Probabilistic slope stability analysis of reinforced slopes by finite element method

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ABSTRACT
A probabilistic slope stability analysis tool for geosynthetic reinforced slopes (and embankments) that combines shear strength reduction method (SSR) with probability theory was developed using a finite element method (FEM) source code for unreinforced slopes. The original numerical model in combination with the shear strength reduction method and Monte Carlo simulation (MC) was first used to compute probability of failure and factor of safety for simple unreinforced slopes with purely cohesive and cohesive-frictional soil. The results are compared to those from combined limit equilibrium method (LEM) and probability theory recently published in the literature. The results are shown to be in good agreement. The validated FEM code was then modified to investigate reinforced slope cases. To verify the new code, a general reinforced slope case was examined using the new program and the FEM software package SIGMA/W. Displacements and reinforcement strains calculated using both programs agreed well. The utility of the new code in combination with Monte Carlo simulation and the shear strength reduction method is demonstrated by a number of examples. The general approach applied to reinforced soil slopes is novel and offers a powerful tool to relate conventional notions of factor of safety for reinforced soil slopes to margins of safety described by more meaningful probability of failure.

RESUMÉ
Un outil d'analyse probabilistique de stabilité de pente pour des pentes et remblais renforcés avec des géosynthétiques, qui combine la méthode de réduction de la résistance au cisaillement (SSR) avec la théorie probabilistique, a été développé en utilisant un code source pour des pentes non renforcées avec une méthode d'éléments finis (FEM). Le modèle numérique original, en combinaison avec la méthode SSR et la simulation Monte Carlo (MC), a d'abord été utilisé pour calculer la probabilité de rupture, ainsi que le facteur de sécurité pour des pentes non renforcées simples avec des sols purement cohérents et cohérents-frictionnels. Les résultats sont comparés à ceux de la méthode d'équilibre limite combinée (LEM) et de la théorie probabilistique publiées récemment dans la littérature. Les résultats semblent bien s'accorder. Le code FEM validé a alors été modifié pour investiguer le cas de pentes renforcées. Pour vérifier le nouveau code, un cas général de pente renforcée a été examiné en utilisant le nouveau programme et le logiciel SIGMA/W (FEM). Les déplacements et les déformations renforcés calculés par les deux programmes s'accordent bien. L'utilité du nouveau code, en combinaison avec la simulation MC et la méthode SSR est démontrée par un bon nombre d'exemples. L'approche générale appliquée aux pentes renforcées est novatrice et offre un outil puissant pour relier les notions conventionnelles de facteur de sécurité pour les pentes renforcées aux marges de sécurité décrites par des probabilités de rupture plus significatives.

1 INTRODUCTION

Geosynthetic-reinforced slopes (and embankments) are widely used in geotechnical engineering. Since the early 1980s, conventional deterministic limit equilibrium methods (LEMs) for unreinforced slopes have been modified to include the stabilizing contribution of geosynthetic reinforcement layers. These methods include circular slip (Kitch 1994), log-spiral (Leshchinsky and Boedeker 1989) and two-part wedge (Bathurst and Jones 2001) approaches. The margin of safety is computed as a single-valued critical factor of safety for the reinforced slope. However, due to uncertainty in the magnitude of soil properties (Phoon and Kulhawy 1999), a carefully designed reinforced slope that satisfies a target factor of safety will always have some probability of failure (P_f). Furthermore, it has been frequently demonstrated in the literature that two nominally identical unreinforced soil structures can have the same factor of safety based on conventional deterministic factor of safety analysis methods but have very different probabilities of failure. The same must be true for reinforced slopes. Hence, it can be argued that a proper appreciation of the margin of safety for reinforced slopes is best understood in probabilistic terms.

Although many studies on probabilistic slope stability analysis of unreinforced slopes can be found in the literature, similar investigations for reinforced slope cases are limited. Kitch (1994) carried out probabilistic analyses of two reinforced slope examples with reinforcement layouts initially selected using design charts based on deterministic limit equilibrium methods. Low and Tang (1997a) proposed a limit equilibrium stability model for reinforced embankments on soft ground and a practical reliability evaluation procedure. However, the LEM method used in both studies has the major disadvantage that the type of critical failure surface must be assumed *a priori* (e.g. circular, non-circular or bi-linear). Recently, a probabilistic analysis technique called the Random Finite Element method (RFEM) has been developed by Griffiths and Fenton (2004) to conduct probabilistic stability analysis of slopes. This approach incorporates random field theory and the shear strength reduction (SSR) method within a Monte Carlo (MC) simulation framework. The combined SSR method and RFEM approach makes no assumption regarding the critical failure surface geometry and internal forces between slices which is the case for conventional method of slices approaches. At the time of this study, the RFEM approach has not been used to model reinforced slopes and embankments.

