay size fraction, and normal stress was used to verify the performance of ANN models. Samples were collected by Clifton Associates Ltd. (Clifton) near Saskatoon, SK and tested by Clifton's Materials Testing Laboratory in Regina, SK. The results yielded ϕ'_r values ranging from 5.7º to 16.7º, liquid limits of 52% to 154%, and clay fraction ranging from 49% to 83%.

2.2 ANN models

Four different Multilayer Perceptron (MLP531 90%, MLP741 90%, MLP531 80%, and MLP741 80%) networks were used to predict ϕ'_r. MATLAB was used to program the models. MLP531 90% and MLP741 90% were trained with 86 data sets, and MLP531 80% and MLP741 80% were trained with 75 data sets, which are the minimum number of records to achieve high accuracy in model training. These numbers were selected from trial and error runs during ANN model training. Each data set contained different LL, PI, CF, and normal stress values as they relate to measured residual friction angles.

2.2.1 Multilayer perceptron network

The structure of a multilayer perceptron network (MLP) model consists of a number of hidden layers with several neurons in each hidden layer. The activation functions for hidden layer neurons are tangent hyperbolic, and for the output layer is linear. In fact, the linear function for transformation from the hidden space to the output space will increase the performance of the network (Vaziri et al., 2006). The structure of the MLP network specifications (i.e., the numbers of training samples, simulating samples, neurons in first hidden layer, neurons in second hidden layer, and neurons in output layer) are summarized in Table 1.

Table 1. Summary of MLP networks

Model Name	Number of			
	Input Layer	Hidden Layer	Output Layer	Training/ Testing Samples
MLP531 90%	5	3	1	86/10
MLP741 90%	7	4	1	86/10
MLP531 80%	5	3	1	75/21
MLP741 80%	7	4	1	75/21

3 RESULTS AND DISCUSSION

3.1 Validation and comparison of models Performance

The training and testing data set from Stark and Hussain (2013) were used to evaluate the performance of all four models and the suitability of these models in predicting residual friction angles was evaluated using Clifton's data set. For all of the validation scenarios, statistical parameters such as Standard Error (SE), correlation factor, and Root Mean Square Error (RMSE) were calculated using Microsoft Excel for predicted and measured data. Equations 1 to 3 show the mathematical expression of these statistical parameters.
The standard error (SE) is given by:

$$SE = \frac{s}{\sqrt{n}} \qquad [1]$$

where s is the sample standard deviation, and n is the size of the sample.

The correlation [Correl(X,Y)] factor is given by:

$$Correl(\phi'r_p, \phi'r_m) = \frac{\Sigma(\phi'r_p - \overline{\phi'r_p})(\phi'r_m - \overline{\phi'r_m})}{\sqrt{\Sigma(\phi'r_p - \overline{\phi'r_p})^2 \Sigma(\phi'r_m - \overline{\phi'r_m})^2}} \qquad [2]$$

where ϕ'_{rp} is the predicted residual soil friction angle, ϕ'_{rm} is the measured residual soil friction angle, and $\overline{\phi}'_{rm}$ and ϕ'_{rp} are the samples average.

The root mean square (RMSE) can be expressed by:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\phi'r_{pi} - \phi'r_{mi}\right)^2} \qquad [3]$$

where N is the number of verifying points (samples) in the verification area.

3.2 Stark training and testing dataset results

Figures 2 to 4 show comparison of measured and predicted ϕ'_r for training and testing data sets; the statistical parameters are summarized in Tables 2 and 3.

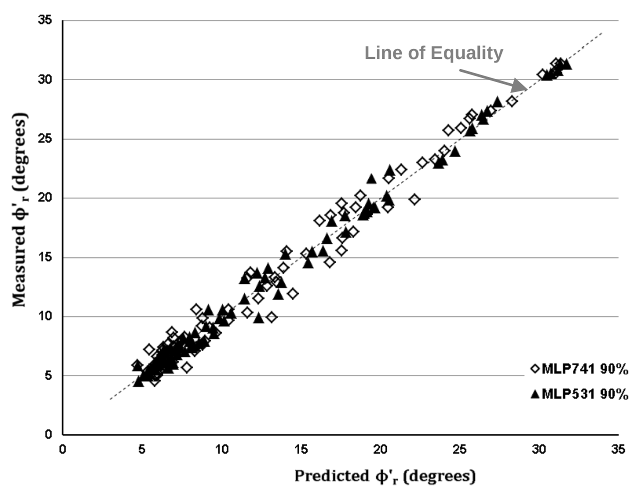

Figure 2. The predicted and measured ϕ'_r comparison on training sets for MLP741 90% and MLP531 90%; Stark and Hussain (2013) training data set.

