Inverse Modelling of Desorption Tests to Establish the Hydraulic Conductivity of Unsaturated Québec Granitic Sand

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ABSTRACT
The Multistep outflow method (MSO) has been widely used to determine the hydraulic conductivity functions of unsaturated soils. Accurate determination of these parameters is important to predict the subsurface flow and water transport in soils. Inverse modeling methods can be used to estimate the hydraulic conductivity of unsaturated soils. These methods can be applied to the existing expressions of flow of water in unsaturated soils to fit the experimental data. In this study, inverse modeling combined with the multistep outflow data was used to estimate the unsaturated hydraulic conductivity of three reconstituted samples of the Québec Valcartier granitic sand. These tests were performed in laboratory under controlled initial and boundary conditions. During each test, soil matric suctions were measured at the bottom and within the soil sample using the transducers and microtensiometers. The commonly used models of van Genuchten (1980) and Mualem (1976) were used in this study.

RÉSUMÉ
La méthode multi-étape a été largement utilisé pour déterminer les fonctions de conductivité hydraulique des sols non saturés. La détermination précise de ces paramètres est important pour prédire l'écoulement souterrain et le transport de l'eau dans les sols. Méthodes de modélisation inverses peuvent être utilisées pour estimer la conductivité hydraulique des sols non saturés. Ces procédés peuvent être appliqués aux expressions existantes de débit d'eau dans les sols non saturés pour être compatibles avec les données expérimentales. Dans cette étude, la modélisation inverse combinée avec les données de sortie en méthode multi-étape a été utilisé pour estimer la conductivité hydraulique non saturée de trois échantillons reconstitués de sable granitique Québec Valcartier. Ces essais ont été réalisés en laboratoire dans des conditions initiales et aux limites contrôlées. Lors de chaque essai, succions matricielle du sols ont été mesurées au fond et dans l'échantillon de sol en utilisant les transducteurs et microtensiometers. Les modèles couramment utilisés de van Genuchten (1980) et Mualem (1976) ont été utilisés dans cette étude.

1 INTRODUCTION

Direct measurements of the unsaturated hydraulic conductivity is difficult and time consuming. Zachmann et al. (1981), Zachmann et al. (1982), Kool et al. (1985) and Parker et al. (1985) used the outflow experiments combined with the parameter estimation methods as a relatively accurate and efficient approach to determine the soil hydraulic functions.

Inverse modelling can be used as a numerical approach to estimate the soil hydraulic functions using Richards (1931) equation for flow of water in unsaturated rigid porous media.

This paper presents the results of the performed MSO experiments carried out on three reconstituted samples of the Québec Valcartier granitic sand. *Hydrus-1D* was used to perform inverse modelling by fitting the MSO obtained data using the van Genuchten (1980) - Mualem (1976) models, in order to determine the unsaturated hydraulic conductivity of the tested materials. Measured and optimized van Genuchten (1980) model parameters are presented for each soil sample. The optimized saturated hydraulic conductivities are compared with the Fillion (2008) measured results of saturated hydraulic conductivity for the same reconstituted soil samples of the Québec Valcartier granitic sand.

2 MATERIALS AND METHODS

2.1 Soil Samples

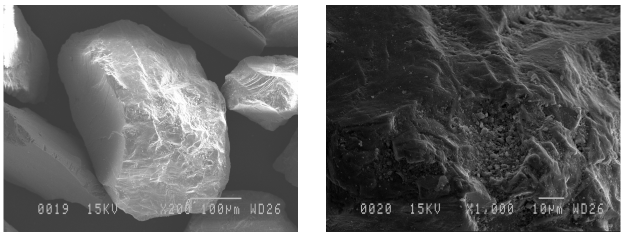
This study was conducted on three reconstituted samples of the Québec Valcartier granitic sand as a part of the projects from NSERC/Hydro-Quebec industrial research chair on the optimization of life cycle of embankment dams at Laval University (H. Siahdashti, 2014). The soil used in this investigation can be classified as crushed sand (Figure 1), and was collected from a quarry near Québec City, Canada. Energy Dispersive Spectroscopy (EDS) proved the presence of Silica as the main component of the studied soil. Al, Ca, Na, O were also traced, that can be due to the existence of different types of Feldspars (Figure 2).