In the current study an existing RFEM program (Griffiths and Fenton 2004) was expanded to allow for probabilistic analysis of reinforced slope cases. Although the expanded code is capable of modeling random fields, in this investigation it was restricted to single random variable cases as a first step to validate the code. This means that all soil elements in the slope model are assigned the same soil property values in each Monte Carlo simulation run. To prevent confusion, the source code is referred to as a FEM code, hereafter.

The original unmodified FEM code was first validated by investigating unreinforced slope cases. Analysis results using the modified FEM source code were then compared to results of FEM analysis of reinforced slopes using a commercial software package. Results are shown to be in good agreement. Finally, the utility of the expanded code is demonstrated using a number of reinforced slope examples.

In this paper, probabilistic slope stability analyses using LEM methods together with probability theory are referred to as probabilistic limit equilibrium methods (PLEM). The FEM approach combined with MC simulation is referred to as PSSR, and incorporates the shear strength reduction method (SSR).

2 VERIFICATION OF THE FEM SOURCE CODE

2.1 Probabilistic slope stability study of unreinforced cohesive slopes

2.1.1 Analysis using PLEM with closed-form solution

Results using the PSSR method are first compared to PLEM results where the latter are taken from probabilistic stability design charts for simple unreinforced purely cohesive soil slopes developed by Javankhoshdel and Bathurst (2014). They combined probability theory and Taylor's slope stability equation (Eq. 1) for cohesive soils (Taylor 1937). Taylor's slope stability equation is expressed as:

$$F_s = \frac{\mu_{su}}{\mu_\gamma H N_s} \quad [1]$$

Here F_s is the mean factor of safety computed using mean values of S_u and γ (μ_{su} and μ_γ), slope height H and slope stability number N_s . F_s was used in Eq. 2 together with coefficients of variation of undrained shear strength (COV_{su}) and total unit weight (COV_γ) to calculate probability of failure, P_f.

$$P_f = p[F_s < 1] = \Phi\left\{\frac{\ln\left(\sqrt{\frac{1+COV_{su}^2}{1+COV_\gamma^2}}/F_s\right)}{\sqrt{\ln\left[\left(1+COV_{su}^2\right)\left(1+COV_\gamma^2\right)\right]}}\right\} \quad [2]$$

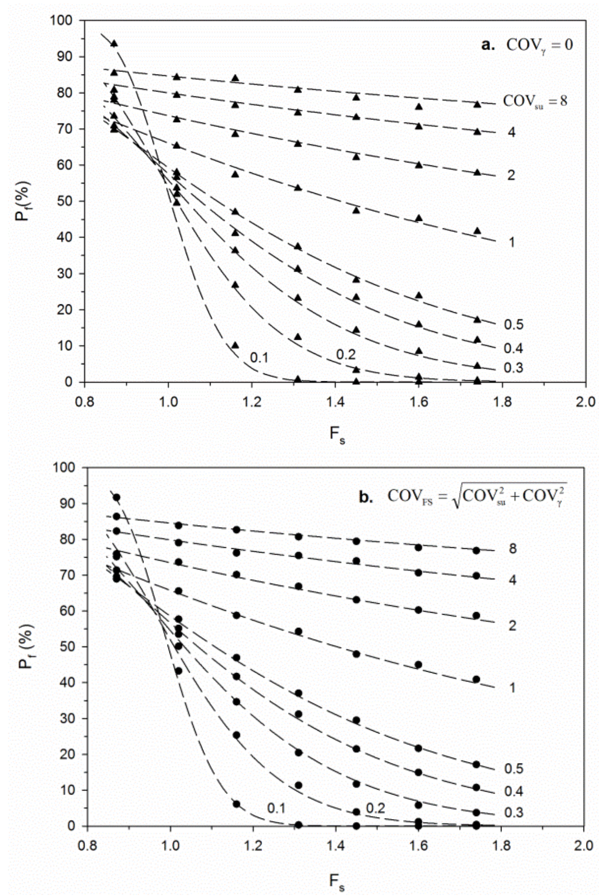

Fig. 1 Probability of failure (P_f) versus (deterministic) mean factor of safety (F_s) for cohesive soil slopes with lognormal distribution of undrained shear strength (S_u) and unit weight (γ) computed using PLEM with closed-