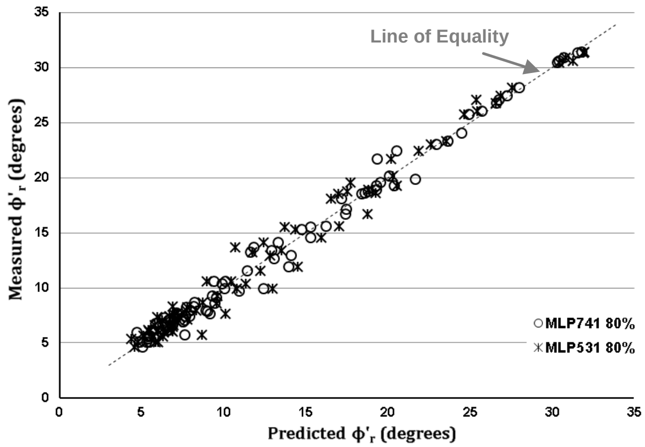

Figure 3. The predicted and measured ϕ'_r for MLP741 80% and MLP531 80% comparison; Stark and Hussain (2013) training data set.

Table 2. Statistical parameters for Stark and Hussain (2013) training data set.

Model Name	Statistical Parameters		
	RMSE	SE	Correlation Factor
MLP531 90%	0.53	0.76	0.995
MLP741 90%	1.09	1.13	0.990
MLP531 80%	1.01	1.14	0.990
MLP741 80%	0.71	0.82	0.995

The results of the training data set yielded a correlation factor of no less than 0.99 in the four models. The RMSE ranged from 0.53 (MLP531 90%) to 1.14 (MLP741 80%) and the SE ranged from 0.76 (MLP531 90%) to 1.14 (MLP531 80%). MLP531 90% and MLP 741 80% had the strongest correlation factors (0.995) whereas the other two models had correlation factors that were lower (0.990).

The results of the testing data set yielded correlation factors between 0.986 (MLP531 80%) and 0.992 (MLP531 90% and MLP741 90%). The RMSE for the test set ranged from 1.03 (MLP741 80%) to 1.3 (MLP531 80%); the SE ranged from 1.02 (MLP741 80%) to 1.25 (MLP531 80%). The results are shown in Figure 4 and Table 3.

Upon training and testing the models, we tested the models using a data set of prairie soils provided by Clifton and the results were checked with the empirical method proposed by Stark and Hussain (2013). The statistical analysis results are listed in Table 4.
The results of the model show greater correlation factors for the Clifton data set than those predicted by Stark and Hussain (2013) method. The greatest correlation factors were obtained from MLP741 80% (0.935) and MLP531 90% (0.919).

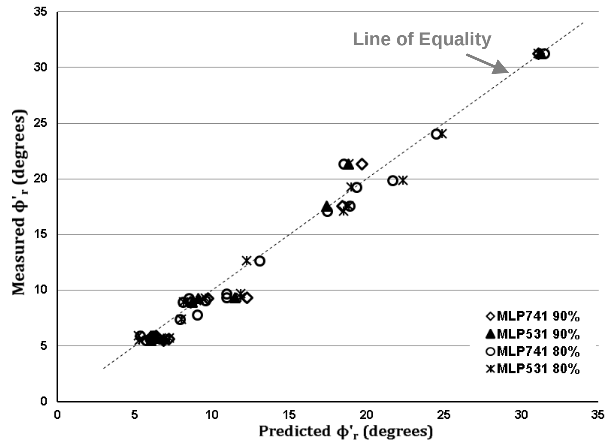

Figure 4. The predicted and measured ϕ'_r comparison for all models; Stark and Hussain (2013) test dataset.

Table 3. Statistical parameters for Stark and Hussain (2013) testing data set.

Model Name	Statistical Parameters		
	RMSE	SE	Correlation Factor
MLP531 90%	1.14	1.18	0.992
MLP741 90%	1.30	1.19	0.992
MLP531 80%	1.29	1.25	0.986
MLP741 80%	1.03	1.02	0.991

Table 4. Statistical parameters for Clifton data.

