# Interpreting pumping test in stratified confined aquifer using the equivalent model

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ABSTRACT
Aquifer tests are commonly used to determine the hydrodynamic characteristics and boundary limits of stratified confined aquifers. However, the interpretation of the resulting data is made based on very simplistic assumptions lacking real-world validation. Heterogeneous and anisotropic layered aquifers are frequently treated using homogeneous anisotropic equivalent models whose neither spatial nor temporal validity have been assessed. This paper proposes a new approach to allow for a more reliable and appropriate application in such conditions, thus, overcoming the shortcomings of these models.

RÉSUMÉ
Les essais d'aquifère sont couramment utilisés pour la détermination des caractéristiques hydrodynamiques des aquifères ainsi que la détection et la localisation des frontières qui les limitent. Cependant, l'interprétation des données fournies par ces essais restent limitées aux méthodes disponibles qui se basent sur des hypothèses très simplistes de la réalité. Dans le cas des aquifères hétérogènes et anisotropes notamment les aquifères stratifiés, l'approche la plus utilisée est celle du modèle équivalent homogène et anisotrope dont la validité spatiale ou temporelle n'a fait, jusque-là, l'objet d'aucune évaluation quantitative ou qualitative. Cet article propose une nouvelle méthode pour l'utilisation fiable de ces méthodes dans de telles conditions.

## 1 INTRODUCTION

Pumping and recovery tests in homogeneous and isotropic confined aquifers are commonly interpreted using Theis (1935) and Cooper-Jacob (1946) methods. Most aquifers consist of many layers of material, each with its own characteristics. They are called stratified or multilayered aquifers. Therefore, they are neither homogeneous nor isotropic. However, for ease of use and simplicity, they are routinely analysed using the approach of the homogeneous anisotropic equivalent model (HAEM). Though often useful, such an approach may introduce excessive errors. Chenaf and Boukemidja (2012) have discussed the spatial and temporal validity of this approach using equivalent storage coefficient and hydraulic conductivity values. The present work uses Finite Element Method (FEM) to improve the storage coefficient of the HAEM. This, in turn, would allow for a more appropriate consideration and interpretation of the resulting data from these models.

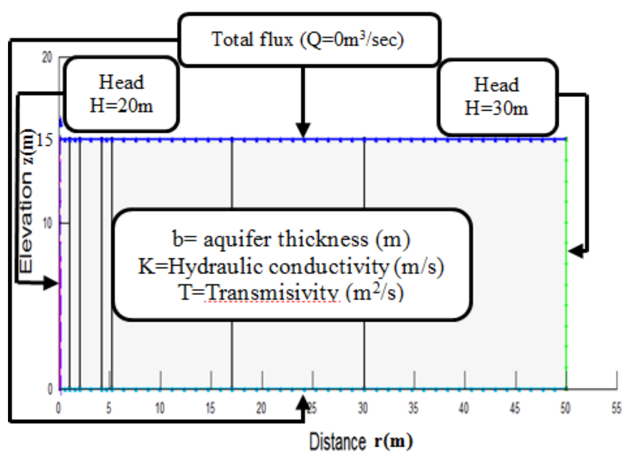


Figure1. Conceptual Model

## 2 NUMERICAL CODE – FINITE ELEMENT METHOD USED

A code using the finite element method is used for a mathematical simulation of aquifer tests in confined aquifers. The analyses are carried out in both steady and transient state for homogeneous isotropic, homogeneous anisotropic and stratified models. The appropriateness of this code was previously assessed in several studies (Chenaf and Chapuis, 1999, Chapuis et al. 2001).

## 3 PUMPING AND RECOVERY TESTS

The interpretation methods considered herein are those of Dupuit (1863) and Thiem (1906) for steady state and the methods of Theis (1935) and Cooper-Jacob (1946) for the transient one. Figure 1 shows a conceptual schema of the analyzed cases.

Thiem's equation is given as:

$$Q=\frac{2\pi Kbs(r)}{\ln R/r} \quad [1]$$

By analogy to heat flow, the local diffusion equation with polar coordinates (of axisymmetric analysis) is given by:

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}=\frac{S}{T}\frac{\partial h}{\partial t} \quad [2]$$

Where

T is the transmissivity and S the storage coefficient , s is the drawdown, t is the pumping time and r the radial distance from the axis of the pumping well.