The selected soil was oven dried and sieved prior to reconstitution of the three tested samples. The grain size distribution of the three studied samples is shown in Figure 3. Table 1 summarizes the reconstituted particle size gradation of the studied materials.

2.2 Multistep Outflow Experiment

The initially saturated soil samples were placed in a manufactured 6-cm height plexiglass Tempe Cell (Inspired by Tempe Cell from Soilmoisture Equipment

Corp., 2011). Figure 4 depicts a schematic sketch of the designed Tempe Cell. The boundary conditions were fully controlled in order to obtain valid experimental data for the inverse modelling process. Pneumatic pressure was applied to the top of the soil sample, and was increased in multiple steps. Outflow was measured during each MSO test and was recorded along with the soil suction at the bottom and within the soil sample. Two microtensiometers were inserted into the upper and lower 1.5 cm of the Tempe Cell to measure the soil suction within the soil sample. By using the recorded data and the water content at the end of the each test, soil water retention curve was determined for each soil sample.



(a) x200 (b) x1000

Figure 1. Electronic photos of a 160 μm particle of the Québec Valcartier granitic sand

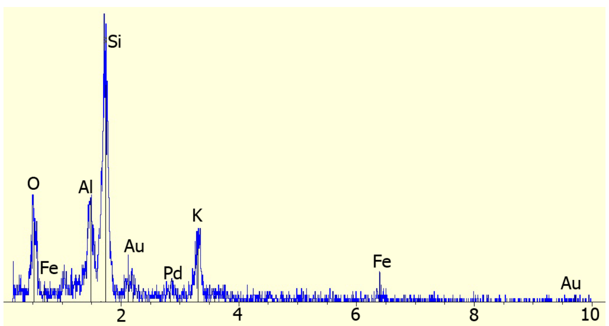


Figure 2. EDS result of a 160 μm particle of the Québec Valcartier granitic sand

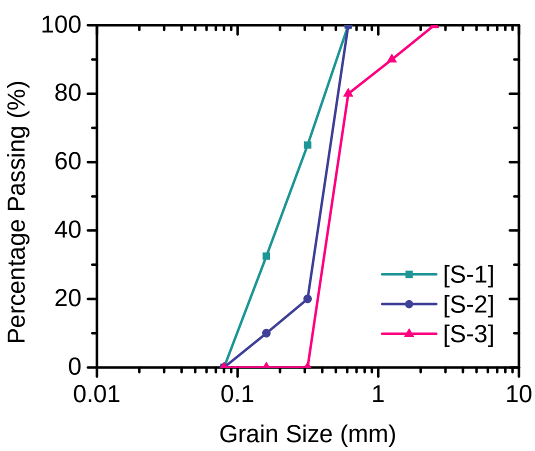


Figure 3. Grain size distribution of the three tested reconstituted granitic sand samples

Table 1. Characteristics of the tested reconstituted samples

Soil Sample	d_{10}	d_{30}	d_{50}	d_{60}	C_c	C_u
	mm	mm	mm	mm	-	-
S-1	0.10	0.16	0.24	0.30	0.85	3.00
S-2	0.17	0.35	0.40	0.43	1.68	2.53
S-3	0.35	0.40	0.47	0.50	0.91	1.43

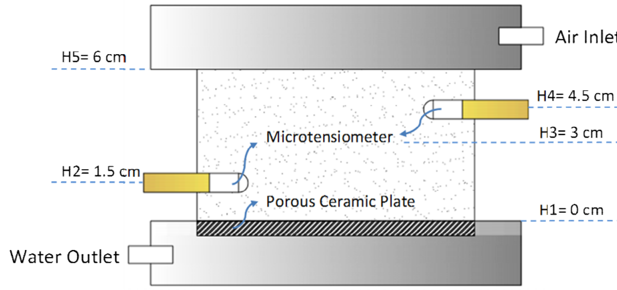


Figure 4. Schematic sketch of the Tempe Cell used for this study (Copied from Siahdashti et al. 2015)