Model Name	Statistical Parameters		
	RMSE	SE	Correlation Factor
MLP531 90%	1.67	1.73	0.919
MLP741 90%	2.31	2.37	0.842
MLP531 80%	2.24	2.28	0.854
MLP741 80%	1.55	1.55	0.935
Stark & Hussein (2013)	2.56	2.19	0.815

In addition to yielding the greatest correlation factors, MLP741 80% also had the lowest SE (1.55) of the test sets. The Stark and Hussain (2013) empirical method yielded the lowest correlation factor (0.815) of the models tested.

Figure 5 illustrates the comparison of model MLP741 80% and the output from Stark and Hussain (2013) empirical method. As shown, both MLP741 80% and the Stark and Hussain (2013) model can predict ϕ'_r more accurately for lower values of ϕ'_r (i.e., ϕ'_r < 11º) as expressed by the data distribution near the line of equality.

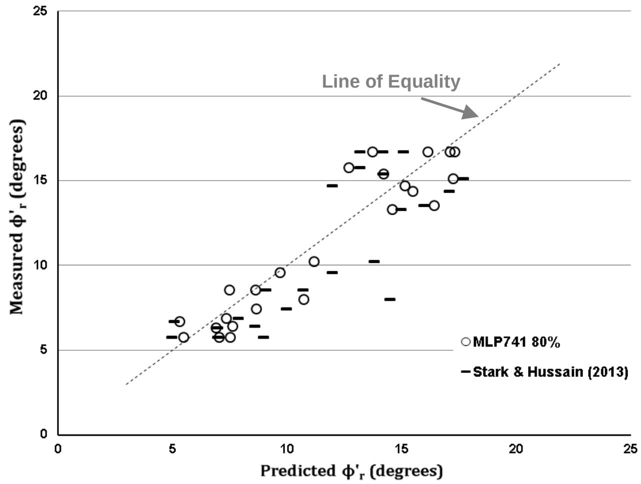

Figure 5. The measured and predicted ϕ'_r for Clifton data from the MLP741 80% model and Stark and Hussain (2013) empirical method.

4 CONCLUSIONS AND RECOMMENDATIONS

This study investigated the reliability of using ANNs to predict ϕ'_r values by training and testing four models using data from Stark and Hussain (2013). Correlation factors were determined for the test and training sets of the models, which yielded the strongest correlation for MLP741 80% (0.991) and no correlation factor less than 0.98. The model was validated by testing prairie soils and comparing the results to prairie soils provided by Clifton Associates Ltd. and comparing the results to those obtained by empirical method proposed by Stark and Hussain (2013). A correlation factor of 0.930 was obtained from the Clifton data set, which is greater than the correlation factor obtained from the Stark and Hussain (2013) method (0.815).

We recommend that a larger data set be used to train the ANN models with soil data from the Prairie Provinces to obtain more accurately trained models, which could be used to predict soil strength parameters and applied to geotechnical engineering practice.

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REFERENCES

Collotta, T., Cantoni, R., Pavesi, U., Ruberl, E., and Moretti, P. C. 1989. A correlation between residual friction angle, gradation and the index properties of cohesive soils. *Geotechnique*, 39(2), 343-346.

Detwiler, L. 2014. Using Backpropagation Neural Networks for the Prediction of Residual Shear Strength of Cohesive Soils. *UVM Honors College* Senior Theses. Paper 12.

Erzin, Y. 2007. Artificial neural networks approach for swell pressure versus soil suction behaviour. *Canadian Geotechnical Journal*, 44, 1215-1223.

Goh, A. T. C. 1994. Nonlinear modeling in geotechnical engineering using neural networks. *Australian Civil Engineering Transactions*, CE36(4), 293-297.

Gardner, M. W. and Dorling, S. R. 1998. Artificial neural networks (The multilayer perceptron) - A review of applications in the atmospheric sciences. *Atmospheric Environment*, 32(14/15), 2627-2636.

Hussain, M. 2010. Analysis and behavior of preexisting landslides. Ph.D. Thesis submitted in partial fulfillment of requirements for Doctoral Degree, *University of Illinois at UrbanaChampaign*, USA, 308 p. Available at: https://www.ideals.illinois.edu/handle/2142/16071.

Kanji, M. A. 1974. Relationship between drained friction angles and Atterberg limits of natural soils. *Geotechnique*, 24(4), 671–674.

KHANLARI, G.R., HEIDARI, M., MOMENI, A.A., ABDILOR, Y. 2012. Prediction of shear strenght parameters of soils using artificial neural networks and multivariate regression methods. *Engineering geology*, p.11-18.

