

Centrifuge modelling of shallow foundations on firm over soft layered clay

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Challenges from North to South
Des défis du Nord au Sud

ABSTRACT

In practice shallow foundations are rarely simply founded on a homogenous soil beds hence there are uncertainties surrounding the mode of failure that will prevail and the bearing capacity factor that should be adopted in design. A series of centrifuge model tests on surface strip footings resting on a layered soil bed having firm clay overlying soft clay of varying thickness are presented. Plane strain test conditions enabled the use of Digital Image Correlation (DIC) to determine the relative displacement of the soil and reveal the foundation failure mechanics. Complementary numerical modelling is also conducted which verifies the model experimental data. General shear and punch failure were observed and the dominant failure mechanism shown to depend on the ratio of the upper layer thickness and footing width.

RÉSUMÉ

Dans la pratique, les fondations sont rarement construites sur des couches de sol homogène. Il y a donc des incertitudes entourant le mode de rupture qui prévaudra et le facteur de capacité portante qui devrait être adopté dans la conception. Une série d'essais à la centrifugeuse sur des semelles filantes superficielles reposant sur un sol stratifié composé d'une couche d'argile raide recouvrant une couche d'argile molle d'épaisseur variable est présentée. Les conditions des tests de déformation plane ont permis l'utilisation de la corrélation d'image numérique (DIC) afin de déterminer le déplacement relatif du sol et de révéler les mécanismes de rupture de la fondation. En complémentarité, une modélisation numérique est aussi réalisée afin de vérifier les données du modèle expérimental. Les ruptures générales par cisaillement et par poinçonnement ont été observées et le mécanisme dominant variait en fonction du ratio d'épaisseur entre la couche de sol superficielle et la semelle.

1 INTRODUCTION

The ultimate bearing capacity of surface strip footings resting on a single layer of homogeneous undrained clay (Figure 1) has been studied by numerous investigators, with practitioners generally adopting Terzaghi's (1943) expression to compute ultimate bearing capacity. In reality however, soil strength profiles beneath footings are rarely homogeneous and may increase or decrease with depth or consist of distinct layers having significantly different properties. While the effect of increasing strength with depth on bearing capacity has been addressed by several researchers, notably Davis & Booker (1973), rigorous solutions to the problem of footings resting on layered clays are less well established.

Meyerhof and Hanna (1978) described an investigation into the ultimate bearing capacity of foundations on layered soil under inclined loads. Two layers were considered, a dense or stiff layer overlying a weak deposit and a loose or soft layer overlying a firm deposit. The analysis identified different modes of failure and compared the results of model tests on circular and strip footings on layered sand and clay soils. Michalowski and Shi (1995) considered the bearing capacity of a strip footing over a two-layered foundation soil implementing a kinematic analysis to determine the average limiting pressure under the footing for a granular soil over clay. They reported that the depth of the collapse mechanism was dependent on the strength of the clay and thickness of the granular layer. Dimensionless design charts with respect to the internal friction angle of sand were presented. Burd and Frydman (1997) described results of

a similar investigation of a rigid shallow footing resting on sand over clay and illustrated the mode of failure and developed further supporting design charts. In particular, the results demonstrated that the shear strength of the clay has an important influence on the mechanisms of load spread.

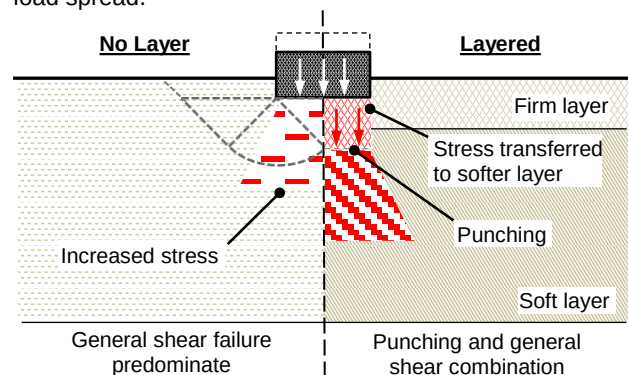


Figure 1. Shallow foundation supported by a layered soil.