2.3 Inverse Modeling Method

One dimensional water flow through the unsaturated rigid soils can be simulated using the Richards (1931) equation. This equation can be written as follows:

$$\partial \theta(h)/\partial t = \partial/\partial z\left[K(h)\,\partial h/\partial z - K(h)\right] \qquad [1]$$

where h is the soil matric suction [L], t is the time [T], z is the vertical distance taken positive upward [L], and $K(h)$ is the hydraulic conductivity [LT^{-1}].

van Genuchten (1980) model to describe soil water retention function combined with Mualem (1976) model was used to describe the soil hydraulic functions. These equations can be written as follows:

$$\theta(h) = \theta_r + (\theta_s - \theta_r)\left[1 + |\alpha h|^n\right]^{-m} \qquad [2]$$

$$K_r(S_e) = S_e^{\,l}\left[1 - \left(1 - S_e^{1/m}\right)^m\right]^2 \qquad [3]$$

where $\theta(h)$ is the given water content [L^3L^{-3}], θ_s and θ_r are the saturated and residual water contents [L^3L^{-3}], respectively, h is the soil matric suction [L], and α [L^{-1}], n [–], and m [–] are the empirical fitting parameters, where $m = 1 - 1/n$. S_e is the effective degree of saturation [–] and is equal to $(\theta - \theta_r)/(\theta_s - \theta_r)$, K_r is the relative hydraulic conductivity [–] and is the ratio of the hydraulic conductivity at any given effective degree of saturation and the saturated hydraulic conductivity, $K(S_e)/K_{Sat}$. l is an empirical fitting parameter [–], which was set to 0.5 as it was proposed by Mualem (1976).

Hydrus-1D software was used to solve the Richards equation and optimize the parameters of the van Genuchten (1980)–Mualem (1976) models (Šimůnek et al. 2005). Appropriate initial boundary conditions are:

$$\begin{aligned} h(z,t) &= h_i(z) & t=0,\ 0<z<L \\ q(z,t) &= 0 & t>0,\ z=L \\ h(z,t) &= h(z,t) - h_a & t>0,\ z=0 \end{aligned} \quad [4]$$

where h_i is the initial pressure head, q is the flux density, *z=0* is the bottom of the porous plate, z=L is the top of the soil and *h (z, t)* is the water pressure head at the bottom of the porous plate, h_a is the pneumatic gas pressure applied to the top of the soil specimen *(at z=L).*

3 RESULTS AND DISCUSSION

3.1 Multistep Outflow Tests

Figure 5 depicts a typical cumulative outflow and matric suction data of the multistep outflow experiments versus time (S-3 material). As it is illustrated, pressure was applied in multiple incremental steps. Higher suctions were applied to the soil sample after reaching a steady state outflow condition. As an example, for S-3 material, at around 2.5 *kPa* a jump was noticed in the cumulative outflow curve that indicates a major release of pore water at this matric suction (release of more than 60% of its pore water).

Soil water retention curves for S-1, S-2 and S-3 are illustrated in Figure 6, Figure 7 and Figure 8, respectively. The experimental data points were fitted using the van Genuchten (1980) model (VG). A good agreement between the experimental points and fitted curve is observed for all three soil samples. Soil water retention curves for all three tested reconstituted medium sand samples have a sharp slope. This sharp slope can be explained by uniform grain size distribution (Table 1), and thus uniform pore size distribution; pores that will desaturate at the same soil suction.