Lupini, J. F., Skinner, A. E., Vaughan, P. R. 1981. The drained residual strength of cohesive soils. *Geotechnique*, 31(2), 181–213.

Maier, H. R. and Dandy, G. C. 2000. Neural networks for the prediction and forecasting of water resources variables: A review of modelling issues and applications. *Environmental Modelling & Software*, 15(2000), 101-124.

Mesri, G., and Cepeda-Diaz, A. F., 1986. Residual shear strength of clays and shales. *Geotechnique*, 36(2), 269–274.

Mesri, G., and Shahien, M., 2003. Residual shear strength mobilized in first-time slope failures. *J. Geotech. Geoenviron. Eng.*, 129(1), 12–31.

Najjar, Y. M. and Ali, H. E. 1998. CPT-based liquefaction potential assessment: A neuronet approach. *Geotechnical Special Publication*, ASCE, 1, 542-553.

Peters N.; Lamb K. N. 1979. Experiences with alluvial foundations for earth dams in the Prairie provinces. *Canadian Geotechnical Journal* 16(2): 255-271

Rani Sudha, Phani Kumar Vaddi, N.V.Vamsi Krishna Togati. 2013. Artificial Neural Networks (ANNS) For Prediction of Engineering Properties of Soils. *International Journal of Innovative Technology and Exploring Engineering* (IJITEE) ISSN: 2278-3075, Volume-3, Issue-1

Shahin, M.A., M.B. Jaksa, and Maier, H.R. 2001. Artificial neural network applications in geotechnical Engineering. *Australian Geomechanics*, Vol. 36, No. 1, pp. 49-62.

Shahin, M. A., and Indraratna, B. 2006. Modeling the mechanical behavior of railway ballast using artificial neural networks. *Canadian Geotechnical Journal*, 43(1), 1144-1152.

Sivakugan, N., Eckersley, J. D., and Li, H. 1998. Settlement predictions using neural networks. *Australian Civil Engineering Transactions*, CE40, 49-52.

Skempton A.W. 1964. The long term stability of clay slopes. *Geotechnique*, 14, 77-101

Skempton. A. W. 1977. Slope stability of cuttings in brown London clay. Proc., *9th Int. Conf. on Soil Mech. and Found. Eng.*, Vol. 3. Tokyo, 26 1-270.

Skempton, A. W. 1985. Residual strength of clays in landslides, folded strata and the laboratory. *Geotechnique*, 35(1), 3–18.

Stark, T. D., and Eid, H. T. 1994. Drained residual strength of cohesive soils. *J. Geotech. Geoenviron. Eng.*, 120(5), 856-871.

Stark, T. D., Choi, H., and McCone, S. 2005. Drained shear strength parameters for analysis of landslides. *J. Geotech. Geoenviron. Eng.* 131(5): 575-588.

Stark, T. D., and Hussain, M. 2013. Empirical correlations: Drained shear strength for slope stability analyses. J. Geotech. Geoenviron. Eng., 10.1061/(ASCE)GT.1943-5606.0000824, 853–862

Tizpa, P., Jamshidi Chenari, R., Karimpour Fard, M., and Lemos Machado, S. 2014. ANN prediction of some geotechnical properties of soil from their index parameters. *Arabian Journal of Geosciences,* 10.1007/s12517-014-1304-3, 2911-2920.

Tiwari, B., Marui, H. 2005. A New Method for the Correlation of Residual Shear Strength of the Soil with Mineralogical Composition. *J. Geotech. Geoenviron. Eng, ASCE.*, 131(9), 1139–1150.

Ural, D. N., and Saka, H. 1998. Liquefaction assessment by neural networks. *Electronic Journal of Geotechnical Engrg.*, http://geotech.civen.okstate.edu/ejge/ppr9803/index.html.

Vaziri, N. Hojabri, A. Erfani, A. Monsefi, M. Nilforooshan, B. 2007. Critical heat flux prediction by using radial basis function and multilayer perceptron neural network: a comparison study. *Nucl. Eng.* Des. 237, 377–385.

Voight, B. 1973. Correlation between Atterberg plasticity limits and residual shear strength of natural soils. *Geotechnique*, 23: 265—267.

Wesley, L.D. 2003. Residual strength of clays and correlations using Atterberg limits. *Geotechnique*, 53(7), 669-672.