Merifield et al. (1999) applied numerical limit analysis to evaluate the undrained bearing capacity of a rigid surface footing resting on a two-layer clay deposit using finite elements in conjunction with the upper and lower bound limit theorems of classical plasticity. Parametric studies were presented with respect to the undrained shear strength of each layer and their respective thickness. The authors observed the

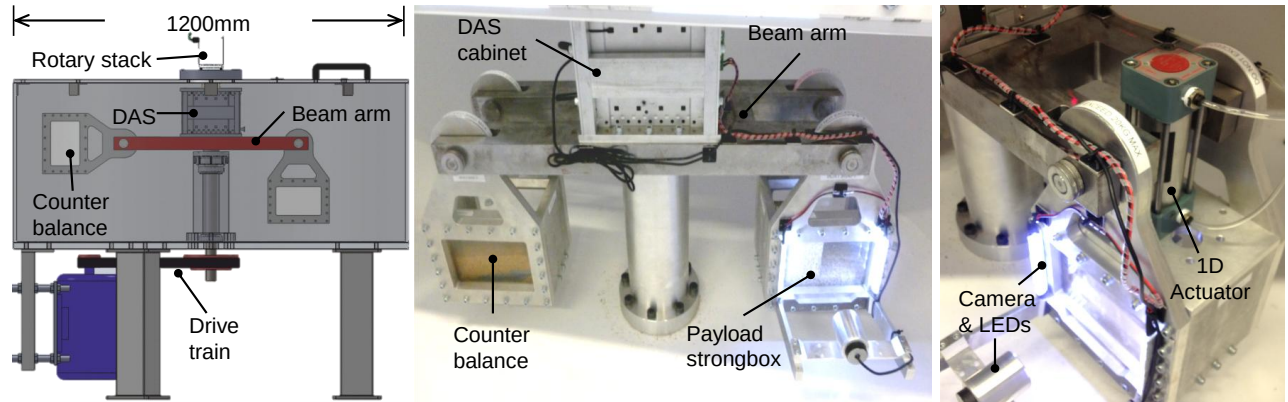


Figure 2. University of Sheffield small scale centrifuge facility.

occurrence of different failure mechanisms for the footing which they report to be a function of (i) the ratio of the soil strength of the upper and lower layer, (ii) ground profile configuration i.e. soft soil overlying firm or firm soil overlying soft and (iii) the thickness ratio of each layer. While it is evident that several investigations have been conducted using analytical and numerical methods, these are not correlated and validated with actual observations from field or laboratory tests.

More recently the aspect of bearing capacity on firm over soft layered soils has been focused on the offshore sector. For example, Baglioni et al. (1982) and Yuan et al. (2005) report on a comprehensive investigation relating to scud pan and jacket foundations in layered soils. Sudden punch failure was observed which poses great risk for design engineers when assessing the likely capacity for either temporary or permanent installations.

This paper reports on a series of centrifuge model tests that considered the bearing capacity of a shallow foundation supported on layered soil. Firm clay overlying a soft clay layer was appraised for a range of layer thickness combinations to evaluate impact on bearing capacity and mode of failure.

2 EXPERIMENTAL PROGRAMME

2.1 Centrifuge facility

The University of Sheffield small scale centrifuge was utilised in this research. The centrifuge has a nominal radius of 0.5 m and is capable of accelerating a payload of 20 kg, measuring 160 mm wide x 125 mm high x 80 mm depth, at 100 gravities (100g). The payload incorporates a viewing window which provides plane strain visualisation of the test package. The centrifuge is equipped with onboard wireless data acquisition, 2 MP camera and LED illumination for image capture, and a 2 port hydraulic rotary union for in-flight control of a 2 kN dual acting pneumatic vertical actuator. Full specification of this centrifuge is described by Black et al. (2014) and is summarised in Table 1.

In all modelling and simulation it is vital that small scale model conforms to appropriate scaling relationships to provide similitude with the full scale prototype. Centrifuge scaling laws are discussed in detail by Garnier

et al. (2007) and those observed in the current investigation are summarised in Table 2. Prototype stress conditions were achieved by applying an acceleration of 50g on the small scale models.

Table 1. Centrifuge specification summary.

Specification	Description
Radius (effective)	0.5 m (0.44 m)
Maximum payload	20 kg at 100g (2g-ton)
Maximum acceleration	100g at 20kg (≈ 425 RPM) 150g at 10kg (≈ 525 RPM)
Size of payload	W = 160 mm; H = 125 mm D = 80 mm
User interfaces	2 port 10bar hydraulic union, 4 way electrical 24A slip ring
Data acquisition	8 Ch AI, 2 MP image capture, wireless communication

Table 2. Centrifuge scaling laws.