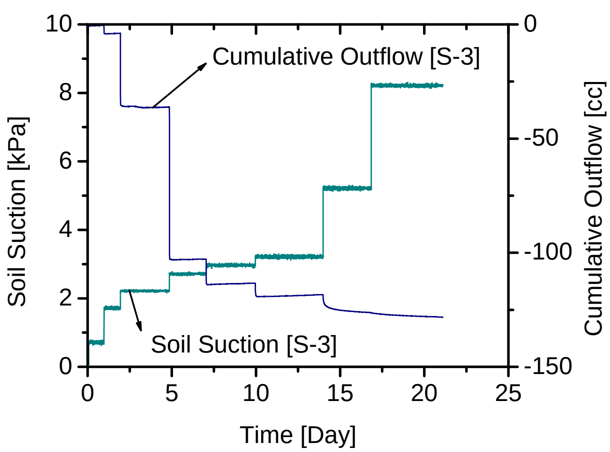


Figure 5. Measured soil matric suction and cumulative outflow versus time, [S-3]

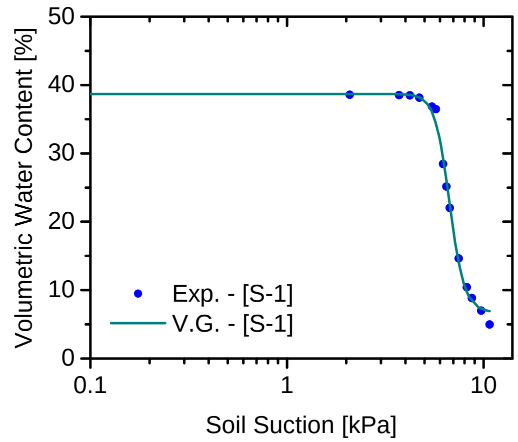


Figure 6. Experimental and VG fitted soil water retention data, [S-1]

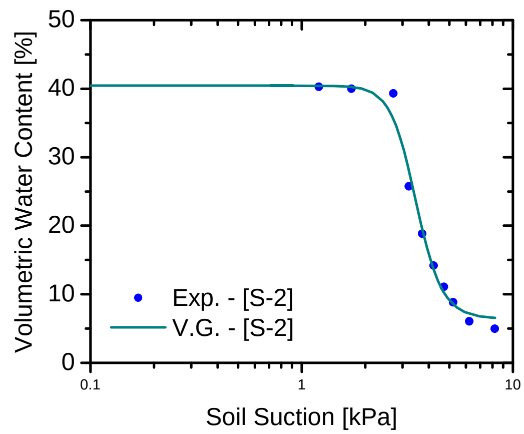


Figure 7. Experimental and VG fitted soil water retention data, [S-2]

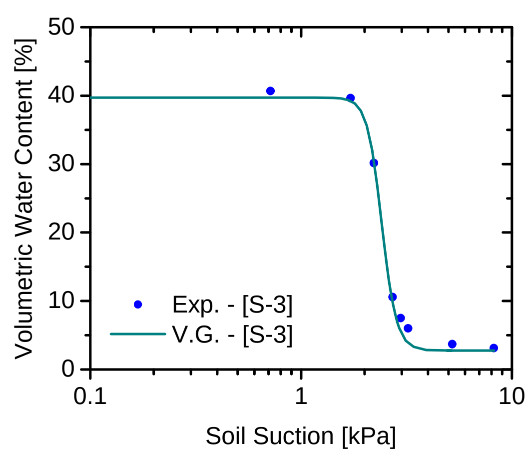


Figure 8. Experimental and VG fitted soil water retention data, [S-3]

3.2 Inverse Modelling

Table 2 summarizes the initial and optimized values of VG model parameters for the three reconstituted samples of the Québec Valcartier granitic sand. θ_s was set equal to the experimentally obtained values of saturated water content of each sand sample, which were 0.387, 0.405 and 0.397 for S-1, S-2 and S-3, respectively. l was set to 0.5 for all three tested sand samples as proposed by Mualem (1976). θ_s and l were kept fixed for the inverse modelling.