form solution (dash lines) and PSSR (symbols) approaches

Fig. 1a and Fig. 1b present two different series of cases. The first one considers the total unit weight γ to be deterministic (no variability) and hence COV_γ is equal to zero. For the second series of cases, both undrained shear strength S_u and total unit weight γ are considered as uncorrelated random variables. Therefore the coefficient of variation of factor of safety is due to the variability in random variables S_u and γ, and is calculated as:

$$COV_{FS} = \sqrt{COV_{su}^2 + COV_\gamma^2} \quad [3]$$

The dash curves in Fig. 1a and Fig. 1b are the closed-form solutions using Eq. 2. The data show that as the mean factor of safety decreases for any constant level of variability in random variables, the probability of failure also decreases, which is expected.

2.1.2 Analysis using PSSR

Fig. 2 shows the simple slope geometry used in the simulations. In order to compare PSSR outcomes with PLEM results, two different groups of probabilistic analysis were conducted. In the first group the undrained shear strength S_u was the only random variable, while in the second group both the undrained shear strength S_u and the unit weight γ of the soil were treated as random variables. The mean value of the unit weight was 20 kN/m^3. All random variables are assumed to have lognormal distribution. For deterministic stability analysis, the choice of Young's Modulus (E) and Poisson's ratio (ν) has little influence on stability analysis outcomes (Griffiths and Lane 1999), hence parameters E and ν were taken as 100 MPa and 0.3, respectively. The mean value of S_u was varied from 30 to 60 kPa with an increment of 5 kPa. For the first group of analyses the undrained shear strength S_u was the only random variable and it was determined that a total number of 1000 Monte Carlo simulations was sufficient to give a consistent estimate of probability of failure. However, for the second group with two random variables, 2000 Monte Carlo simulations were needed to obtain a reliable result. A parametric study to investigate the influence of number of Monte Carlo simulations on probability of failure outcomes related to two random variables was also performed. Since cohesion (c) and friction angle (ϕ) are the most important parameters in slope stability analysis, they were taken as the random variables in the parametric study. Fig. 3 shows that for cases with two random variables, 2000 Monte Carlo simulations will give similar estimates of probability of failure.

The simulation results for the first and second groups are plotted as symbols in Fig. 1a and Fig. 1b, respectively. The PLEM results based on the closed-form solution and PSSR outcomes can be seen to agree very well.

2.2 Probabilistic slope stability study of unreinforced cohesive-frictional (c-ϕ) soil slopes

Javankhoshdel and Bathurst (2014) also developed stability design charts for simple unreinforced c-ϕ soil slopes using PLEM (LEM with MC simulation). In these charts both cohesion c and friction angle ϕ were considered as random variables having lognormal distributions. The SVSlope software package (Fredlund and Thode 2011) was used to carry out circular slip (simplified Bishop's method) analyses together with the Floating Method option for probabilistic analyses. The solid lines in Fig. 4 show numerical results for $\mu_c / (\mu_\gamma H \tan\mu_\phi) = 0.2$ and $\mu_\phi = 30°$. These curves are general and apply also to for different combinations of μ_c, μ_γ and H, as long as $\mu_c / (\mu_\gamma H \tan\mu_\phi) = 0.2$. In their analysis, 4500 Monte Carlo simulations were used in each case. To check the PSSR approach for cohesive-frictional soil slopes, a series of analyses were carried out based on the slope with the geometry shown in Fig. 2. The slope gradient was varied from 0 to 1.5 to obtain different mean values of factor of safety. Apart from the slope angle, the slope model has the same geometry and boundary conditions as those shown in Fig. 2. Parameters used in the program are shown in Table 1. Again, based on the results in Fig. 3, a total of 2000 Monte Carlo simulations were used in PSSR analyses.