Theis (1935) proposed a solution of the equation based on the well function, W(u), where u is the argument of the function of the well. Theis' solution was simplified by Cooper-Jacob (1946).

$$S=\frac{4}{(r^2/t)}.u.T \quad \text{and} \quad T=\frac{Q}{4\pi s}W(u) \quad [3]$$

The Cooper-Jacob's method is based on Theis' formula as applicable for values of $u = r^2S/4Tt$ less than 0.01 only, transmissivity T and storage coefficient S are given by Cooper-Jacob as:

$$T=\frac{2{,}30Q}{4\pi\Delta s/\text{cycle}} \quad \text{and} \quad S=\frac{2{,}25Tt_0}{r^2} \quad [4]$$

Where :

$\Delta s$/cycle and $t_0$ are the slope and intercept at origin of Cooper-Jacob's line, respectively. By considering the anisotropy ratio $n = K_h/K_z$, the intercept at origin $t_{0n}$ of Cooper Jacob's line of anisotropic aquifer becomes:

$$t_{0n}=\frac{Sr^2}{2{,}25\,T_n}=\frac{Sr^2}{2{,}25\,T_{isotrop}*n}=t_0/n \quad [5]$$

In aquifer test, subsequent phase to the pumping phase is recovery. Theis' equation for a homogeneous isotropic confined aquifer in recovery is given by:

$$s'_{isotrop}=\frac{2{,}30\,Q_{isotrop}}{4\pi T_{isotrop}}\log\left(\frac{S'}{S}\frac{t}{t'}\right)=\Delta s'_{isotrop}\log\left(\frac{S'}{S}\frac{t}{t'}\right) \quad [6]$$

Other methods such as those of Chenaf and Chapuis (2002) can also be used.

In recovery, the residual drawdowns of homogeneous and anisotropic aquifer, are given by:

$$s'_{anisotrop}=\frac{2{,}30\,Q_{anisotrop}}{4\pi T_{anisotrop}}\log\left(\frac{S'}{S}\frac{t}{t'}\right)=s'_{isotrop} \quad [7]$$

Where :

$Q_{anisotrop}=Q_{isotrop}*n$ and $T_{anisotrop}=T_{isotrop}*n$,

## 4 NUMERICAL MODELS OF PUMPING TESTS IN HOMOGENEOUS ISOTROPIC CONFINED AQUIFER

### 4.1 STEADY STATE

In a case of a confined aquifer with a thickness b=15m, a boundary recharge of $h_0$ = 30m located at R = 50m, a pumping well radius $r_w$ = 0.10 m screened through the entire thickness of the aquifer, and with a pump producing a minimum well hydraulic head of $h_w$ = 20 m, well drawdown $s_w$ is, therefore, equal to 10 m. The of hydraulic conductivity coefficient is $K = 10^{-4}$ m / s which is constant throughout the aquifer. Figure 1 shows a conceptual scheme of the analyzed cases with boundary conditions.

For the isotropic case, the theoretical value of flow rate from the well $Q_W$ calculated by Thiem's equation is equal to $1.612\times10^{-3}$ $m^3$/ s, while a finite element (FE) discretization with secondary nodes gives a difference of 0.012% between theoretical and numerical flow rate. This FE grid will here-on be considered for all simulations that follow, for its better accuracy on the calculation of the flow rate.

Six (6) anisotropic models are analyzed with respective values of the coefficient of anisotropy n=Kh / Kz of 5, 10, 100 and 1000. In confined aquifer, the vertical hydraulic conductivity coefficient, $K_Z$ does not affect the flow of water. It is therefore kept constant in all models. Only the horizontal hydraulic conductivity coefficient $K_h = K_z$ n is variable and takes the following values $5\times10^{-4}$ m / s, $10^{-3}$ m/s, $10^{-2}$ m/s and $10^{-1}$ m/s. The drawdown distribution in the homogeneous anisotropic aquifer are identical to the respective distribution in the homogeneous isotropic aquifer regardless of the value of n. However, the flow rates in all models are proportional to the flow rate of the homogeneous isotropic model ($1.612 \times 10^{-3}$ $m^3/s$) and the coefficient of proportionality is the ratio of anisotropy n, thus confirming equations 7.