Fillion (2008) measured the saturated hydraulic conductivities of the same reconstituted soils samples of Québec Valcartier granitic sand. Table 3 presents the porosities of the tested materials and their corresponding saturated hydraulic conductivities. Fillion (2008) measured results of the saturated hydraulic conductivities of the same soil samples and their porosities are also presented in Table 3. Inverse modelling estimated hydraulic conductivities of S-1 and S-2 sands are higher than Fillion (2008) measured results. This increase can be explained by their higher porosities compare to the porosities of Fillion (2008) samples. Porosity of S-3 is almost the same in both studies, but the estimated saturated hydraulic conductivity is 12% less than the Fillion (2008) measured result. This difference can be explained by experimental errors or bias in the parameter optimization.

Table 3. Inverse modelling optimized and Fillion (2008) measured values of saturated hydraulic conductivities

Soil Sample	Inverse Modelling		Fillion (2008)	
	n	K_{sat}	n	K_{sat}
	-	(cm/hr)	-	(cm/hr)
S-1	0.387	26	0.353	18
S-2	0.405	112	0.373	76
S-3	0.397	340	0.396	385

Figure 9 and Figure 10 compare the experimentally measured (observed) and fitted soil suctions at the bottom of the S-1 sand and the cumulative outflow from the S-1 sand, respectively. As it is seen, there is a good agreement between the observed and inverse modelling fitted data. As it can be noticed in Figure 9, in order to be able to optimize the experimental data with the inverse modelling soil suctions below the air entry value were omitted and not considered during the optimization. This was also done by Watabe et al. (2000).

A typical comparison of the measured and fitted soil water retention data is depicted in Figure 11. As it is seen, there is a good correspondence between the measured and fitted results of S-2 sand.

Figure 12 and Figure 13 show the determined unsaturated and relative hydraulic conductivities of S-1, S-2 and S-3 sands. It was observed that the unsaturated hydraulic conductivity for S-3 sand has a sharper slope compare to S-1 and S-2 sands. Unsaturated hydraulic conductivities of S-1 and S-2 decrease with similar slopes. As it is seen, soils with higher saturated hydraulic conductivities will start to desaturate at lower soil suctions.

Considering the fairly good agreement observed between the measured and optimized saturated hydraulic conductivities of the tested materials, a decent accuracy for the determined unsaturated hydraulic conductivities can be expected.

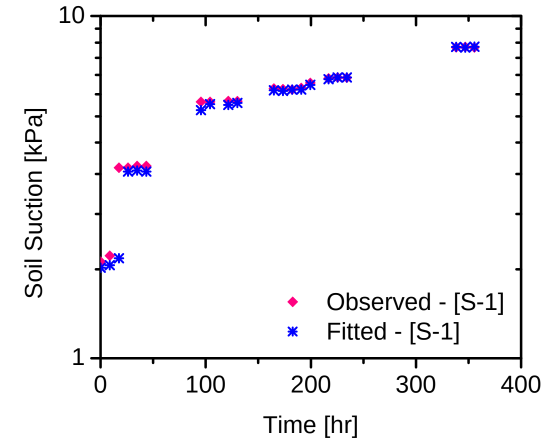


Figure 9. Measured and fitted soil suction of the bottom of the soil sample versus time, [S-1]

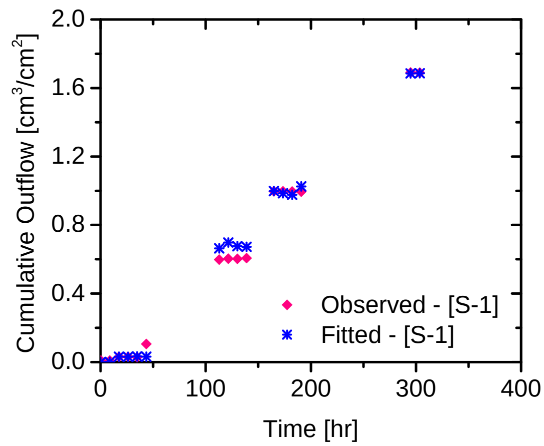


Figure 10. Measured and fitted cumulative outflow from the soil sample versus time, [S-1]

Table 2. Inverse modelling initial and optimized values for van Genuchten (1980) model