Fig. 4 shows that both programs give very similar results for different combinations of COV_c and COV_ϕ.

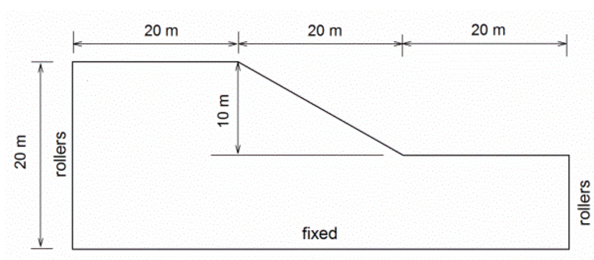

Fig. 2 Test problem for cohesive soil slope

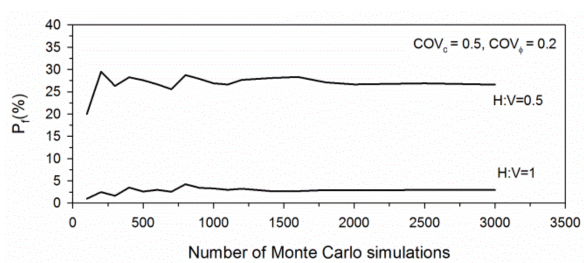

Fig. 3 Number of Monte Carlo simulations versus probability of failure (P_f) for c-ϕ soil slope examples with two random variables

3 VERIFICATION OF MODIFIED FEM CODE

3.1 Methodology

With the exception of some minor modifications, the FEM source code is Program 6.2 found in the book by Smith and Griffiths (1998) was used in the current study. The bulk of the code is for SSR analysis. The original program is for two-dimensional slope stability analysis of unreinforced slopes with elastic-perfectly plastic soils governed by Mohr-Coulomb failure criterion. Eight-node quadrilateral elements are used. The bottom of the slope foundation is fixed in both horizontal and vertical directions. The vertical boundaries on both sides are fixed in the horizontal direction. The gravity "turn-on method" is used in the analysis. For more details, see Griffiths and Lane (1999).

Table1. Input parameters for PSSR analyses with c - ϕ soil slopes

H:V	0 - 1.5
H	10 m
Cohesion c	23 kPa
Friction angle ϕ	30°
Unit weight γ	20 kN/m^3
Young's modulus E	100 MPa
Poisson's ratio	0.3

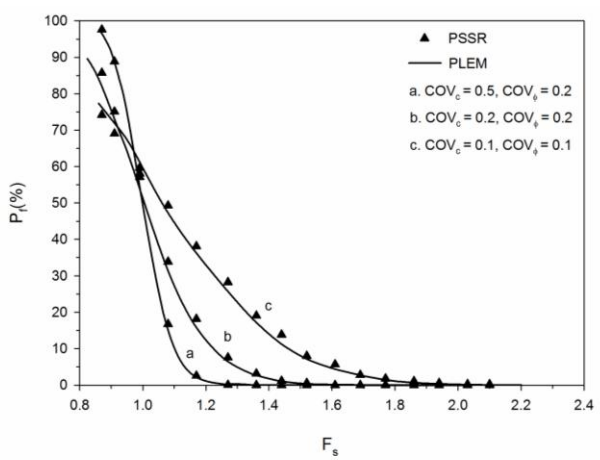

Fig. 4 Probability of failure (P_f) versus (deterministic) mean value of factor of safety (F_s) with a range of COV for strength parameter values (comparison between PLEM and PSSR approaches)

In this study, three-node beam elements were added to the program to model reinforcement layers. The beam elements have no bending stiffness, Therefore, the simplified beam element, which is also called a bar/rod element, has only one degree of freedom. There is no interface between the reinforcement and soil (i.e. the soil and reinforcement elements are perfectly bonded). In the SSR analysis, the axial stiffness of the reinforcement is constant and only the soil strength parameters are reduced.

The modified source code was first checked by comparing the computed reinforcement strains and displacements with results from the software package Sigma/W (Geo-Slope Ltd. 2014) for the same stable slope in Fig 5.

The first step was to select a reinforced slope case. The second step was to select convergence tolerance limits for both programs so that both will give similar and consistent numerical outcomes. This preliminary exercise is necessary because the two FEM programs use different convergence criteria to identify numerical equilibrium.