Parameter	Scaling law (Model/Prototype)
Gravity (m/s^2)	$1 \cdot N$
Length (m)	$1/N$
Area (m^2)	$1/N^2$
Volume (m^3)	$1/N^3$
Density (kg/m^3)	1
Stress (kN/m^2)	1
Unit Weight (kN/m^3)	$1 \cdot N$
Strain	1

2.2 Soil Type and Properties

Kaolin clay was used in this investigation. Samples were prepared by consolidating slurry mixed with de-aired water at 1.5 times the Liquid Limit. Consolidation pressures were ramped up to 200 kN/m² and 400 kN/m² to produce consolidated homogeneous blocks of clay that were representative of soft and firm soil respectively. Several specimens were extracted for undrained strength assessment using unconsolidated undrained (UU) triaxial tests. Confining pressures of 50 kN/m² to 200 kN/m² were considered as this reflected the ground stress range in the centrifuge experiments. The soft and firm consolidated clay blocks had an undrained strength (c_u) of 20 kN/m² and 40 kN/m² respectively (Figure 3). These strengths were also verified by hand vane shear measurements obtained at the end of testing.

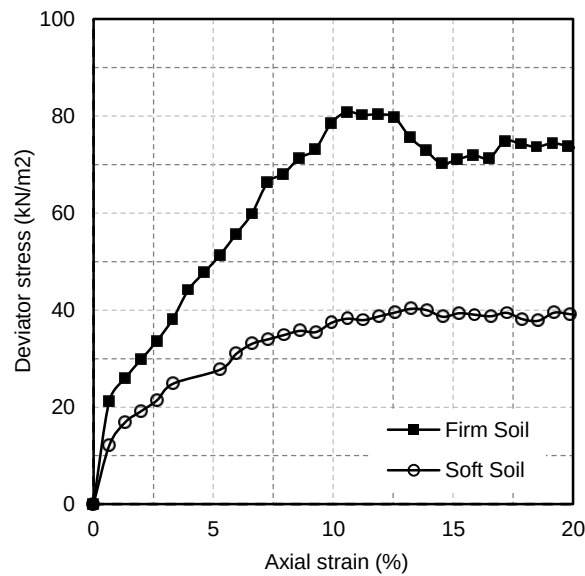


Figure 3. Strength assessment for soft and firm soil.

Liquid and plastic limit tests were also conducted and determined to be 70% and 34% respectively, yielding a Plasticity Index of 36%, such that the soil could be described as a high plasticity clay. The unit weight of the soft and firm soil was determined as 16.6 kN/m³ and 17.2 kN/m³ respectively.

2.3 Model preparation and procedure

Consolidated clay blocks were removed from the consolidation system in preparation for model construction. Soil layers of varying thickness of soft and firm soil were required to make a composite layered sample. Side cutting templates and a wire saw were used to trim the block to the desired layer thickness. The required model layer combinations were configured and the sample was placed back into the consolidation press under a nominal 100 kN/m² for a short period to ensure 'knitting' of the interface boundary between the upper and lower layer. The now combined block sample was then

carefully extracted and the side faces textured with 'flock' to provide sufficient contrast variation in colour to enable image correlation processes to be used for soil displacement tracking. The front viewing window was lightly greased to reduce interface friction and the sample was located into the strongbox with the rear side plate reattached.

The mass of the counterweight was established and both the payload and counterweight were mounted into position on the beam. All electrical cables for camera and LED illumination were terminated and secured. The vertical actuator, complete with a model strip foundation of width (B) 20 mm, were then mounted onto the support rails of the payload. The footing was positioned on the soil surface and then locked into position using a screw clamping arrangement. During spin-up of the centrifuge an upward stress was applied to the lower actuator chamber, jacking it against the clamping screw, holding it in position on top of the soil. This prevented premature loading of the soil during initial spin-up until the desired gravity level had been reached.

Once the internal safety related checks were complete the centrifuge containment lid was locked and centrifuge was accelerated to 50 gravities at an equivalent radius of 1/3 the model height which minimised stress related errors in the model. The image acquisition system was initiated with optimum parameters previously determined, and set to capture 1 frame a second. Stress was applied in a ramped loading by increasing stress in the upper chamber of the vertical actuator at 15 kN/m² per minute.

2.4 Test programme

Five centrifuge tests were conducted which considered footing tests on a homogeneous block of firm soil (T1) and also for a firm overlying soft soil layered combination. A summary of the test configuration and corresponding prototype conditions for the applied acceleration of 50g is shown in Table 3 and Figure 4. Note, the upper and lower layer properties are referred to with the relevant subscript indicator, i.e. shear strength of upper and lower layer are c_{u1} and c_{u2} respectively.

Table 3. Test programme and parameters.