Parameter	Initial values			Optimized values		
	S-1	S-2	S-3	S-1	S-2	S-3
θ_r	6.77E-02	6.43E-02	2.74E-02	5.46E-03	5.28E-02	3.21E-02
K †	30	80	300	25.79	112.19	339.98
A	1.48E-02	2.90E-02	4.10E-02	1.54E-02	2.98E-02	4.10E-02
N	11.85	7.39	12.77	8.40	6.42	14.00

† *cm/hr*

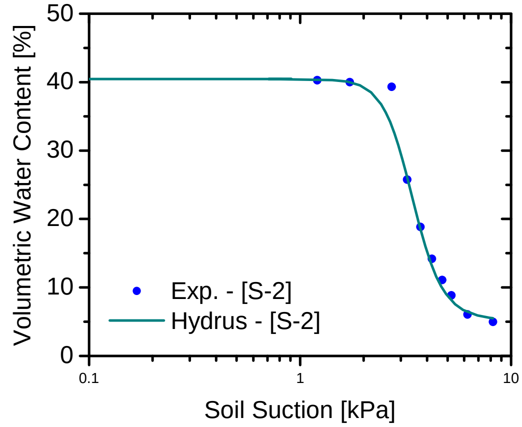


Figure 11. Measured and inverse modelling fitted soil water retention data, [S-2]

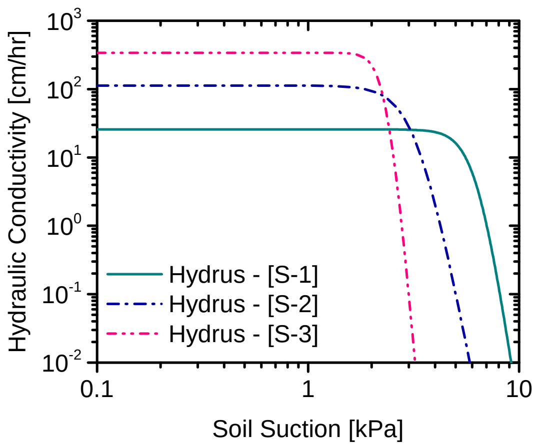


Figure 12. Determined unsaturated hydraulic conductivity [S1], [S-2] and [S-3]

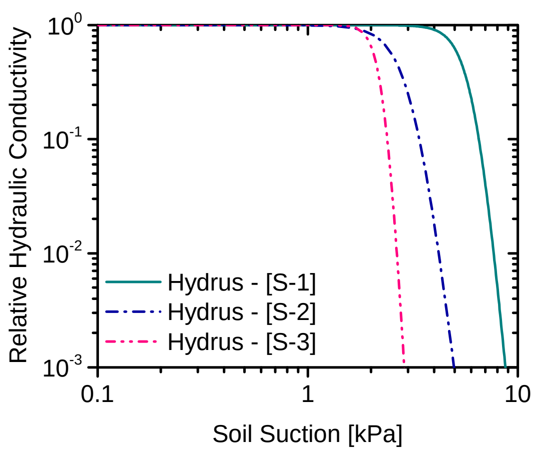


Figure 13. Relative hydraulic conductivity, [S1], [S-2] and [S-3]

4 CONCLUSIONS

MSO experiments were carried out on three reconstituted samples of the Québec Valcartier granitic sand. Experimentally obtained soil water retention curves were determined and fitted with the van Genuchten (1980) model.

MSO obtained data were used to perform inverse modelling to determine the unsaturated hydraulic conductivity of the tested sand samples. Measured and fitted soil suctions and the cumulative outflow of the soil samples were compared, and a good agreement between the two results was observed. Inverse modelling fitted soil water retention curves were also compared with the measured data and there was a good correspondence between the two results. Inverse modelling optimized saturated hydraulic conductivities of the tested materials were compared with the Fillion (2008) measured results. S-1 and S-2 with higher porosities compare to the Fillion (2008) results showed a slightly higher saturated hydraulic conductivities. Determined unsaturated hydraulic conductivities of the three tested materials were also compared.

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