3.2 Reinforced slope model for code verification

In this investigation a slope with a slope face angle of 45°, slope height of 5 m and a horizontal backfill surface and fore slope was assumed. The unfactored friction angle of the soil was assumed as 30°, and the unit weight of the soil as 18 kN/m^3. To avoid numerical instability during finite element analyses, the soil cohesion was assigned a value of 1 kPa. The candidate reinforcement was taken as a uniaxial geogrid with properties reported by Walters et al. (2002). The axial stiffness of the reinforcement is 600 kN/m and the tensile strength at rupture is 72 kN/m. Using the prescribed slope geometry, soil properties, and reinforcement properties described above, the number of reinforcement layers and spacing between reinforcement layers was determined using the LEM-based design charts by Bathurst and Jones (2001). Assuming a nominal factor of safety equal to 1.3, the design friction angle of soil was 23.9° and the minimum ratio of reinforcement length to slope height was found to be L/H = 0.81, which leads to a minimum reinforcement length of 4.05 m. For convenience, the length of the reinforcement was taken as 5 m. It was determined that a minimum of 5 layers of reinforcement at 1 m vertical spacing was required to achieve a factor of safety of 1.3 using the charts by Bathurst and Jones (2001). The geometry of the reinforced slope is shown in Fig. 5. The matching FEM mesh is shown in Fig. 6.

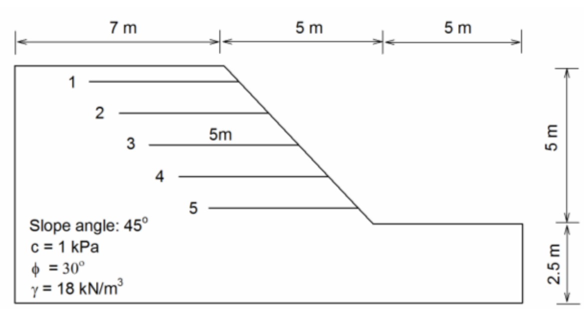

Fig. 5 Geometry of reinforced slope model

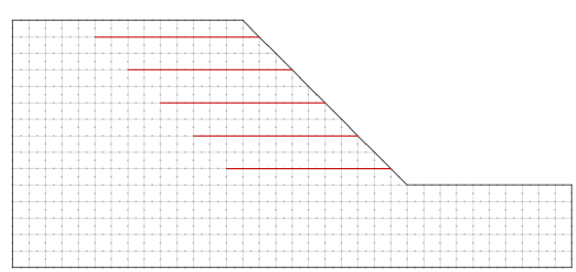

Fig. 6 FEM mesh for reinforced slope model

3.3 Convergence criterion limit for modified FEM code

A convergence criterion is implemented in the original and modified FEM code developed by the writers to determine when the model is in equilibrium (converged). A solution is deemed to have converged if the relative change between the largest nodal displacements of two successive iterations is smaller than a specified tolerance. Fig. 7

shows that as the convergence tolerance decreases, computed reinforcement strains are more consistent. The difference between results based on the convergence tolerance of 10^{-6} and 10^{-5} (relative displacement) is not visually distinguishable. There is a negligible difference between the results for convergence tolerance of 10^{-4} and 10^{-5}. Although the result of convergence tolerance of 10^{-5} is slightly more accurate, a higher computation cost due to more iteration steps is required for negligible benefit. Therefore, a convergence tolerance of 10^{-4} was used in all simulation runs using the modified FEM code.

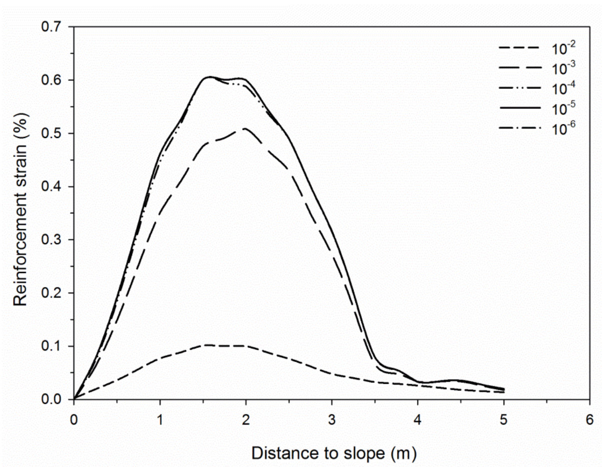

Fig. 7 Influence of convergence tolerance on distribution of reinforcement strain (layer 3)