Test No.	Layer 1		Layer 2	
	Thickness H ₁ mm: [*m]	c_{u1} kN/m ²	Thickness H ₂ mm: [*m]	c_{u2} kN/m ²
1	80 [4.0]	40	0	N/A
2	40 [2.0]	40	40 [2.0]	20
3	20 [1.0]	40	60 [3.0]	20
4	15 [0.75]	40	65 [3.25]	20
5	10 [0.5]	40	70 [3.5]	20

Note: *prototype for acceleration N = 50g
Footing Width: B = 20mm [1.0]

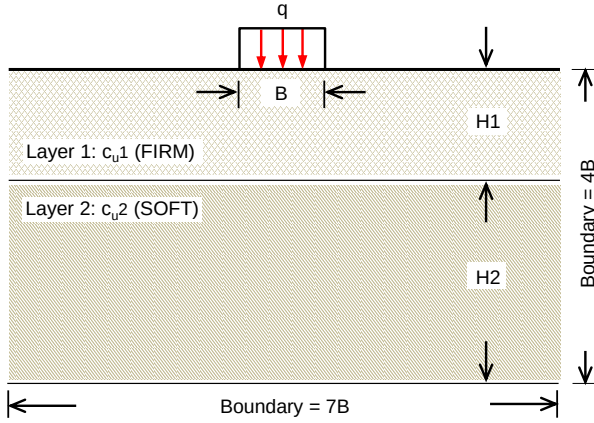


Figure 4. Model test configuration.

The four layer thicknesses considered (10, 15, 20 and 40 mm) provided normalised thickness ratios, H_1/B , of 0.5, 0.75, 1 and 2 respectively. In tests on uniform blocks of clay H_1/B was equal to 4.

Using the coefficient of consolidation (c_v) values derived during final consolidation stage, undrained loading conditions were achieved by ensuring that the rate of foundation displacement (v) was sufficient to ensure the dimensionless velocity ($V=vD/c_v$) was greater than 30 (Finnie and Randolph, 1994).

3 RESULTS

3.1 Load settlement response

Figure 5 presents the bearing capacity against normalised settlement (s/B) response for the 20 mm wide strip footing tested at an accelerated gravity of 50g resting on firm clay overlying soft soil. It is observed that the initial stiffness of each footing is similar at approximately 75 kN/m² up to $s/B = 2\%$. This initial consistency reflects the similar soil strength and resistance provided in the upper layer in each test. Beyond $s/B = 2\%$ divergence in the bearing resistance response is observed, especially for the cases of the homogeneous soil bed ($H_1/B = 4.0$) and that containing the thinnest firm layer ($H_1/B = 0.5$). The bearing capacity response for these two extreme conditions provides the resistance envelope for the other test configurations considered. In the case of $H_1/B = 4.0$ the maximum bearing capacity was 210 kN/m² at the point of failure compared to that of 150 kN/m² for $H_1/B = 0.5$. In the case of $H_1/B = 2.0$ a similar maximum bearing capacity was recorded as that in the uniform bed, albeit with a slightly reduced stiffness response over the full displacement range. Tests $H_1/B = 1.0$ and 0.75 exhibit consistent responses up to $s/B = 6\%$ at which point the bearing capacity of the latter reduces quickly as the footing penetration advances. The consistency in the bearing capacity response provides some confidence in the reliability of the results obtained from the centrifuge tests and the successful test methodology implemented.

3.2 Bearing capacity factor

In the absence of surcharge pressure, the ultimate bearing capacity, q_u , of a strip footing on an infinite uniform purely cohesive soil can be expressed as

$$q_u = N_c \times c_u \quad [1]$$

where c_u is the undrained shear strength and N_c is the bearing capacity factor. Equation 1 is valid for a homogeneous soil condition; however in practice non-homogeneous layered soil conditions are often encountered. In such cases it is necessary to consider the full layered soil profile when determining the footing bearing capacity as it is governed by the strength ratio of the two layers and the relative thickness of the top layer to that of the foundation width. In this respect several authors have postulated a modified bearing capacity factor to evaluate this more complex bearing problem. Brown and Meyerhof (1969) published bearing capacity factors based on experimental studies expressed by the following equation

$$N_c = 1.5 \left(\frac{H}{B} \right) + 5.14 \left(\frac{c_{u2}}{c_{u1}} \right) \quad [2]$$