## 4.2 UNSTEADY STATE

The previous models are also analyzed in unsteady state by considering the boundary conditions as functions of time. Thus, the hydraulic head at r = 50 m remains constant over time and equal to h (t) = 30 m. The pumping flow is q (t) =$-1.612 \times 10^3 m^3/s/ml$. The upper and lower limits are considered impervious, therefore, no flow; total flow Q (t) = 0. The storage coefficient S is represented by the change in volumetric water content as a function of pore water pressure θ (u). It is equal to $7.36 \times 10^{-3}$.

For the isotropic case, the Cooper-Jacob graph gives, at a given r, a log-linear part defined by a slope Δs /cycle-log and intercept at the origin $t_0$. For the anisotropic case, the drawdown s versus log of time at different radial distances r are identical to those of the homogeneous and isotropic aquifer with values of intercepts at the origin satisfying Eq. N 5 for each ratio n (Table 1).

Table 1. Values of the times $t_{0n}$ at r =0.45 m

| n (anisotropy ratio) | $t_0(s)$ |
|---|---|
| 1 | 0,442 |
| 10 | 0,442 $10^{-1}$ |
| 100 | 0,442 $10^{-2}$ |
| 500 | 0,884 $10^{-3}$ |

## 5 RECOVERY ANALYSES

The recovery phase is simulated by assuming an instant closure of the pump known as indirect pump closure (Chenaf,1997).

### 5.1 ISOTROPIC AND HOMOGENEOUS MODEL

The initial conditions of the recovery phase are represented by the hydraulic head at time t = $t_A$ located on the linear part of the Cooper-Jacob curve (Figure 2). This time is set equal to 300 s. The residual drawdown s' vs. Log t / t' at different radial distance r, converge to a line of slope Δs' / cycle-log = 1.85 m and intercept at the origin $(t / t')_0 = 1$ (Figure 3). From Eq 6, we obtain S / S' = 1 and T = $10^{-4}$ $m^2$ /s. These values correspond to the values entered into the model.

### 5.2 ANISOTROPIC AND HOMOGENEOUS MODELS

The recovery tests are performed on Mod1-A, Mod1-B, HAEM1-A and HAEM1-B. The residual drawdown curves s' vs. log t / t', for the four values used for anisotropy ratio n (n = 10, 100, 500.1000) are superposed on those of the isotropic homogeneous case, thus confirming Eqs. [8]. The residual drawdowns are not influenced by the anisotropy which, in fact, confirms the theoretical equations 7.

## 6 NUMERICAL MODELS OF PUMPING TESTS IN STRATIFIED AQUIFERS

The homogeneous anisotropic equivalent model HAEM of the stratified model defined by its thickness $b_{HAEM}$ equal to the sum of layer thicknesses $b_j$, the horizontal and vertical hydraulic conductivity coefficients $K_h$ and $K_v$, expressed, respectively, in terms of hydraulic conductivities of different layers, by the following classical equations:

$$b_{AHEM} = \sum_{j=1}^{m} b_j \qquad [8]$$

$$K_h = \frac{\sum_{j=1}^{m} b_j K_j}{\sum_{j=1}^{m} b_j} \quad \text{and} \quad K_v = \frac{\sum_{j=1}^{m} b_j}{\sum_{j=1}^{m} \frac{b_j}{K_j}} \qquad [9]$$

Where m is the total number of layers, $b_j$ and $K_j$ are the thickness and hydraulic conductivity of the layer j, respectively.

In unsteady state the same two groups (A and B) of stratified models that were considered in Chenaf and Boukemidja (2012) are analyzed. Mod1-A and Mod1-B are used to illustrate the methodology.