3.4 Convergence criterion limit for Sigma/W FEM program

The reinforced slope shown in Fig. 5 was also analyzed using Sigma/W. To be consistent with the former program procedure, the gravity turn-on method was used. During calculations, Sigma/W computes the difference between the displacements at each node from two successive iterations. If the difference is within a specified tolerance (minimum displacement), the solution is deemed to have converged. For example, consider two successive displacements such as 1.23×10^{-6} m and 1.23×10^{-7} m. These two numbers have the same number of significant digits and the difference is 1.11×10^{-6} m. If a minimum difference of 1.0×10^{-5} m is specified, then the solution is deemed to have converged. Sigma/W provides the user with an option to select the significant digit number from 1 to 8 to obtain different accuracy. Fig. 8 shows that, the combination of significant digit number and minimum difference has a large influence on the accuracy of the results. For a smaller minimum difference value and a larger significant digit number, a higher accuracy can be obtained. In this particular problem, five significant digits were used and the minimum displacement difference specified was 10^{-6} m.

3.5 Comparison of results using modified FEM and Sigma/W

Fig. 9 shows that the strains, horizontal and vertical displacements of reinforcement computed using both programs agree well.

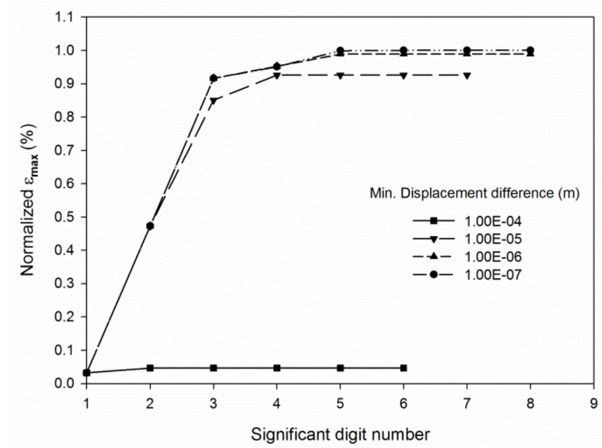

Fig. 8 Influence of significant digit number and minimum displacement difference on normalized maximum reinforcement strain (layer 3)

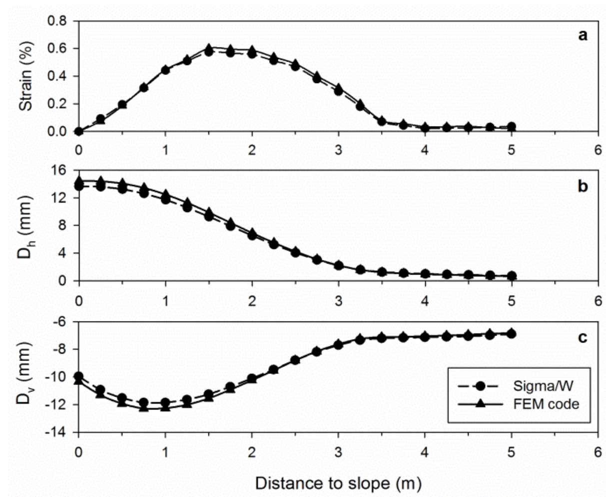

Fig.9 Comparison of analysis results using modified FEM code and Sigma/W program (layer 3). (a) Reinforcement strain. (b) Reinforcement horizontal displacement D_h. (c) Reinforcement vertical displacement D_v.