where H is the thickness of the upper layer, B the footing width, c_{u1} and c_{u2} the shear strength of the upper and lower layer. Merifield et al. (1999) calculated the upper and lower bound bearing capacity factors of layered clays under strip footings by employing the finite element method in conjunction with the limit theorems of classic plasticity, and proposed a simplified modified bearing capacity approximation N_c^* as

$$N_c^* = \left(\frac{q_u}{c_{u1}} \right) \quad [3]$$

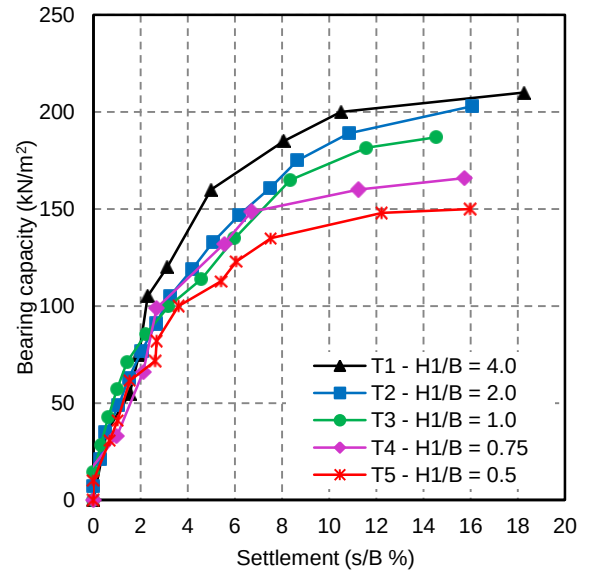


Figure 5. Foundation bearing capacity response.

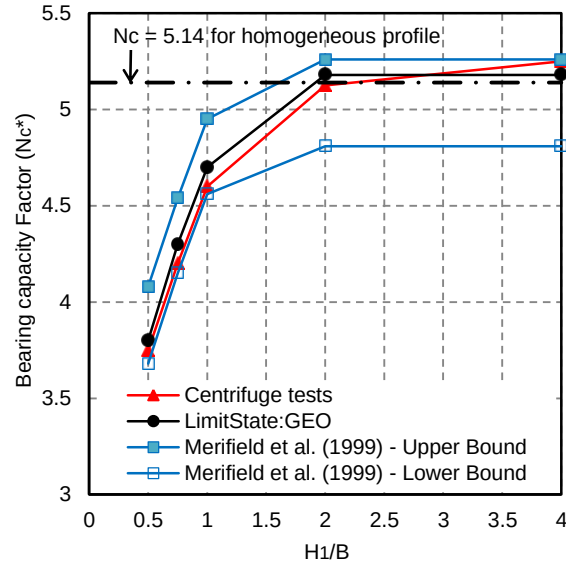


Figure 6. Bearing capacity factor comparisons.

The bearing capacity factors for the current centrifuge model tests was determined using equation 3 and are presented in Figure 6. It is clearly evident that the bearing capacity factor is influenced by the depth of the upper layer and its relative thickness to the width of the footing. These values are also correlated with upper and lower solutions by Merifield et al. (1999) and yield good agreement. In addition, Test 1 ($H_1/B = 4.0$) represents a uniform soil strength sample and thus should conform to the classical theoretical bearing capacity factor ($\pi+2$) (Terzaghi and Peck 1948). The bearing capacity factor in Test 1 was determined to be 5.25, approximately 2% over this theoretical value which could be due to either (i) some residual interface friction at the soil-window boundary or, (ii) increased resistance being mobilised in the soil as the penetration advances due to increased self-weight stresses, as reported by Davis and Booker (1973).

3.3 Numerical evaluation

Numerical analysis of the layered footing problem was carried out using LimitState:GEO, developed by Smith and Gilbert (2007), which uses the theory of discontinuity layout optimization (DLO). Discontinuity Layout Optimization (DLO) is used as an alternative to finite element analysis and implements a system of connected nodes to identify the most critical failure mechanism. The program optimizes the layout of failure planes using linear programming to minimise the internal energy dissipated along them and give the best upper bound solution. More information on discontinuity layout optimization can be found in Smith and Gilbert (2007).

Analysis was carried out using the implementation of DLO within the software LimitState:GEO Version 3.2a (LimitState, 2014). The geometry of the problem was modelled at prototype to represent the model test configurations outlined in Table 3. The soil strength

properties for the firm and soft soil layers were as determined by the UU triaxial tests.