Soil layers in MOD1-A have the same soil water characteristic curve θ (u) that is equal to the equivalent homogeneous model, HAEM. Therefore, storage coefficient value for each layer and for the HAEM are:

$$S_{J\ MODi\text{-}A} = m_w * \gamma_W * b_J \qquad [10]$$

$$S_{HAEM} = m_w * \gamma_w * b_{HAEM} \qquad [11]$$

Where:
$S_{HAEM}$ is the storativity of HAEM
$S_{J\ MODi\text{-}A}$ is the storage coefficient of the MODi-A
$\gamma_w$ : Volumic weight of water,
$m_w$ : the slope of the curve θ(u) defined as :
$m_w = \Delta\theta / \Delta u_w$ [12]

Models of Group B noted MOD1- B are formed of layers with the different water characteristic curve θ (u). The equivalent model HAEM-B has a characteristic curve defined as follow:

$$S_i = m_{wi} b_i \gamma_w \qquad [13]$$

$$S_{HAEM} = \sum_{i=1}^{j} S_i = \gamma_w \sum_{i=1}^{j} m_{wi} b_i$$

$$m_{wHAEM} = \frac{\sum m_{wi} b_i}{b_{HAEM}} \qquad [14]$$

$$b_{HAEM} = \sum_{i=1}^{j} b_i$$

Where: $S_{HAEM}$ is the storativity of HAEM, $S_J$ is the storativity of the layer I of the considered model, $\gamma_w$ : Volumic weight of water, $m_{wi}$ and $m_{wHAEM}$: the slope of the curve θ(u) defined as $m_w = \Delta\theta/\Delta u_w$ for the layer I and the HAEM, respectively, and $b_i$ and $b_{HAEM}$, thicknesses of the layer I and the HAEM, respectively.

Six (6) stratified models formed of 2 to 5 layers are analyzed in steady state. Each layer j has a thickness bj ranging from 1 to 7 m such that the total thickness b = Σbj of each model is 15m. Each layer is considered isotropic ($K_{HJ} = K_{VJ} = K_j$) $K_j$ hydraulic conductivity value varying between 1.4 $10^{-4}$ and $6x10^{-5}$ m/s as Σkjbj = $10^{-3}s^{-1}$. For each analyzed model, the value of horizontal hydraulic conductivity, and its total thickness $K_{HJ}$ are respectively:

$$K_{hj}=10^{-4} \text{ m/s} \qquad [15]$$

$$\sum_{j=1}^{m} b_j=15\text{m} \qquad [16]$$

## 6.1 STEADY STATE

In steady state, the storage coefficient does not impact the calculation of the flow rate and heads, so, the MODi-A and et MODi-B give the same results, the equivalent model HAEM represent the MODi unconditionally.
In fact, for any elevation z from the aquifer base, the variation of drawdowns versus the radial distance r, for the MODi and HAEM are superposed as shown in figure 5.

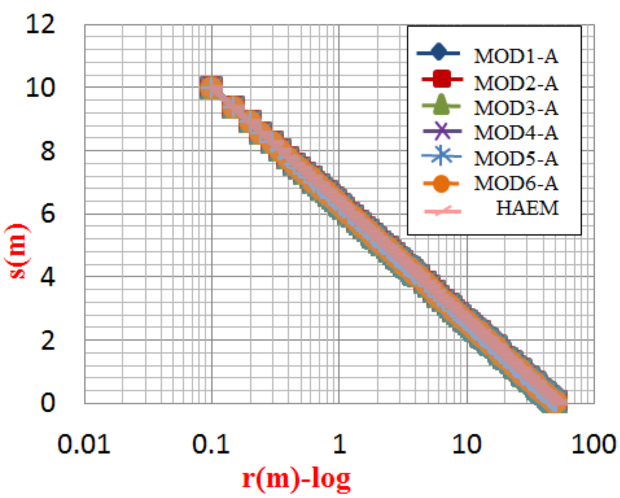


Figure 2: s vs log r at the elevation z=0.5b; in steady state

## 6.2 UNSTEADY STATE

In the transient state, the function θ (u) is very important. Considering the two approaches MODi-A and MODi-B, in which storage coefficient and slope $m_w$ are defined by Eq. 11 to 14.
The equivalent model HAEM1-A has a total thickness b = 15m and a characteristic curve θ (u) with slope $m_w$ = 5x $10^{-5}$ , the resulting storage coefficient (Eq. 10) $S_{AHEM}$ of $7.36x10^{-3}$. The unit flow rate of equivalent model is $q_{HAEM}$ =- 1.612x10 $m^3/s/ml^{-3}$, this flow is imposed at the well for all stratified models MODI.