3.6 Determination of factor of safety

The iteration ceiling adopted in the modified FEM program to satisfy the convergence criterion limit was 4000 for cases when the program was searching for the reduction in soil strength (reduction factor) required to bring the model (slope) to failure. If the algorithm is unable to converge within 4000 iterations, a rapid increase in nodal displacements occurs indicating that the slope has failed. The corresponding shear strength reduction factor used by the program at failure is the factor of safety of the reinforced slope. For the simple reinforced soil slope used here, the modified FEM program gave a factor of safety of 1.5, which is very close to the result calculated by LEM (F_s = 1.506, Bishop's method). It can be seen that the FEM result is higher than the nominal factor of safety (F_s = 1.3) adopted in the original design. This can be explained by noting that the design charts were developed by first searching for the critical two-part wedge failure surface of the unreinforced slope. Hence, the critical failure mechanisms are constrained to two-part wedge geometries and are decoupled from the reinforcement tensile loads that are added afterwards and computed as the sum of the forces required to keep the slope in horizontal equilibrium. The design charts ensure that all reinforcement layers extend beyond the critical unreinforced failure mechanism. Hence, the design charts consider only internal stability of the reinforced soil zone. For these reasons the design charts give a conservative (safe) estimate of factor of safety compared to LEM and FEM predictions.

3.7 Failure features of reinforced and unreinforced slopes

Fig. 10 shows that for the unreinforced slope the plastic deformation (yield) zone is small. Fig. 10a shows that the failure surface is shallow and passes above the toe. In Fig. 10b, a deformation (shear) band is visually detectable. Fig. 11 shows that the addition of reinforcement layers increases the size of the plastic deformation zone. As a result, the stability of the slope is increased.

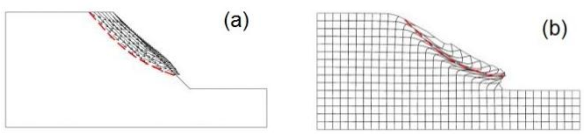

Fig. 10 Failure features of unreinforced slope with F_s = 0.79. (a) Plastic displacement vectors. (b) Deformed mesh. (c = 1 kPa, ϕ = 30°, γ = 18 kN/m^3)

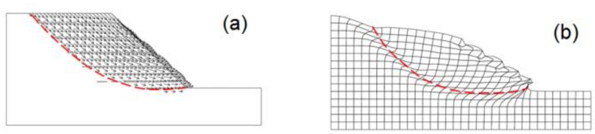

Fig. 11 Failure features of reinforced slope with F_s = 1.5. (a) Plastic displacement vectors. (b) Deformed mesh. (c = 1 kPa, ϕ = 30°, γ = 18 kN/m^3, J = 600 kN/m)

4 PROBABILISTIC STABILITY STUDY OF REINFORCED SLOPES USING MODIFIED FEM CODE

The reinforced slope model shown in Fig.5 was used for probabilistic study in this section. During the calculations, only soil friction angle was varied while reinforcement properties and slope geometry were held constant. Fig. 12 shows results of simulations using the modified FEM code with MC simulation to investigate the influence of mean soil friction angle and random variability of soil friction angle on probability of failure. The plots show that for reinforced and unreinforced slopes with the same nominal soil properties and geometry, the reinforced slope has a higher factor of safety and lower probability of failure.

For example, for the case of an unreinforced slope with a friction angle of 35 degrees, the factor of safety is 0.92, which means the slope has failed (Fig. 12a). On the other hand, a reinforced slope with the same soil properties has a factor of safety as high as 1.79.

Results of probabilistic analyses are shown in Fig 12b. For an unreinforced slope with a friction angle of 35 degrees and a coefficient of variation of 0.2, the probability of failure is 69%. If the slope is constructed with 5 layers of reinforcement, the probability of failure decreases to 0.6% in this example, which is a very large improvement. For the same unreinforced slope but with a coefficient of variation of 0.5, the probability of failure is 66%. If the slope is constructed with the same number of reinforcement layers as before, the probability of failure will decrease to 19%. Again, the reinforcement reduces the probability of failure by a large amount. However, the effect of reinforcement on reducing P_f values is also influenced by amount of variability in the estimate of soil friction angle. For slopes with a low coefficient of variation of ϕ (COV_ϕ = 0.2), the difference between probability of failure values of reinforced and unreinforced slopes is relatively large, especially for friction angles in the range of 25 to 40 degrees. However, for slopes with COV_ϕ = 0.5, the change in P_f is relatively small. This is consistent with the expectation that reinforcement is less effective in stabilizing a slope if the slope has high variability in soil properties.