To permit direct comparison of the numerical results with those obtained in the centrifuge tests, a suite of verification tests were conducted to benchmark accuracy and configure optimum programme properties (i.e. such as nodal density). In this respect, layered tests were configured having $c_{u1}/c_{u2} = 1.0$ with the computed bearing capacity factor compared to the well-established closed form bearing capacity solution $\pi+2$. The result shown in Figure 7 is 5.18, which compares favourably. During these tests the optimum nodal density for the boundary value problem was also determined to be 3345 with a scale factor of 500.

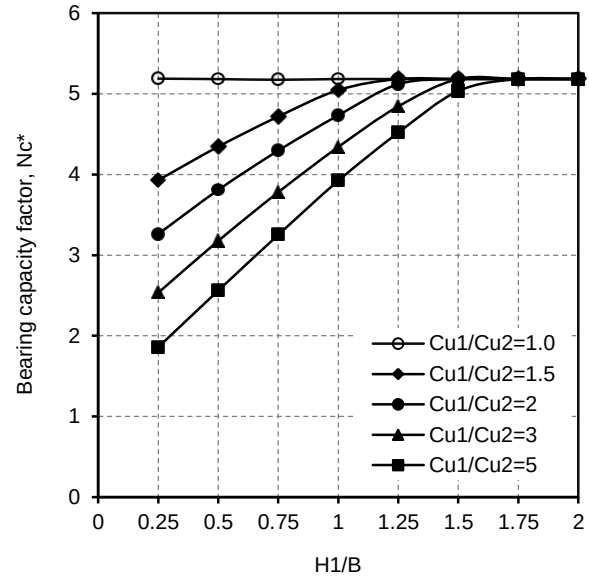


Figure 7. Bearing capacity factor for varying soil strengths.

A parametric study was deployed to investigate the impact of soil shear strength ratio and layer thickness. It is evident in Figure 7, as expected, that greater difference in strength between the upper and lower layer yielded significantly reduced bearing capacity factors. This compares well with work by Michalowski (2002) and Merifield et al. (1999). These observations serve to reinforce the importance of determining a suitable bearing capacity factor for complex layered soil conditions as failure to do so would have catastrophic consequences on the foundation stability if the classical value $\pi+2$ were inappropriately used.

The data corresponding to $c_{u1}/c_{u2} = 2.0$ was compared against the bearing capacity factors determined from the experimental centrifuge tests. This is presented in Figure 6 where it can be seen the DLO numerical results shown excellent agreement and remain within the upper and lower bound solutions of Merifield et al. (1999).

3.4 Mode of failure

Figure 8 illustrates the failure slip planes for the strong-overlying soft clay profile as determined by numerical