### 6.2.1 MODELS MODI-A

The 6 models simulated in steady state are analyzed in transient one. The storage coefficient given by Eq 11, and constant slope $m_w = 5x10^{-5}$, (equivalence approach 1).

For all the analyzed models MODi-A, the results show that

**a)** The sum of flow rates of the layers forming each aquifer is equal to the flow rate of the HAEM.

$$\sum_{j=1}^{m} q_j = q_{HAEM} \qquad [18]$$

**b)** Cooper-Jacob's curves (s(r,t) vs log t) at different values of r (∀r), are superposed to the curves of the AHEM at z=0,5b , as shown on figure 6 for the MODi-3.
**c)** The values of slope Δs/cycle-log and intercept at the origin $t_0$ of the Cooper-Jacob line are respectively 1,85 m and 0,442 s. These values are the same as those of AHEM.
For the different models analyzed the variation of drawdown(s) versus time (t), at different elevation and distance (z and r), is superposed on the equivalent model from r = 7m (∀z)

Thus, the HAEM offers an appropriate equivalence for the Modi-A for a distance from the axis of the pumping wells greater than or equal to 10.5 m. Both models Modi-A and its HAEM give the same values of T, S and equivalent flow rates and are: S = 7.36 10-3 and q = 1.618 10-3 $m^3$ / s, respectively.
d) The steady state is obtained after a pumping time t = 10000 sec. This is the same time as it took the homogeneous isotropic model with the same characteristics of HAEM (q, S and T).

So when performing a pumping test in a stratified confined aquifer (MODi-A), it is recommended to install monitoring piezometer in the middle of the thickness of aquifer (z=0,5b), and to use directly Eqs 3 and 4 to determinate the aquifer properties which are equal to those of the HAEM at this elevation ($S=7.36x10^{-3}$, $Q=1.618x10^{-3}$ $m^3/s$)

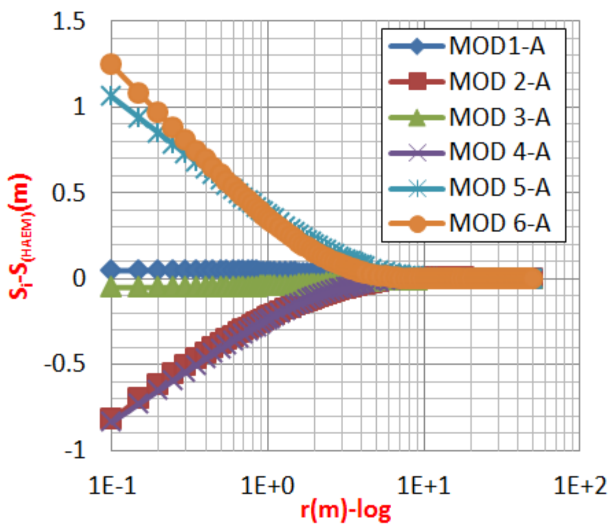


Figure 3: ($s_i$-$s_{HAEM}$) at z = 0.5b and t = 600s for MODi-A.(b = 15m)

The difference si-$s_{HAEM}$ between drawdowns MODi-A and drawdown of the HAEM at t=600 s is represented in function of the radial distance r and the elevation z = 0.5 b for the first six models MODi-A. Figures 3 shows that the curves overlap perfectly from r = 10.8 m (r = 0.7 b with b = 15m in this case). From this distance the difference ($S_I$-$S_{HAEM}$) is zero and therefore HAEM-A represent appropriately MODi-A.

6.2.2 MODELS MODI-B

Each layer j forming the MODi-B is defined by a different water content curve as the storage coefficient is the same in all layers (Eqs. 13 and 14). As an example, the model MOD3-B, is composed of three layers with a thickness $b_j$ and slope of the water content curve $m_{wj}$ (Eq.14)

Figures 4 shows drawdown curves for MOD1-A and MOD1-B compared to their respective equivalent Models HAEM1-A and HAEM1-B. These drawdowns are taken at radial distances r = 0.1m, 1.4m, 7m and 13 m and the elevation z = 0.5b. The curves of models A and B are superposed to their corresponding HAEM-A and HAEM-B at z = 0.5b. The curves have all the same slopes, thus the same transmissivity values.