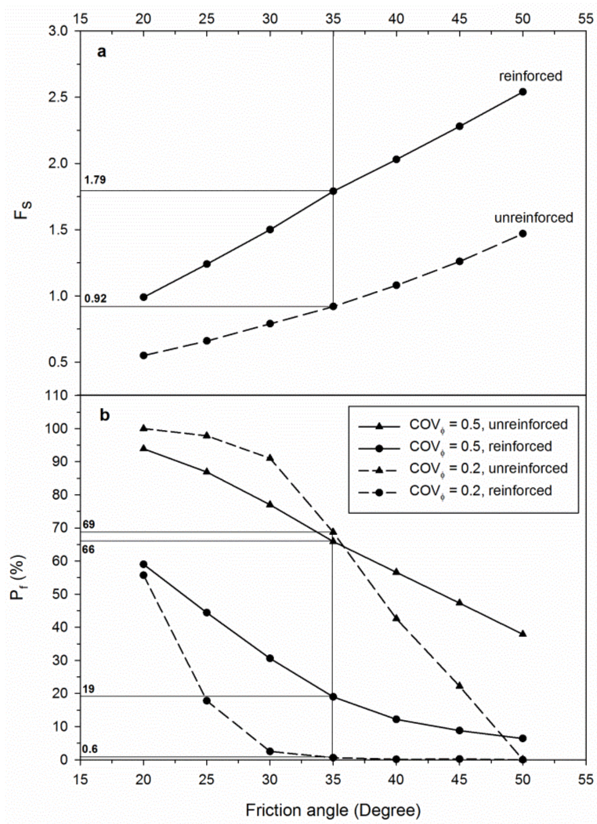

Fig. 12 (a) Factor of safety (F_s) and (b) probability of failure (P_f) versus nominal (mean) friction angle for unreinforced and reinforced soil slopes and different COV for soil friction angle

A series of probability of failure versus factor of safety curves are plotted in Fig.13 for the case of 45° slopes and (mean) friction angle from 20 to 60 degrees. The larger value was selected to detect trends in data outcomes. The plot shows that for slopes with COV_{ϕ} = 0.2 and factor of safety equal to 1.79, the probability of failure of the unreinforced slopes is 2.9% while the P_f value for the reinforced slope cases is 0.6%. For the slopes with COV_{ϕ} = 0.5 and the same factor of safety, the probability of failure of the unreinforced slopes is as high as 32%; however, for the reinforced slopes the P_f value decreases to 19%. In both cases, the reinforcement plays an important role in reducing the potential for failure of the slope.

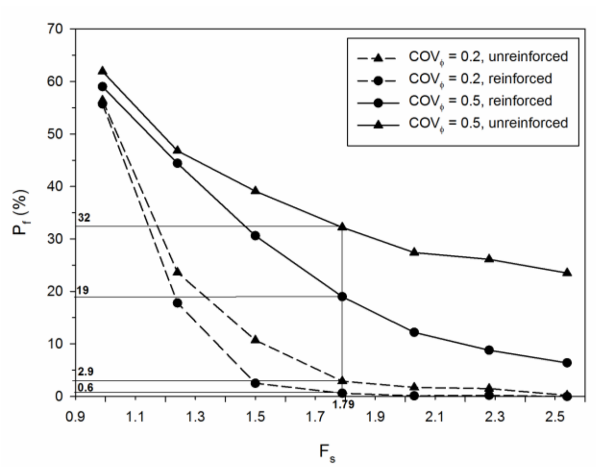

Fig. 13 Probability of failure (P_f) versus factor of safety (F_s) for unreinforced and reinforced slopes

5 CONCLUSION

A numerical tool for probabilistic stability analysis of geosynthetic reinforced slopes has been developed based on additions to the FEM source code for unreinforced slopes developed by Griffiths and Fenton (2004). The modified FEM code has been validated against a commercial code and then used for probabilistic stability analysis of reinforced slopes. This was done as a first step to validate the new code which has a module to investigate spatial variability and is the subject of ongoing research by the writers.

Results presented in the current study show that the reinforcement can increase the factor of safety and decrease the probability of failure of a slope. For a cohesionless soil slope with low variability of soil friction angle, the reinforcement can reduce the probability of failure to effectively zero. For a slope with a high variability of soil friction angle, the reinforcement can still reduce the probability of failure by a large amount.

The advantage of the new numerical tool is that it offers a powerful tool to relate conventional notions of factor of safety for reinforced soil slopes to margins of safety described by more meaningful probability of failure.

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