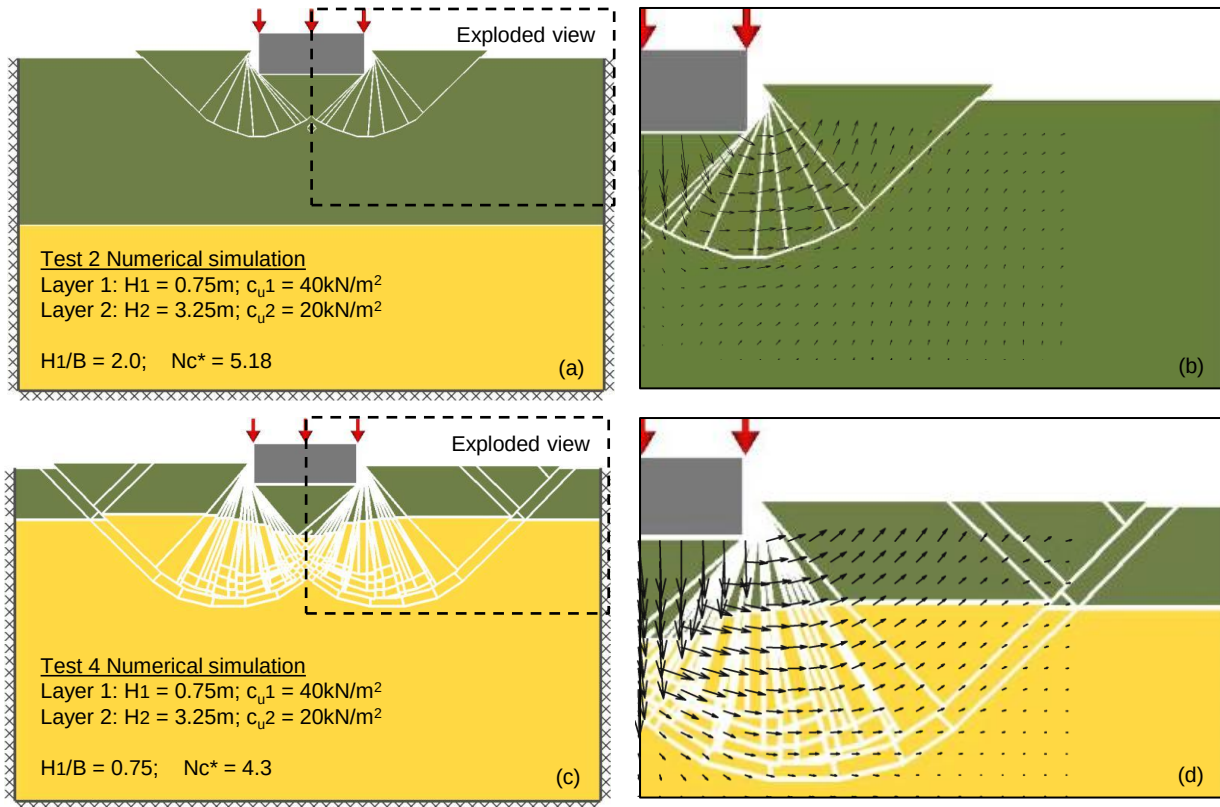


Figure 8. Failure mode comparison in LimitState:Geo numerical model and soil displacement in experimental tests.

analysis for Test 2 and Test 4, $H_1/B = 2.0$ and 0.75 respectively. It is observed that the failure mechanism is fully contained in the upper firm layer in T2, extending to approximately $1B$ in the vertical and horizontal direction away from the foundation. The failure mode is that of general shear failure and a similar mechanism was also prevalent in the non-layered homogeneous sample. Reflecting on the bearing capacity factor N_c^* presented in Figure 6, the failure mode observations offers further reassurance for the similar bearing capacity factors reported of 5.25 in Test 1 and 5.12 in Test 2.

As the thickness of the upper layer reduces a change in the failure mode characteristics is evident. Referring to Test 4 in Figure 8c, the mechanism propagates from the upper firm layer and penetrates deep into the underlying soft layer. Also apparent is that the width of the mechanism is significantly greater extending almost to the boundary edge of the test package.

As part of the experimental model tests images of the exposed soil surface were captured for the purpose of identifying soil displacement behaviour. Image processing was conducted using GeoPIV as developed by White et al. (2003), which is a specially adapted form of digital image correlation (DIC) for geotechnical applications. An interrogation mesh containing patches 60 pixels with spacing of 15 pixels was used to analyse the image data set. Vector trajectories of each patch centre showing the indicative soil movement direction are superimposed over the numerically predicted slip plane for Test 2 and Test 4 in Figure 8b and 8d. It is clear that there is strong

correlation between the extents of the mechanism observed in the physical model experiments and those from the numerical study. The DIC observations of Test 4 revealed that the soil immediately beneath the foundation moved downward in unison with foundation as settlement advanced. This was accompanied with noticeable displacement of the firm-soft interface boundary which became more pronounced when the firm overlying layer was thinner. As the settlement reached a critical depth the plug of soil restrained beneath the footing was observed to penetrate fully into the soft underlying soil rapidly signaling the onset of foundation failure. This observation is synonymous of punching failure that has been reported by Merifield et al. (1999). The authors state that full punching shear is characterised by a complete vertical separation of the top layer, which acts as a rigid column of soil that punches into the lower layer. This was confirmed by displacement mechanics observed in the centrifuge model tests.

The DIC results indicate that a complex relationship exists between general, local and punching failure and ratios c_{u1} / c_{u2} and H_1/B . Failure generally occurs by either partial or full punching shear through the upper layer followed by yielding of the bottom layer (Merifield et al. (1999). In the current study full punching shear was evident for the H_1/B ratio of 0.5 , while for $0.5 \geq H_1/B \leq 1.0$ a transition between full and partial punching shear occurred. Beyond $H_1/B \geq 2.0$ low levels of partial punching were detected only at large foundation displacements with general shear being the common failure mode from this

layer thickness onwards. For ratios $H_1/B > 2$, the failure was contained fully in the upper layer. These limits on the mode of failure are similar to those reported by Meyerhof and Hanna (1978) and Chen (1975) would suggested that reductions on bearing capacity could manifest up to depth ratios H/B approximately 2.5.