As shown in Figure 5, the difference si-$s_{HAEM}$ between the respective drawdowns MODi-B and HAEM at t= 600 s at radial distance r and elevation z = 0.5 b for the first six models MODi-B, show that for the curves overlap perfectly from r≥10.8 m (r = 0.7 b with b = 15m in this case). From this distance the difference ($S_I$-$S_{HAEM}$) is zero and therefore the approach of MODi-B is also appropriately applicable.

In conclusion HAEM-A and HAEM-B represent correctly MOD-A and MOD-B if pumping test data are collected in monitoring wells that are installed at a distance from the pumping well and at an elevation z that simultaneously satisfy the following both condition: r≥b and z = 0.5b

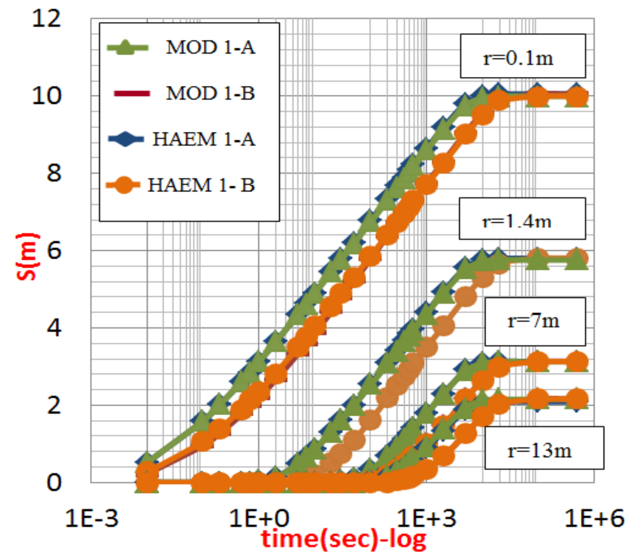


Figure 4: s Vs. Logt for Mod1-A, Mod1-B, HAEM1-A and HAEM1-B,

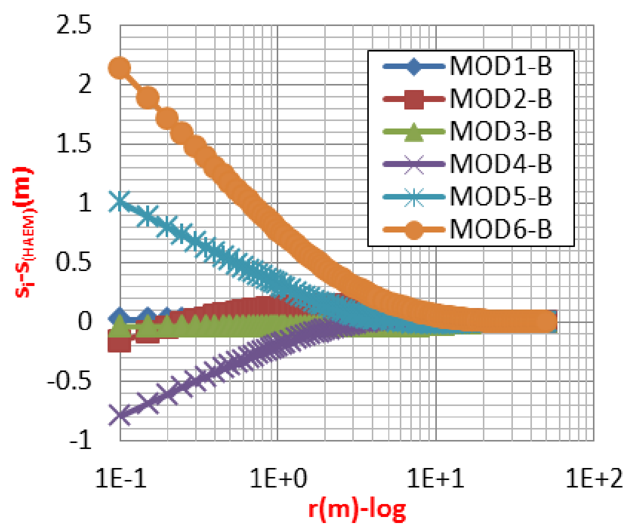


Figure 5. ($s_i$-$s_{HAEM}$) at z = 0.5b and t = 600s for MODi-B (b = 15m)

INFLUENCE OF THE AQUIFER LENGTH ON THE RESULTS

All previous analyses have been made to an aquifer length of R = 50m (description of the analyzed model is in section 4.1). The influence of the aquifer length on the previous result was investigated. The six previous models, were analyzed with different lengths of R = 30, 40 and 80 m. The difference of drawdowns $s_i$-$s_{HAEM}$ is determined for each model and for each R. This difference remains the same for all models. As a result, the length of aquifer does not affect the condition of validity of the equivalence approach 1: r ≥ 0.7 b.

EFFECT OF AQUIFER THICKNESS ON RESULTS

The influence of the aquifer thickness b on the previous result (condition of validity of the equivalence approach 1: $r \geq 0.7$ b (b: thickness of the aquifer)) was investigated. The six models Modi-A are analyzed with three different thicknesses, respectively b = 5, 15 and 20 m. The difference in drawdowns $s_i$-$s_{HAEM}$ is determined for each model and for each thickness b. The distance for each respective thickness between MODi-A and its equivalent model is 3.5 m for b = 5 m, 10.8 m for b = 15 m, and 14 m for b = 20 m which corresponds to 70% of the total thickness b. For example, Figure 11 shows the difference ($s_i$-$s_{AHEM}$) at time t = 600 s for a thickness b = 5 m. For the six models analyzed, the curves are superposed from r = 3.5 m or $r \geq 0.7$b.

7. NUMERICAL SIMULATED MODELS IN RECOVERY OF STRATIFIED AQUIFER

MODI-A and MOD1-B and their corresponding equivalent Models HAEM-A and HAEM-B are analyzed in recovery, as explained in section 3. The distribution of residual drawdown versus the ratio t / t '(s' vs t / t'), for MOD1-A and MOD1-B compared with the HAEM1-A and HAEM1-B, respectively, is in Figures 6 and 7.

Curves (s' vs t / t ') for different radii r, converge to a theoretical line which passes through 1, with a slope of Δs' = 1.85 m. This value is the same as the one obtained for the isotropic homogeneous case (having the same characteristics of AHEM). These results are found at an elevation z = 0.5 b. Thus for all models, for any elevation z (taken in this study respectively equal to 0.15 b, 0.25 b, 0.5 b, 0.75 and 0.9 b), the curves of residual drawdown s' vs. t / t' of Modi-A are superposed on those of HAEM, from r = 7m, representing 0.7 b, as found for pumping. Figures 9 and 8 show ($s'$-$s'_{(HAEM-A)}$) MOD1-A and ($s_1$-$s_{(HAEM-B)}$) for Mod1-B with radial distance at four different times t = 10, 100, 500 and 10000 s and at the elevation z = 0.5b. The HAEM models represent well the stratified system at a distance from a pumping well equal to 70% the aquifer thickness. The length of aquifer and thickness have the same influence on the results of recovery as for the pumping phase.

The results show that for the entire duration of pumping and recovery, the HAEM model is more representative of stratified confined models for radial distances $r \geq 0.7$b. This is observed for any elevation z in the aquifer. For monitoring tests of stratified confined aquifers, it is recommended to install piezometers at distances $r \geq 0.7$b from the well as their measuring points are at mid-thickness of the stratified aquifere (elevation z = 0.5b)