CONCLUSIONS

A series of five centrifuge models of surface strip footings founded on firm overlaying soft soil profiles of different heights were tested using the 2gT centrifuge at the University of Sheffield. The thickness of the top firm soil layer was varied from 0.5 to 4 times the strip footing width. The initial stiffness of the top firm layer in all the experiments was similar up to a normalised settlement of 2% of footing width. Beyond this region, an effect of firm layer thickness on the mobilized bearing capacity was observed. The modified bearing capacity factor for multi-layer soil proposed by Brown and Meyerhof (1969) was calculated for the different firm to soft layer soil thicknesses. These factors were compared against the upper/lower bound capacity factors proposed by Merifield et al. (1999) and numerical results from LimitState:GEO, good agreement was observed. LimitState:GEO was further used to predict the failure modes of different firm to soft layer soil thicknesses. Experimental slip planes were derived using GeoPIV and a good fit was achieved against the predicted failure planes. The results highlight the effect of multi-layered soils on bearing capacity and the important design considerations needed for non-homogeneous soils.

ACKNOWLEDGEMENTS

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REFERENCES

- Baglioni V. P, Chow G. S, Endley S. N, 1982. Jacking-up foundation stability in stratified soil profiles. *Proc. 14th Offshore Technical Conference*, 4: 363-369
- Black, J.A. Baker, N. and Ainsworth, A. 2014. Development of a small scale teaching centrifuge. *Physical Modelling in Geotechnics - Proceedings of the 8th International Conference on Physical Modelling in Geotechnics* 2014, ICPMG 2014, 187.
- Brown, J.D. and Meyerhof, G.G. 1969. Experimental study of bearing capacity in layered clays, *Proc. 7th Int. Conf. on Soil Mechanics and Foundation Engineering*, Mexico, Vol. 2, 45–51.
- Burd, H. J. and Frydman, S. 1997. Bearing Capacity of Plane-strain Footings on Layered Soils. *Can. Geotech. J.* Vol. 34, 241-253.
- Davis, E.H. and Booker, J.R. 1973. The effect of increasing strength with depth on the bearing capacity of clays. *Géotechnique* 23: 551-563.
- Chen, W. F. 1975. Limit analysis and soil plasticity. Amsterdam: Elsevier.
- Finnie, I.M.S., & Randolph, M.F. 1994. Punch-through and liquefaction induced failure of shallow foundations on calcareous sediments. *Proc. Int. Conf. on Behavior of Offshore Structures*, BOSS '94, Boston: 217-230.
- Garnier, J. Gaudin, C. Springman, S.M. Culligan, P.J. Goodings, D.J. Konig, D. Kutter, B.L. Phillips, R. Randolph, M.F. Thorel, L. (2007), *Catalogue of scaling laws and similitude questions in geotechnical centrifuge modelling* 7 (3), *International Journal of Physical Modelling in Geotechnics*, 1–23, ISSN: 1346-213X, E-ISSN: 2042-6550.
- LimitState. 2014. LimitState:GEO Manual Version 3.2.a, June 2014 edn, LimitState Ltd.
- Merifield, R. S., Sloan, S. W. and Yu, H. S. 1999. Rigorous Plasticity Solutions for the Bearing Capacity of Two-layered Clays. *Geotechnique*, 49, no. 4, 471-490.
- Meyerhof, G. G. and Hanna, A. M. 1978. Ultimate bearing capacity of foundations on layered soils under inclined load. *Can. Geotech. J.* 15, 565-572:
- Michalowski, R. L. 2002. Collapse Loads over Two-layer Clay Foundation Soils. *Soils and Foundations*, Vol. 42, No. 1, 1-7.
- Michalowski, R.L. and Shi, L. 1995. Bearing capacity of footings over two-layer foundation soils. *ASCE Journal of Geotechnical Engineering*, 121, 421-428.
- Smith, C.C. and Gilbert, M. 2007. Application of discontinuity layout optimization to plane plasticity problems. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 463, no. 2086, 2461-2484.
- Terzaghi K, 1943. Theoretical Soil Mechanics. John Wiley, New York.
- Terzaghi K, Peck R. B. 1948. Soil Mechanics in Engineering Practice. John Wiley, New York.
- Yuan F. F, Luan M. T, Yan S. W, 2005. Study on bearing behavior of foundations on multi-layer subsoil. *Frontiers in Offshore Geotechnics. Proceedings of the International Symposium. on Frontiers in Offshore Geotechnics (IS-FOG 2005)*, 19-21 Sept 2005, Perth, WA, Australia.
- White, D., Take, W., and Bolton, M., 2003, Soil Deformation Measurement Using Particle Image Velocimetry (PIV) and Photogrammetry, *Geotechnique*, Vol. 53, No. 7. 619–631.