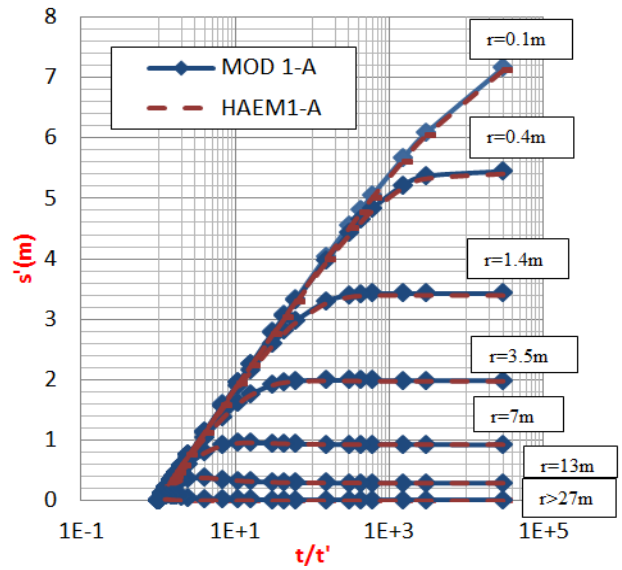


Figure 6: s' vs logt/t' for Mod1-A and HAEM1-A

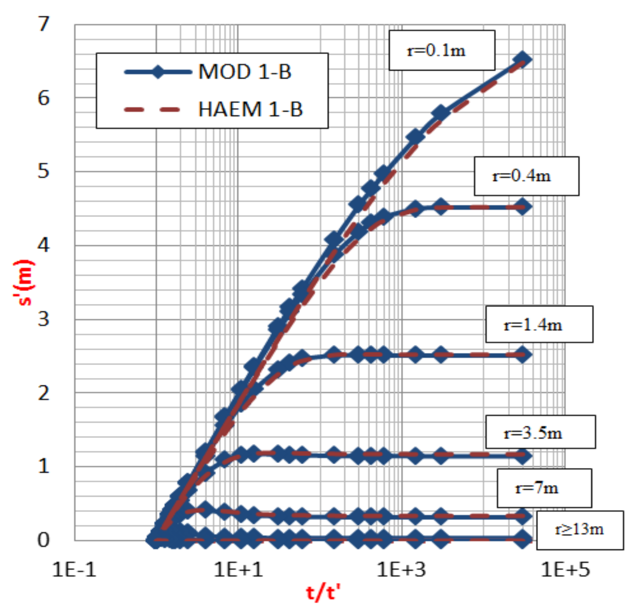


Figure 7: s' vs logt/t' for Mod1-B and HAEM1-B

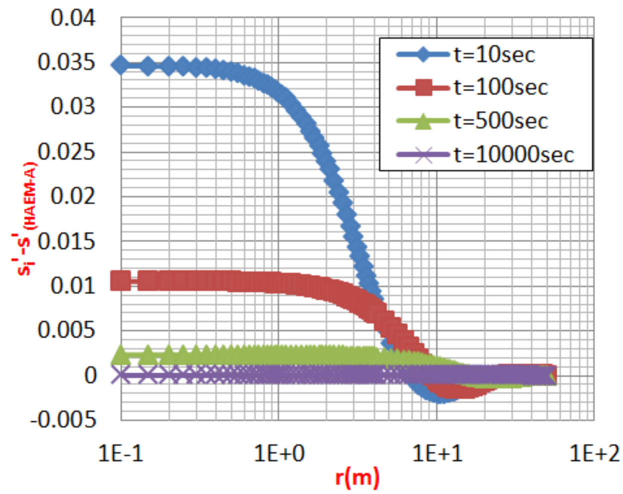


Figure 8. ($s'_i$-$s'_{HAEM}$) at z = 0.5b MODi-A (b = 15m)

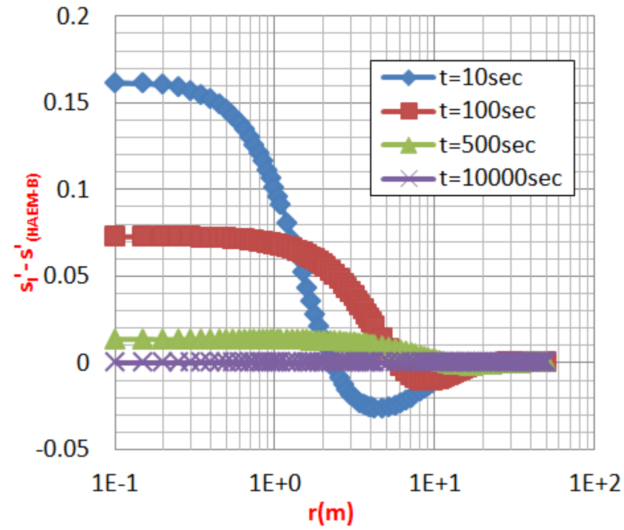


Figure 9. ($s'_i$-$s'_{HAEM}$) at z = 0.5b MODi-B (b = 15m)

## 8. CONCLUSIONS

The equivalent model approach is applicable only for data collected in the middle of the height of the aquifer and at a radial distance of pumping wells equal to 70% of the thickness of the aquifer, or 0.7b. So, in the general case of an aquifer test, it will always be necessary to install monitoring wells or piezometers in the middle of the thickness of aquifer (z = 0.5b) and at a radial distance of 0.7b. It is, thus, shown that the common practice to extract drawdown data arbitrarily at the pumping well is inappropriate and could lead to significant errors in estimating the hydrodynamic parameters S and T. The storage coefficient of the equivalent model can be accurately considered as the sum of the storage coefficients of the individual layers and the horizontal transmissivity as the sum of the transmissivity of the individual layers.

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